Study of the Effects of OTA Imperfections in the Second Order Gm-C Filters

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ABSTRACT: *With the help of two integrator loop configuration the effects of OTA imperfections in the second order Gm-C filters is studied. We assume the single stage CMOS OTA in the investigations and the most important OTA imperfections in this case are their input resistance and input capacitances. We found that the several considered circuits have similar properties concerning these imperfections and it is shown that the gyrator biquad has the best behaviour.*

Keywords: Gm-C Filters, Operational Transconductance Amplifiers (OTA), Imperfections, Biquads

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1. Introduction

The active filters based on operational transconductance amplifiers (OTA) and capacitors, known as Gm-C filters, are still widely used in analog signal processing [1,2]. The frequency domain of their implementation is very wide – it ranges from Hertz area [3] up to several hundreds of MHz [4,5] and even to GHz [6]. They attract with several benefits: easy for integration; versatile configurations satisfying different requirements; wide frequency tuning range, covering more than one decade when necessary [3,4,6]. These filters are known for long time, however they are still object of investigations, related to their extending and modifying applications and to the use of the new technologies for their realization. The permanent trends for reducing of the supply voltages, reduction of the sizes, requirements for low consumed dc power, etc., change the OTA parameters and as consequence the effects of these parameters in filter circuit.

Most often as second order G_m-C sections (biquads) are used the circuits belonging to the wide class of two-integrator loop filters [1,2]. This is due basically to their versatility – for the most of them one circuit is able to realize every transfer function, which is achieved by applying the input signal or by taking the output signal from different points of the circuit. Other advantages are their abilities for realizing of high Qfactors and for operation at high frequencies.

Usually single stage CMOS OTAs are used for design of G_m-C biquads. Multistage OTAs are appropriate or realizing of high G_ms, which is achieved at the price of reduced frequency bandwidth due to appearance of high-impedance node between the stages [7]. OTA with high Gms are necessary for high frequency filters, which is in contradiction with their limited bandwidth. Thus, it is desirable in such cases to try to design OTAs with high G_ms.

The influence of the single-stage OTA parameters on the behavior of G_m-C sections differs in some extent compared with multistage OTA. The major parasitic parameters of a multistage OTA are its input and output impedances and the frequency dependence of Gm, represented usually by single pole approximation [1]. The basic parasitic parameters of single-stage CMOS OTA are their input capacitances and their output impedances. The frequency dependence of G_m in this case (its increasing with frequency) can be represented by one zero [1]. However it is more appropriate to consider G_m as frequency independent and add a parasitic transition capacitance to be accounted for the frequency dependence – in fact this is *Cgd* of the transistors in input differential pair of the OTA. Since this capacitance is much less than the input capacitance it has less effect.

This paper tries to compare the effects of the OTA parasitic parameters in the most popular Gm-C biquads based on two integrator loop configuration. The goal is to estimate how these influences change the possibilities of the circuits for realization of high Q-factor filters and their ability for operation in wide frequency range. The second chapter, describes shortly the considered circuit if ideal OTAs are used. Third chapter considers the effect of OTA output resistances on the behavior of the circuits, and in the fourth chapter is discussed the changes due to the input capacitances of the amplifier – so called excess phase effect.

2. Short Description of the Considered Circuits

The filter circuits, which will be investigated here, are given in Figure 1. They represent most of the two-integrator loop biquads [1,2] and their single-ended versions are shown for simplicity. All circuits are able to realize different biquads (low-pass, high-pass, band-pass, with complex zeros) depending on the way of applying the input signal and taking the output signal. The transfer functions of the circuits can be written in the general way:

$$H(s) = \frac{N(s)}{s^2+d_1 s+d_0} = \frac{N(s)}{s^2+\frac{\omega_{p0}}{Q_{p0}}s+\omega_{p0}^2}, \tag{1}$$

where $N(s)$ is either first or second order polynomial or a constant, depending on the section type. The parasitic effects, considered here, concern basically the denominator of the transfer function and are significant at high pole Q-factor. For this reason their influence on the numerator $N(s)$ will be not considered and inputs and outputs are shown in Figure 1 for the case of band-pass sections – circuits, which most often require high Q-factors.

The coefficients of the transfer function denominators and the pole parameters are summarized in Table 1. In all formulas a is ratio of identically marked g_ms, while b differs in Figure 1(a) and Figure 1(d):

$$a = \frac{g_{m3}}{g_{m4}};\ b = \frac{g_{m5}}{g_{m6}} \text{ in (a)};\ \ b = \frac{g_{m5}}{g_{m4}} \text{ in (d)}. \tag{2}$$

3. Influence of OTA Output Conductances

OTA output impedances consist typically of parallel connected conductance and capacitance. The output capacitances in the considered circuits are connected in parallel either to the integrator capacitors C_1 and C_2 or to the input capacitance of the same or another OTA. In the first case they can be considered as parts of C_1 or C_2; in the second case their influence should be considered together with the influence of the OTA input capacitances. For this reason the effect of output impedances will

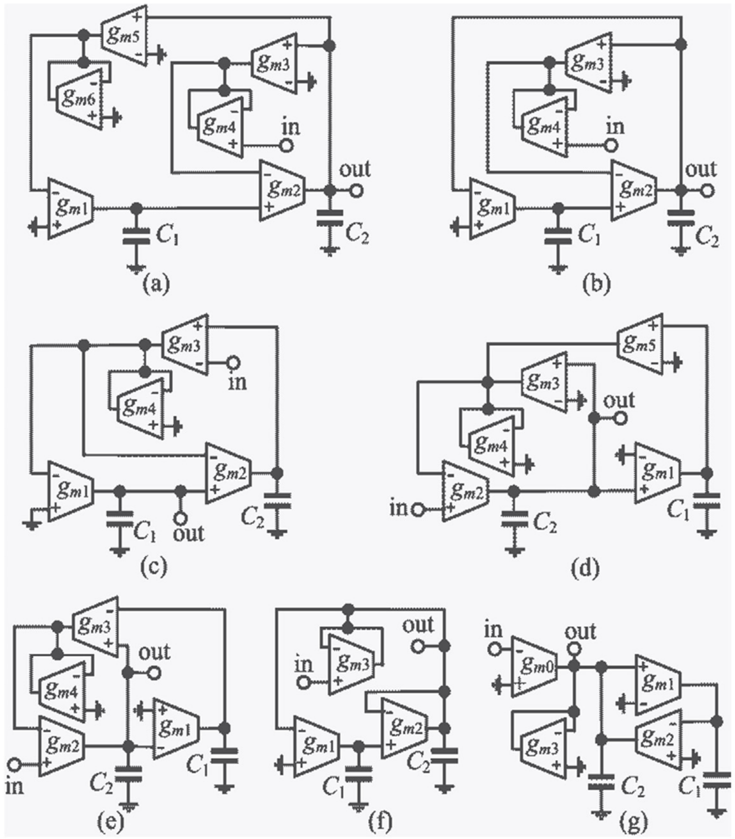

Figure 1. Two-integrator loop biquads [1]: (a), (b), (c) distributedfeedback configurations; (d), (e) summed-feedback configurations; (f) Tow-Thomas Gm-C circuit; (g) gyrator band-pass biquad

	d_1	$d_0 = \omega_{p0}^2$	Q_{p0}
Fig. 1(a) Fig. 1(d)	$a\frac{g_{m2}}{C_2}$	$b\frac{g_{m1}g_{m2}}{C_1C_2}$	$\frac{1}{a}\sqrt{b\frac{g_{m1}}{g_{m2}}\frac{C_2}{C_1}}$
Fig. 1(b)	$a\frac{g_{m2}}{C_2}$	$\frac{g_{m1}g_{m2}}{C_1C_2}$	$\frac{1}{a}\sqrt{\frac{g_{m1}}{g_{m2}}\frac{C_2}{C_1}}$
Fig. 1(c) Fig. 1(e)	$a\frac{g_{m2}}{C_2}$	$a\frac{g_{m1}g_{m2}}{C_1C_2}$	$\sqrt{\frac{1}{a}\frac{g_{m1}}{g_{m2}}\frac{C_2}{C_1}}$
Fig. 1(f)	$\frac{g_{m2}+g_{m3}}{C_2}$	$\frac{g_{m1}g_{m2}}{C_1C_2}$	$\sqrt{\frac{C_2}{C_1}}\frac{\sqrt{g_{m1}g_{m2}}}{g_{m2}+g_{m3}}$
Fig. 1(g)	$\frac{g_{m3}}{C_2}$	$\frac{g_{m1}g_{m2}}{C_1C_2}$	$\frac{\sqrt{g_{m1}g_{m2}}}{g_{m3}}\sqrt{\frac{C_2}{C_1}}$

Table 1. Transfer Function Parameters of the Biquads In Figure 1

be considered as effect of output conductances only. The inspection of the circuits in Figure 1 shows that only output conductances g_{o1} and g_{o2} of OTAs g_{m1} and g_{m2} affects the circuit parameters, since they are in parallel to C_1 and C_2. The other output conductances are in parallel to OTAs, connected as resistors, and can be absorbed in these resistors.

After these observations the analysis can be done easily – it is enough to replace sC_1 by $sC_1 + g_{o1}$ and sC_2 by $sC_2 + g_{o2}$ in the expressions for the transfer functions. Proceeding in this way, the following expression for the transfer function of the circuits in Figure 1(a) and 1(d) is derived:

$$H(s) = \frac{N(s)(1+\omega_1/s)(1+\omega_2/s)}{s^2+s\left(\frac{\omega_{p0}}{Q_{p0}}+\omega_1+\omega_2\right)+\omega_{p0}^2+\frac{\omega_{p0}}{Q_{p0}}\omega_1+\omega_1\omega_2}, \tag{3}$$

where $\omega_1 = g_{o1}/C_1$ and $\omega_2 = g_{o1}/C_2$. The changes due to output conductances appear in three places. It increases pole frequency however this increase is small. Both terms which are added to $\omega_{p0}{}^2$ are much smaller since the ratios ω_1/ω_{p0} and ω_2/ω_{p0} are

$$\frac{\omega_1}{\omega_{p0}} = \frac{g_{o1}}{\sqrt{bg_{m1}g_{m2}}}\sqrt{\frac{C_2}{C_1}},\quad \frac{\omega_2}{\omega_{p0}} = \frac{g_{o2}}{\sqrt{bg_{m1}g_{m2}}}\sqrt{\frac{C_1}{C_2}}. \tag{4}$$

They are significantly less than 1 if $g_{o1,2} << g_{m1,2}$ (well-designed OTA) and also high-Q circuits are considered.

It seems that the multipliers $1 + \omega_1/s$ and $1 + \omega_2/s$ increase the gain to infinity at very low frequencies. However they present in the formula only because the change of the numerator is not considered. The numerator $N(s)$ is also function of C_1 and C_2. The capacitors in the numerator will produce the same terms in a way, which will cause canceling with the terms coming from the denominator. Therefore in the final expression for $H(s)$ these two multipliers will not exist.

The most important effect of the output conductances is in the first order term in the denominator. The quantities ω_1 and ω_2, which are added to ω_{p0}/Q_{p0}, can be of the same range as ω_{p0}/Q_{p0} and limit the maximum achievable Q-factor. If assume $(\omega_{p0}/Q_{p0}) = 0$, i.e. $Q_{p0}\to\infty$, then the pole Q-factor is determined by the sum $\omega_1 + \omega_2$ and is equal to

$$Q_{p\,max} \approx \frac{1}{\frac{g_{01}}{g_{m1}}\frac{a}{b}Q_{p0}+\frac{g_{02}}{g_{m2}}\frac{1}{a}\frac{1}{Q_{p0}}}. \tag{5}$$

Formulas (3), (4) and (5) are derived for Figure 1(a) and 1(d), however they are valid for all circuits except the gyrator one (Fig. 1(g)), if set $b = 1$ for Fig. 1(b); $b = a$ for Figure 1(c) and 1(e); and $b = a = 1$ for Figure 1(f). Both quantities Q_{p0} and $Q_{p\,max}$ determine the real pole quality factor Q_p according the formula

$$\frac{1}{Q_p} = \frac{1}{Q_{p0}} + \frac{1}{Q_{p\,max}}. \tag{6}$$

These results can be interpreted in the following way: If Q_{p0} is specified then exists a parameter $Q_{p\,max}$, dependent on Q_{p0} and on the ratios $g_{o1,2}/g_{m1,2}$, which limits Q_p. Since Q_{p0} is ratio of $g_m s$ and of capacitors and can't be done infinitely large, the real Q_p of the circuit is always combination of Q_{p0} and $Q_{p\,max}$. The value of $Q_{p\,max}$ is necessary to be high if high Q_p is needed – it should be of the range of Q_{p0} or higher. Since Q_{p0} enters in the denominator of $Q_{p\,max}$, this requirement leads to hard demand on the ratio g_{o1}/g_{m1} – it should be less than $1/Q_{p02}$.

The expressions for ω_p and Q_p for the gyrator circuit are

$$\omega_p^2 = \left(1 + \frac{g_{o1}g_{23}}{g_{m1}g_{m2}}\right)\omega_{p0}^2 \approx \omega_{p0}^2;\; Q_p \approx \sqrt{\frac{g_{m1}g_{m2}}{g_{o1}g_{23}}}\frac{1}{\sqrt{\frac{\tau_{o1}}{\tau_{o2}}}+\sqrt{\frac{\tau_{o2}}{\tau_{o1}}}}, \tag{7}$$

where $g_{23} = g_{o2} + g_{m3}$, $\tau_1 = g_{o1}/C_1$, $\tau_2 = g_{o2}/C_2$, and ω_{p0} is given in the last row of Table 1. The maximum pole Q, defined by OTA output conductances only, is achieved when $g_{m3} = 0$, i.e. when the corresponding OTA is missing (in fact this OTA is used only for fixing the desired Q_p); and when $\tau_1 = \tau_2$. Thus the gyrator circuit has more potential for realization of *high-Q* biquads.

4. Limitations From OTA Input Capacitances

Other parasitics, which may affect significantly the behavior of the circuits, are OTA's input capacitances. Those of them, proper adjustment. More critical is the influence of the input capacitances, which appear in parallel to OTAs, connected as resistors: g_{m4} and g_{m6} in Figure 1(a) and g_{m4} in Figure 1(b), (c), (d) and (e). They introduce parasitic capacitances C_{p4} in parallel to g_{m4} and C_{p6} in parallel to g_{m6}, which are equal correspondingly:

- in Fig. 1(a): $C_{p4} = C_{in,2} + C_{in,4}$, $C_{p6} = C_{in,1} + C_{in,6}$;
- in Fig 1(b), (d) and (e): $C_{p4} = C_{in,2} + C_{in,4}$;
- in Fig. 1(c): $C_{p4} = C_{in,1} + C_{in,2} + C_{in,4}$

where $C_{in,k}$ is the input capacitance of OTA g_{mk}.

The capacitances C_{p4} and C_{p6} together with g_{m4} and g_{m6} form parallel RC circuits. They introduce additional phase shift (so called excess phase) in the gains of the voltage multipliers, created with the help of g_m and g_{m6} (for example of the multipliers g_{m3}/g_{m4} and g_{m5}/g_{m6} in Figure 1(a)). In fact the coefficients a and b in the formulas for d_1 and d_0 in Table 1 are not frequency independent and the following formulas are valid

$$a = \frac{a_0}{1+s/\omega_4};\ \ b = \frac{b_0}{1+s/\omega_6} \text{ in (a)};\ \ b = \frac{b_0}{1+s/\omega_4} \text{ in (d)}, \tag{8}$$

where a_0 and b_0 are the values of these parameters according (2) and frequencies ω_4 and ω_6 are given by

$$\omega_4 = g_{m4}/C_{p4}\,; \quad \omega_6 = g_{m6}/C_{p6}. \tag{9}$$

The replacement of the expressions for a and b changes the formulas for filter transfer functions in the following way:

$$\text{For Fig 1 (a): } H(s) = \frac{N(s)(1+s/\omega_4)(1+s/\omega_6)}{s^2\left(1+\frac{s}{\omega_4}\right)\left(1+\frac{s}{\omega_6}\right)+s\frac{\omega_{p0}}{Q_{p0}}\left(1+\frac{s}{\omega_6}\right)+\omega_{p0}^2\left(1+\frac{s}{\omega_4}\right)}\,; \tag{10a}$$

$$\text{For Fig 1 (b): } H(s) = \frac{N(s)(1+s/\omega_4)}{s^2\left(1+\frac{s}{\omega_4}\right)+s\frac{\omega_{p0}}{Q_{p0}}+\omega_{p0}^2\left(1+\frac{s}{\omega_4}\right)}\,; \tag{10b}$$

$$\text{For Fig 1 (c), (d) and (e): } H(s) = \frac{N(s)(1+s/\omega_4)}{s^2\left(1+\frac{s}{\omega_4}\right)+s\frac{\omega_{p0}}{Q_{p0}}+\omega_{p0}^2}\,. \tag{10c}$$

The terms $(1 + s/\omega_4)$ and $(1 + s/\omega_6)$ appear in the denominator and also multiply the numerator. Their effect in the denominator of (10c) will be considered firstly. Now it is of 3rd degree, i.e. the complex poles of the filter are changed and appears a third pole, which is real. The influence of the term $(1 + s/\omega_4)$ depends basically on the ratio ω_4/ω_{p0}. The numerical investigation shows existence of minimal allowed value of this ratio, at which the pole Q-factor is equal to infinity and below this value $Q_p < 0$, i.e. the circuit is unstable. This is illustrated in Figure 2(a). The minimal value of ω_4/ω_{p0} is equal to Q_{p0}, which can be proved mathematically. At $(\omega_4/\omega_{p0}) = Q_{p0}$ the denominator of (10c) can be written as (11)

$$D(s) = \left(s^2 + \omega_{p0}^2\right)\left(1 + s/\left(\omega_{p0}Q_{p0}\right)\right), \tag{11}$$

i.e. the complex poles are purely imaginary and the circuit is at the boundary of stability. Of course the value of ω_4 in the real circuits should be higher of the limit ($= \omega_{p0}Q_{p0}$) in order to have reserve of stability. For example, if assume 20% allowed increasing of Q_p then ω_4 should be not less than $60\omega_{p0}$, $178\omega_{p0}$ and $589\omega_{p0}$ for Q_{p0} = 10, 30 and 100 correspondingly – significantly stronger requirement than the limit $\omega_{p0}Q_{p0}$.

The other influences of ω_4 are negligible. Figure 2(b) shows that the relative variation of the frequency of the complex poles is less than 0.1% for values of ω_4/ω_{p0}, for which the circuit is stable. The extra real pole is equal to ω_4 as can be concluded from Figure 2(c) and it will cancel with the zero introduced by the multiplier $(1 + s/\omega_4)$ in the numerator.

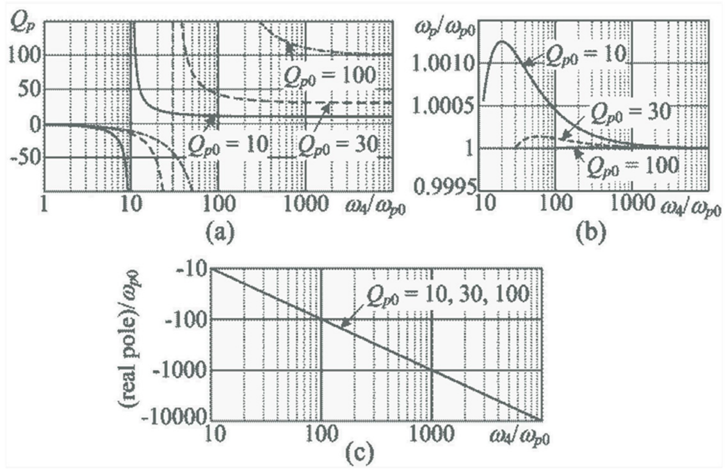

Figure 2. Influence of the ratio ω_4/ω_{p0} for the circuits in Figure 1(c), (d) and (e): (a) change of the pole Q-factor; (b) change of the pole frequency; (c) extra real pole

The considerations above are valid also for the circuit in Fig. 2(a) when $\omega_4 = \omega_6$. Then identical multipliers $(1 + s/\omega_4)$ appear in the numerator and denominator of (10a), they cancel themselves and the denominator of (10a) becomes the same as those of (10c). When ω_4 and ω_6 differ, the studying of the transfer function can be done numerically, calculating the poles at different values of ω_4 and ω_6. Elaborating in this way it is found out for moderate ratios ω_4/ω_6 between 0.5 and 2 that the instability is defined from ω_6 only and the limit is $\omega_6/\omega_{p0} \approx Q_{p0}$. Which is interesting, this limit doesn't depend on ω_4. The other influences are insufficient as in the previous case: the frequency of the complex pole pair is stable and two real pole appears, which are far from the complex poles.

The variations of Q and of the pole frequency of the circuit in Figure 1(b) from the parasitic frequency are illustrated in Figure 3. The Q-factor also increases at low ω_4, however it is not so much and the circuit stays stable even for values of ω_4 unrealistically close to ω_{p0}. The change of the pole frequency also is small. The extra real pole, which also appears in this circuit, is far from the pole frequency for $\omega_4 > 10\omega_{p0}$ (normal values of ω_4) and its effect can be neglected.

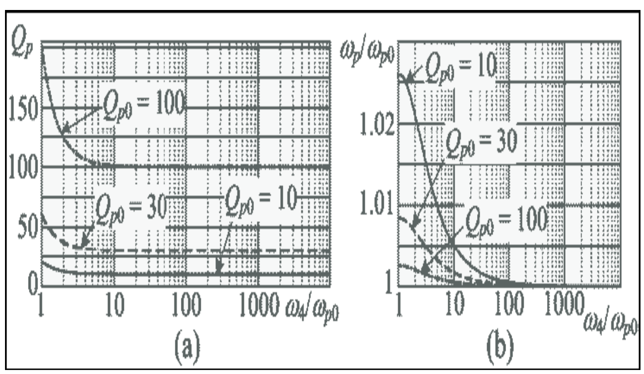

Figure 3. Influence of parasitic frequency $\omega_4 = g_{m4}/C_{p4}$ in Figure 1(b): (a) variation of pole Q-factor; (b) variation of pole frequency.ω_4

Generally, the excess phase due to parasitic capacitances in parallel to OTAs, connected as resistors in Figures 1 (a) to (e), causes the effect, known in the two-integrator loop biquads with other types of amplifiers [1]: increasing of the pole Qfactor, which may bring even to instability of the circuit. In the considered case the reason is the input capacitances of some OTAs. This effect limits the high frequency application of the circuits. High pole frequencies are achieved by large $G_m s$ of OTAs g_{m1} and g_{m2} and small capacitances C_1 and C_2. Large Gms require transistors with larger sizes in single stage OTAs and as consequence – higher input capacitances. The consideration here shows existence of lower limit for the ratio ω_4/ω_{p0}. Thus, raise of ω_{p0} leads to higher g_{m1} and g_{m2}, higher input capacitances, lower ω_4 and as result – approaching the limit.

A way to compensate the effect of excess phase is to add a proper resistor in series to one of the capacitors C_1 and C_2 – the one which is in the loop immediately before g_m4. This resistor together with the capacitor shifts the phase of the voltage over them in opposite direction to the phase shift, caused by g_{m4} and C_{p4}. This approach is necessary to be considered for each circuit separately. It is applied for some circuits [8, 9] and can help partly. Its disadvantage is different dependences of g_{m4} and compensating resistor from temperature, process, etc.

The last two circuits in Figure 1 – the one in (*f*) and the gyrator biquads in (*g*) do not suffer from excess phase. This is because all OTA input capacitances in these circuits are in parallel to integrator capacitors and there is no place, where an extra phase shift can arise. This superior property of both circuits is only when they are realized with single stage CMOS OTAs. For example one of the first papers concerning the excess phase and its compensation is about gyrator circuit [8], of course when gyrator amplifiers are realized in different way.

5. Conclusion

A generalized consideration of G_m-C biquads based on two integrator loop configuration is done concerning the influence of OTA output resistances and input capacitance. These OTA imperfections have the most negative impact on the circuit behavior when OTAs are single-stage CMOS. The effects of both types of imperfections are considered separately – an approximate approach, however it allows more clear characterizations of the influences.

The OTA output resistances reduce pole Q in all circuits. In the most of the circuits except the gyrator biquad Q is defined by ratio between $G_m s$ of some OTAs and the output resistances additionally reduce it. The gyrator circuit is an exception of this rule and its Q can be defined only from the output resistances of the OTAs in the gyrator, which makes the gyrator able to realize higher Q.

OTA input capacitances, when they are in parallel to amplifiers connected as resistors, change the phases of some voltages in the circuit, which causes intolerable increasing of Q and even instability. The effect is more visible when the circuit is intended for operation at higher frequencies. Again the gyrator is an exclusion – it doesn't suffer from this disadvantage.

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