

# Analytical Magnitude Optimum Criterion using the Rotor Field Oriented (RFO) Control Algorithm

Bojan Bankovic, Milutin Petronijevic, Nebojsa Mitrovic, Vojkan Kostic, Filip Filipovic  
Faculty of Electronic Engineering at University of Nis, Aleksandra Medvedeva 14  
Nis 18000, Serbia  
[{bojan.bankovic@elfak.ni.ac.rs}](mailto:{bojan.bankovic@elfak.ni.ac.rs})



**ABSTRACT:** For electric drives, the PI controller is the core component part of the control algorithm. It results in a better wide range application and represent the important part of the engineering learning. Integrated Education Platform for controllers tuning using graphic user interface in dSPACE ControlDesk environment. Analytical Magnitude Optimum Criterion is used for controllers tuning in the case of the Rotor Field Oriented (RFO) control algorithm. For testing we have implemented the dSPACE DS 1103 control board and tested in the laboratory for performance.

**Keywords:** Integrated Education Platform, Controllers Tuning, Rotor Field Oriented Control Algorithm

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## 1. Introduction

Experimental approach in students teaching is important part of education. It is a link between theory and practice, preparing them for real engineering jobs, where they will design, build, implement and test real hardware. Development of power electronics, low-cost microcontrollers and microprocessors such as DSP, FPGA and others enabled modernization of the laboratory environments. This new education environment with a possibility of integration software tools and real time control of experiments have been primary part of many laboratories. On this way a risk of short circuits, device and equipment damages or students injuries is minimize.

In the field of the electrical drives control PI controllers are most common choice since they have a simple structure and offer a satisfactory performance over a wide range of operations. The widely use of PI controllers lie on their effectiveness and efficiency, simple implementation and low costs. The PI controller as a type of PID controllers is often applied in the control algorithm of the induction motor drive, among the various control schemes proposed in literature [1]. In electrical drives control a classical multi loop structure is used with appropriate controller in it. The controlled variables are position, speed, current, voltage and torque.

Magnitude Optimum Criterion method is used to design and tune PI controller gains. The efficiency of the tuning depends on the accuracy of the proposed mathematical model according to the actual operating conditions of the electrical drive [1], [2].

The aim of this paper is to present students a platform for the evaluation of system behavior due to different controller parameters tuning in the high-performance drive with RFO control method, based on Matlab/Simulink environment and the dSPACE system through ControlDesk graphic user interface. Practical application of the analytical approach for controllers tuning is analyzed.

## 2. Mathematical Model for RFO

The aim of the high performance drives is good static and dynamic characteristic during the control of the electromechanical variables such as speed, torque and position. The RFO algorithm is a torque-rotor flux decoupled control. A flux phasor is oriented to the rotor flux field orientation, and control method is based on a closed loop structure. The stator currents of the induction motor are separated into flux and torque producing components by using  $dq$  coordinate transformation. The torque current component lies in the  $q$  axis, while the flux component is on the  $d$  axis. The basic equations for RFO control will be presented in the  $d-q$  synchronous coordinate system:

$$u_{ds} = r_s i_{ds} + \sigma L_s \frac{di_{ds}}{dt} + \frac{L_m}{L_r} \frac{d\psi_{dr}}{dt} - \omega_{dq} \sigma L_s i_{qs} \quad (1)$$

$$u_{qs} = r_s i_{qs} + \sigma L_s \frac{di_{qs}}{dt} + \omega_{dq} \frac{L_m}{L_r} \frac{d\psi_{dr}}{dt} - \omega_{dq} \sigma L_s i_{ds} \quad (2)$$

$$L_m i_{ds} = \psi_{dr} + T_r \frac{d\psi_{dr}}{dt} \quad (3)$$

$$L_m i_{ds} = (\omega_{dq} - \omega) T_r \psi_{dr} \quad (4)$$

$$\sigma = 1 - \left( \frac{L_m^2}{L_s L_r} \right) \quad (5)$$

**where:** subscripts  $s$  and  $r$  are corresponding to the stator and the rotor variables,  $u_d$ ,  $u_q$ ,  $i_d$ ,  $i_q$ ,  $\psi_d$ ,  $\psi_q$  are  $d-q$  components of the voltage, current and flux,  $r$ ,  $L$  and  $L_m$  are the resistance, self-inductance and magnetizing inductance,  $\sigma$  is leakage coefficient and  $T_r$  is rotor time constant. The speed  $\omega$  represent the rotor angular speed of the induction motor while  $\omega_{dq}$  represent the rotation speed of the reference coordinate system.

In order to align the synchronous speed rotating reference frame with the rotor flux it is necessary to know the module and position of the rotor flux space vector. Therefore, the rotor flux position estimator is composed using stator current components and motor parameters. The slip speed is calculated from Eq. (6):

$$\omega_{slip} = \frac{1}{T_r} \frac{L_m}{\psi_{dr}} i_{qs} \quad (6)$$

Assuming that the alignment of the  $q-d$  axis is performed in a way that the  $\psi_{dr} = \psi_r$  and  $\psi_{qr} = 0$ , the electromagnetic torque for induction motor is given with Eq. (7):

$$T_e = \frac{3}{2} p \frac{L_m}{L_r} (\psi_{dr} i_{qs}) \quad (7)$$

**where:**  $T_e$ ,  $T_m$ ,  $J$  and  $p$  are electromagnetic torque, external mechanical torque, inertia and number of poles. Based on the Eq. (6) and Eq. (7) the torque is linearly proportional with slip frequency.

Detailed mathematical model of RFO is well-known and can be found in [3], [4]. Basic scheme, shown in Figure 1. introduces

multi-rate digital control system with SVPWM and inner current loop and outer speed loop. In Figure 1. the main objective of the decoupling circuit is to achieve decoupled control of the motor flux and torque. Eq. (1) and Eq. (2) show that there is no decoupled control between the  $d$ - $q$  components of voltages and currents. To achieve decoupled control, it is necessary to introduce appropriate decoupling elements with Eq. (8) and Eq. (9):

$$u_{ds\_dec} = \frac{L_m}{L_r} \frac{d\psi_{dr}}{dt} - \omega_{dq} \sigma L_s i_{qs} \quad (8)$$

$$u_{qs\_dec} = \omega_{dq} \frac{L_m}{L_r} \psi_{dr} + \omega_{dq} \sigma L_s i_{ds} \quad (9)$$

where are  $u_{ds\_dec}$ ,  $u_{qs\_dec}$  compensation parts which must be added in a decoupling circuit.

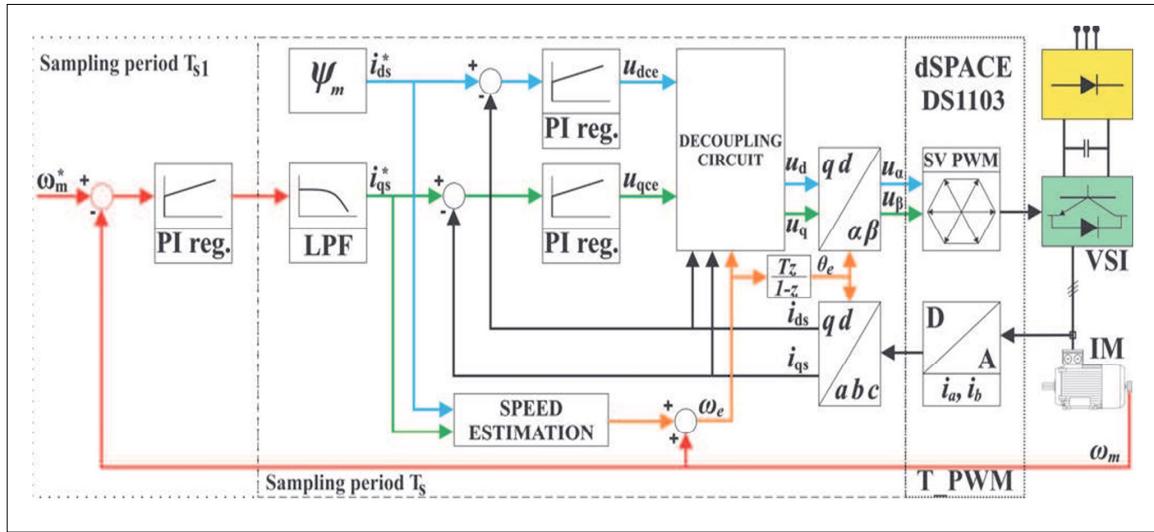


Figure 1. Block diagram of RFO control method

### 3. Tuning the Controllers Parameters

The determination process for an optimal values of controller parameters starts from the selection of appropriate method for their calculation. There are a variety of methods based on theoretical analysis but our focus will be on controller tuning using the magnitude optimum criterion. This criterion can be applied to the current and speed control loop through the technical and symmetrical optimum methods. Motor and inverter specification for controller parameters calculation are given in the Table 1.

#### 3.1 Tuning Current Controller

The analysis of the current and speed control loops is done in the frequency domain. Technical optimum as analytical method for calculation the controller parameters is used. The main objective of the method is to maintain the amplitude of the closed loop frequency response equal to unity  $|W_{CL}(j\omega)| \approx 1$ . The transfer function of the closed loop in general form is given with Eq. (10):

$$W_{CL}(s) = \frac{b_0 + b_1 s^1 + b_2 s^2 + \dots + b_m s^m}{a_0 + a_1 s^1 + a_2 s^2 + \dots + a_n s^n} \quad (10)$$

where  $n \geq m$ . For forcing  $|W_{CL}(j\omega)| \approx 1$  in the widest possible frequency range  $a_i = b_j$  for  $\forall i, j; (i = 0, n)(j = 0, m)$  given in the [1], [2]. The technical optimum is especially suitable for the transfer function of the close loop with twodegree polynomial

Induction motor IM1 parameters	
Nominal power $P_n$	1500 W
Nominal speed $n_n$	2860 o/min
Number of pole pairs $p$	1
Nominal stator voltage $U_{S_{phase}}$	230V
Stator resistance $r_s$	5.45 Ω
Stator Inductance dissipation $L_{ls}$	11.8 mH
Rotor resistance $r_r$	3.18 Ω
Rotor inductance dissipation $L_{lr}$	11.8 mH
Magnetizing inductance $L_m$	441.3 mH
Total inertia $J$	0.0035 kgm <sup>2</sup>
Inverter Danfoss VLT5003	
Nominal voltage $U_{VLT,n}$	3x(380 – 440) VAC
Rated input current $I_{L,n}$	3.8 A
Rated output current $I_{VLT,n}$	4.1 A
Switching frequency $T_{PWM}$	5 kHz

Table 1. Motor and Inverter Parameters

function in the denominator. The transfer function of the current close loop needs to be reduced in the form of the Eq. (11).

$$W_{CL}(s) = \frac{a_0}{a_0 + a_1 s^1 + a_2 s^2} \quad (11)$$

The current control loop with controller is shown on Figure 2.

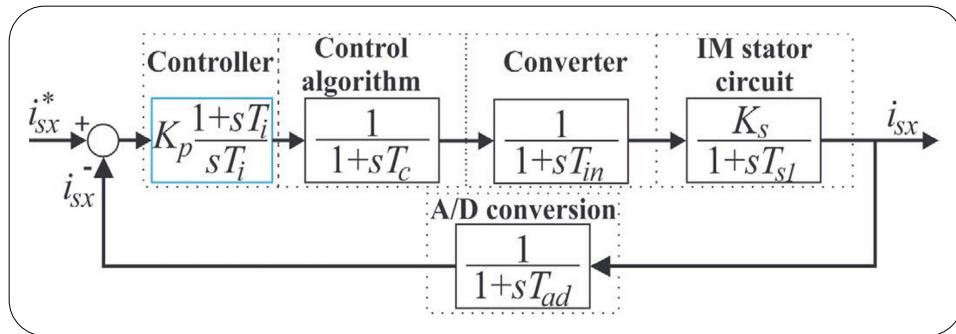


Figure 2. Current control loop

To reduce transfer function in the form of the Eq. (11) the zero  $T_i$  of the controller need to be determine in a such a way that pole zero cancellation between dominant time constant of the object  $T_{si} = \sigma L_s / R_s$  and controller's zero  $T_i$  is achieved. In the case of the current control loop after moving the block of the A/D signal conversion from feedback path in direct path and for  $T_i = T_{s1}$  the open loop and close loop transfer functions are given in Eq. (12) and Eq. (13).

$$W_{OL}(s) = \frac{i_{sx}}{i_{sx}^*}(s) = \frac{K_p}{sT_i} \frac{1}{1+sT_e} K_s \quad (12)$$

$$W_{CL}(s) = \frac{i_{sx}}{i_{sx}^*}(s) = \frac{K_p K_s}{K_p K_s + sT_i + s^2 T_i T_e} \quad (13)$$

Using one equivalent time constant  $T_e = T_c + T_{in} + T_{ad} = 0.3 \text{ ms}$ , and applying the rule described in [1]  $a_1^2 = 2a_0 a_2$ , where are  $a_0 = K_p K_s$ ,  $a_1 = T_r$ ,  $a_2 = T_e T_r$ , the proportional gain is  $K_p = \frac{T_i}{2K_s T_e}$  and  $K_s = \frac{1}{R_s}$ . Finally, the close loop transfer function is given with Eq. (14):

$$W_{CL}(s) = \frac{1}{1+2sT_e} \quad (14)$$

### 3.2 Tuning Speed Controller

The transfer function of the close loop is the third degree polynomial function therefore the symmetrical optimum method is applied. The speed control loop is shown in Figure 3.

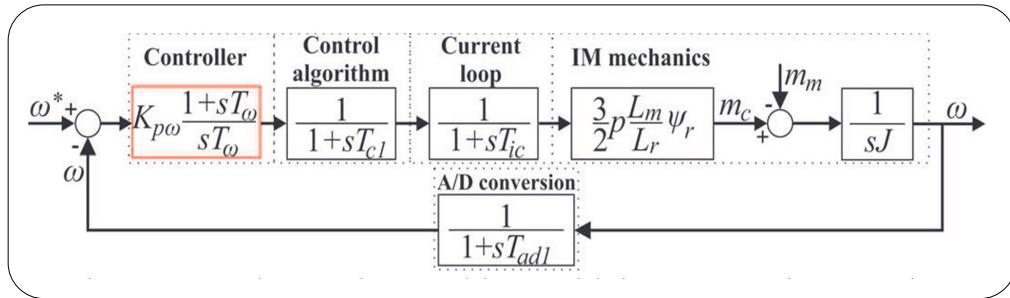


Figure 3. Cascade speed control loop with incorporated current loop

The symmetrical optimum is detailed described in the [1], [5]. The transfer function of the speed close loop needs to be reduced in the form of the Eq. (15).

$$W_{CL}(s) = \frac{a_0 + a_1 s^1}{a_0 + a_1 s^1 + a_2 s^2 + a_3 s^3} \quad (15)$$

Moving the block of the A/D signal conversion in the direct path and using time constant  $T_e' = T_{c1} + T_{ic} + T_{ad1} = 2.6 \text{ ms}$  the transfer functions are given in Eq. (16) and Eq. (17).

$$W_{OL}(s) = \frac{\omega}{\omega^*}(s) = \frac{K_p \omega (1+sT_\omega)}{sT_\omega} \frac{1}{1+sT_e'} \frac{3}{2} p \frac{L_m}{L_r} \psi_r \frac{1}{sJ} \quad (16)$$

$$W_{CL}(s) = \frac{K_p \omega (1+sT_\omega) K_{ma}}{K_p \omega K_{ma} + s K_p \omega T_\omega K_{ma} + s^2 T_\omega K_{mb} + s^3 T_\omega T_e' K_{mb}} \quad (17)$$

The determination of the controller parameters the system of the equations are used:  $a_1^2 = 2a_0 a_2$ ,  $a_2^2 = 2a_1 a_3$ , where are  $a_0 = K_{p\omega} K_{ma}$ ,  $a_1 = K_{p\omega} T_\omega K_{ma}$ ,  $a_2 = T_\omega K_{mb}$ ,  $a_3 = T_\omega T_e' K_{mb}$ . Solving the system of equations lead to proportional gain

$$K_{p\omega} = \frac{4JL_r}{3T_\omega pL_m\psi_r} \text{ and } T_\omega = 4T_e'.$$

#### 4. Experimental Verification Controller Tuning

The experimental system, shown in Figure 4 comprises three main parts: the frequency converter with three-phase induction motor IM1, the control system and the measuring and acquisition system. Frequency converter Danfoss VLT 5003 is modified with a special IPC control card developed in order to build an open architecture for the rapid control prototyping of control algorithms [6], [7], [8]. Motor is loaded with coupled induction motor IM2, which is controlled with Danfoss frequency converter FC302. The control system is based on the dSPACE DS1103 Controller Board connected to host computer with Matlab/Simulink. Two main components of this software are Real Time Interface (RTI) which is the implementation software and ControlDesk which is the experimentation software.

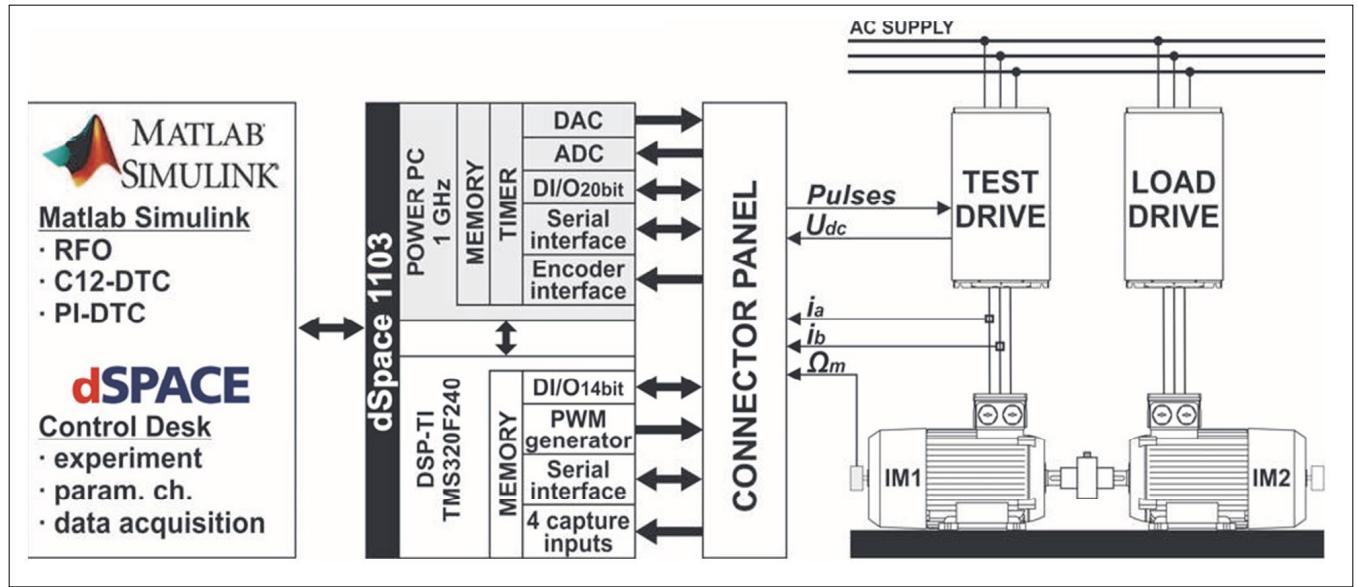


Figure 4. Main parts of the experimental system

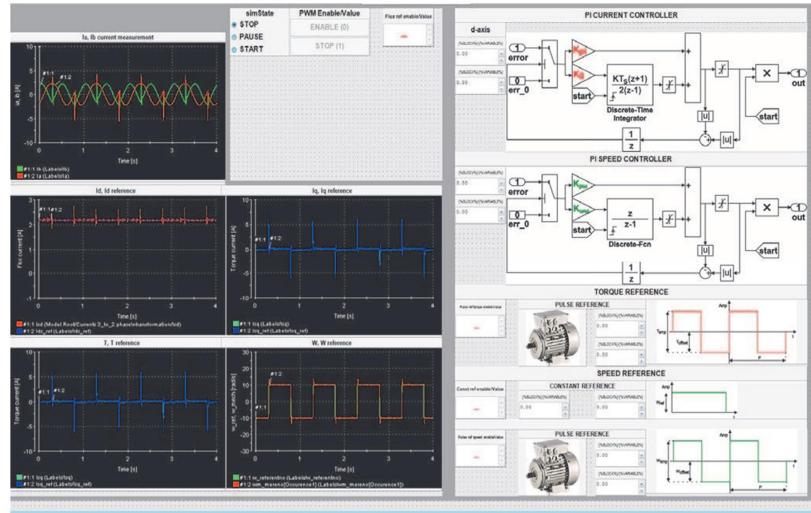


Figure 5. Control panel for controller parameters tuning

The ControlDesk allows users to control and monitor the real-time operation of the experiment using virtual instruments and building a control window [9]. Experiments are guided with ControlDesk virtual control panel shown in Figure 5. Results are monitored via virtual instruments, and besides, graphics can be copied directly into the prepared template for laboratory reports. Signals from the sensors and encoders are connected to the DS1103 connection panel and then processed in Matlab. The photo of complete test setup is shown in Figure 6.

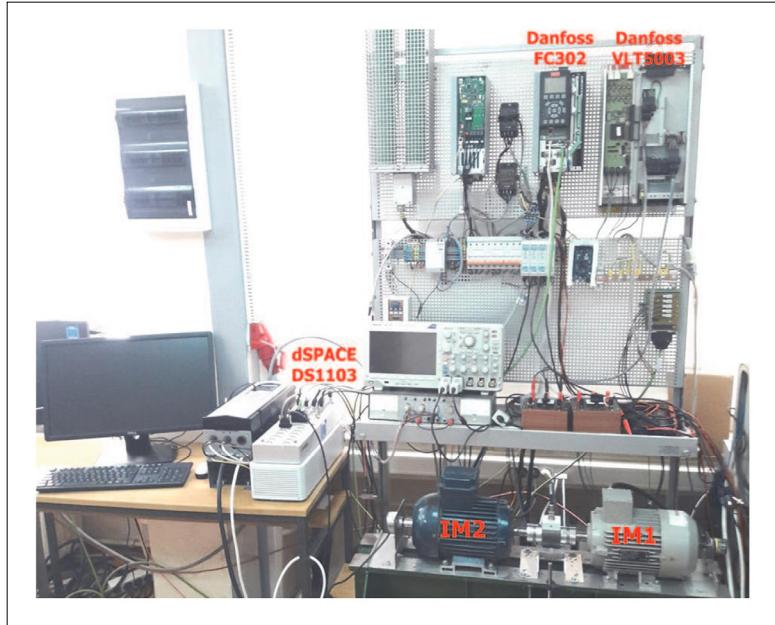


Figure 6. Photo of experimental electric motor drive system

## 5. Results of Controllers Tuning

Setting the current controller is done according to the values of the parameters  $K_{pi\_dig} = 46.6203$  and  $K_{ii\_dig} = 1.09$ . The

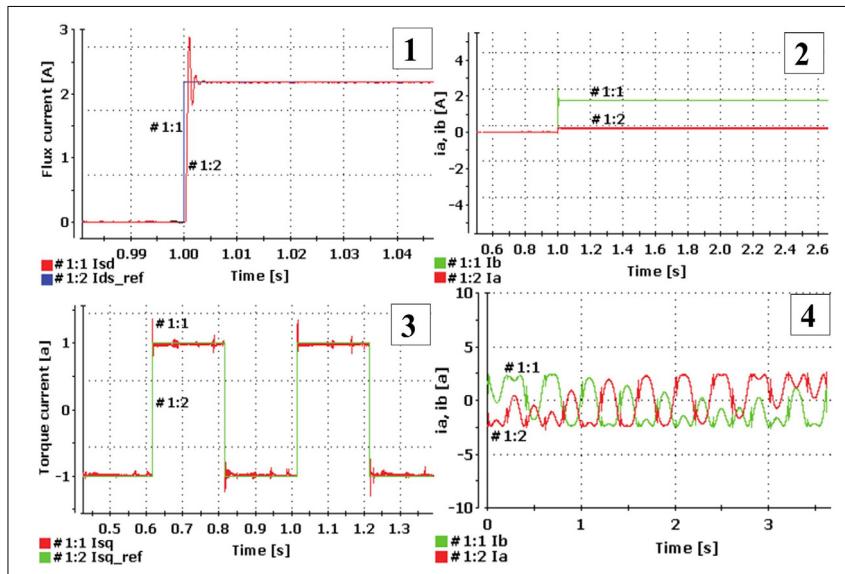


Figure 7. Flux and torque currents response and phase currents

induction motor is magnetizing with the nominal current value  $i_{sd} = 2.182 A$ . Figure 7.1 shows the magnetizing process of the induction motor and Figure 7.2 stator currents which are DC because the motors shaft does not rotate. The dynamic response examination of the system is performed with the rectangular reference  $isq$  amplitude of  $\pm 1A$ . On the Figure 7.3 the current  $i_{sq}$  response is given in relation to the reference, while the stator current  $i_a$  and  $i_b$  are given in Figure 7.4.

Setting the speed controller is done according to the values of the parameters  $K_{pw\_dig} = 0.621821$  and  $K_{ww\_dig} = 0.07772$ . Testing the speed controller is performed with rectangular reference amplitude of  $\pm 10 \text{ rad/s}$  and the period of 1 s. In Figure 8.1 and Figure 8.2 a speed response over the rectangular reference is given. There is overshoot in the response characteristic for the symmetrical optimum method which is in the case of the step reference signal about 43%. The torque current  $i_{sq}$  and the stator currents  $i_a$  and  $i_b$  are given on Figure 8.3 and Figure 8.4.

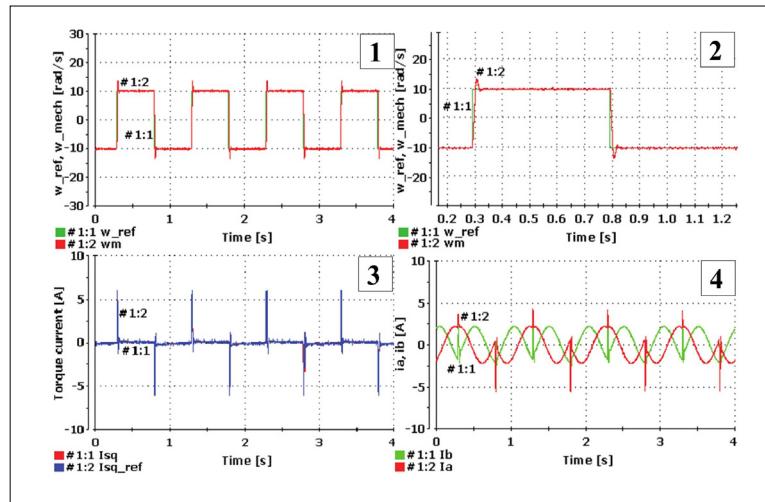


Figure 8. Speed response, torque current and phase currents

#### 4. Conclusion

Integrated Education Platform has a great benefit in education purpose. It represents the connection between the theory and practical application of the acquired knowledge. By working in small groups, students can apply knowledge in the controller parameters setting on the real drive. They can analyse applied analytical or engineering methods, compare them and draw conclusions based on the system's dynamic and static behaviour. Their conclusions may be submitted in a report form for further review.

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