

# Mathematical Model for Aircraft Functioning using GPS

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**ABSTRACT:** *In this work, we have used a mathematical system of the aircraft and a system for automatic guidance using a GPS. Besides, we used simulations for land guidance system when using GPS. We have used Simulink for the simulating the program tools. We have compared the GBAS and ILS for simulations.*

**Keywords:** Automatic Landing, Mathematical Model, Aircraft, Autopilot, GPS, GBAS

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## 1. Introduction

Aircraft landing is one of the most important phases of flight in terms of the technical complexity and security. The airports and the aircraft are equipped with appropriate equipment for landing but it is not enough sufficiently response for the landing problem especially when landing is under low visibility conditions.

This is a constant challenge to finding new solutions to improve the process of landing in terms of overcoming the problems of the low visibility conditions and better safety. The classic instrument landing systems (ILS, MLS) does not provide landing of aircraft in all weather conditions, unlike of those systems the new technology which is based on GPS (DGPS, WAAS, LAAS) provides an opportunity for a new approach to solving the problem of landing offering flexible solutions and application.

Satellite based landing system GBAS has many advantages in terms of ILS [1]. From the perspective of this paper, can appoint

the fact that GBAS provides guidance of aircraft during approach and landing in different curvilinear trajectories defined by control points (Way Points) unlike from ILS which provides guiding of the one fixed path determined by antennas of localizer and provider of the line of landing. Also, GBAS, a one GBAS station on the airport can simultaneously serve all runways, while an ILS is only tied to a one runway. Using GPS and GBAS for landing requires installing the proper equipment at airports, equipping aircraft with additional devices and modifying new algorithms for automatic landing implemented in the autopilot.

This paper is organized as follows. In Section 2, is described a system for automatic landing of aircraft using the GPS. In Section 3 is given a short description of mathematical models of aircraft. In Section 4 is described a concept of guidance system for automatic landing of aircraft using the GBAS. In Section 5 are presents the results of a simulation for the system for automatic landing using MATLAB/SIMULINK. Conclusion is given in Section 6.

## 2. Automatic Landing Systems Using GPS

Ground-Based Augmentation System (GBAS) is a system that provides differential corrections and integrity monitoring of Global Navigation Satellite Systems (GNSS). GBAS provides navigation and precision approach service in the vicinity of the host airport (approximately a 23 nautical mile radius), broadcasting its differential correction message via a very high frequency (VHF) radio data link from a ground-based transmitter. GBAS yields the extremely high accuracy, availability, and integrity necessary for Category 1, and eventually Category 2, and 3 precision approaches [1], [6]. GBAS demonstrated accuracy is less than one meter in both the horizontal and vertical axis. The GBAS architecture is shown on Figure 1.

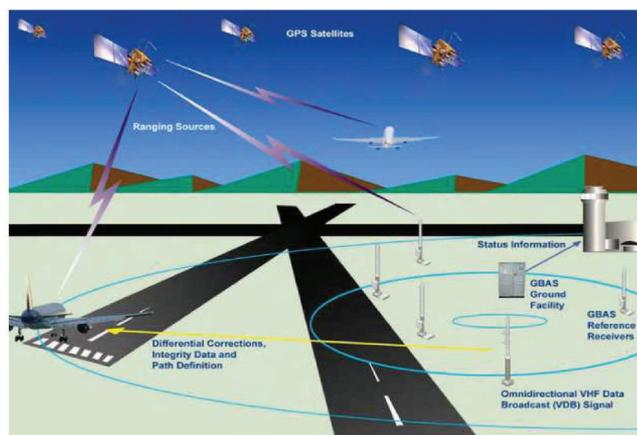


Figure 1. GBAS architecture

Ground-Based Augmentation System (GBAS) is constituted of three subsystems: satellite, ground and avionics. The Satellite subsystem is formed by satellites GNSS (GPS, etc.). This subsystem provides signals for GPS receivers set of aircraft and signals for ground GBAS station.

The ground subsystem is constituted from three or four referent receivers, which collect pseudo ranges for all the primary GNSS satellites in view and computes and broadcasts differential corrections and integrity-related information for them based on its own surveyed position. These differential corrections are transmitted from the ground system via a Very High Frequency (VHF) Data Broadcast (VDB). GBAS also transmit information, which is used to define the reference trajectory.

GBAS Avionic subsystem allows users to receive and decode VDB data and integrate with navigation signals which coming from the satellite constellation.

Existing systems for landing, ILS, have disadvantage and limitations where to the new technology for landing based on GPS, GBAS, should in due time to fully replace them. Some of the benefits from the implementation of GBAS are: reduction of critical and sensitive areas; curved approach; positioning Service; provision of service in several runways in the same airport; provision of several approach glide angles and displaced threshold; guided missed approach; and adjacent airports use.

### 3. Aircraft Mathematical Models

With a view to test the process of landing with simulations is used the complete 6-DOF model of aircraft and linearized models for longitudinal and lateral movements [6], [7]. The complete 6-DOF model is presented in the following system of differential equations written in vector-matrix form:

$$\dot{\mathbf{V}}_B = -\tilde{\boldsymbol{\omega}}_B \mathbf{V}_B + m^{-1}(\mathbf{R}_B^A + \mathbf{F}_B) + \mathbf{g}_B \quad (1)$$

$$\dot{\boldsymbol{\omega}}_B = -\mathbf{I}_B^{-1} \tilde{\boldsymbol{\omega}}_B \mathbf{I}_B \boldsymbol{\omega}_B + \mathbf{I}_B^{-1}(\mathbf{M}_B^A + \mathbf{M}_B^F) \quad (2)$$

$$\dot{\mathbf{q}} = \mathbf{f}(\boldsymbol{\omega}_B, \mathbf{q}) \quad (3)$$

$$\dot{\mathbf{R}}_G = \mathbf{T}_{BG}^T \mathbf{V}_B \quad (4)$$

where:  $\mathbf{V}_B = [u, v, w]^T$  is the velocity vector of the center of mass, CM, in the body fixed coordinate system  $B(P; x, y, z)$ ,  $\boldsymbol{\omega}_B = [p, q, r]^T$  is the angular rate vector,  $p$ ,  $q$  and  $r$  are the angular rates of rolling, pitching and yawing of the body,  $\mathbf{R}_G = [x_0, y_0, z_0]^T$  is a position vector of the center of mass,  $\mathbf{g}_B = [g_x, g_y, g_z]^T$  is the gravitational acceleration;  $\tilde{\boldsymbol{\omega}}_B$  is skew symmetric matrix which is composed of components of the vector  $\boldsymbol{\omega}_B$ ;  $\mathbf{T}_{BG}$  is transformation matrix from  $G$  to  $B$  coordinate systems;  $m$  is the mass and  $\mathbf{I}_B$  is the inertia matrix;  $\mathbf{R}_B^A = [X, Y, Z]^T$  and  $\mathbf{F}_B = [F_x, F_y, F_z]^T$  are aerodynamic and reactive forces,  $\mathbf{M}_B^A = [L, M, N]^T$  and  $\mathbf{M}_B^F = [M_x, M_y, M_z]^T$  are vectors of aerodynamic and propulsive moments, respectively.

Subscripts B and G denote that vector are presented in body ( $B$ ), or in inertial coordinate system ( $G$ ). The matrix  $\mathbf{T}_{BG}$  is the product of the elementary matrices of rotation;  $\mathbf{q}$  is a vector with four components-quaternions which uniquely determines attitude (angular orientation) of the body.

For calculation of the matrix of transition  $\mathbf{T}_{BG}$  and Euler angles  $\mathbf{S} = [\phi, \theta, \psi]^T$  are used quaternion:

$$\mathbf{T}_{BG} = \mathbf{T}_{BG}(\mathbf{q}) \quad (5)$$

$$\mathbf{S} = \mathbf{S}(\mathbf{q}) \quad (6)$$

The vector of state of the model with quaternions is composed of 13 components:

$$\mathbf{x} = [\mathbf{V}_B^T, \boldsymbol{\omega}_B^T, \mathbf{q}^T, \mathbf{R}_G^T]^T = [V_x, V_y, V_z, q, p, r, q_0, q_1, q_2, q_3, x_0, y_0, z_0]^T \quad (7)$$

The linearized models of flight are derived from general 6-DOF model and models are described in the matrix vector form. For longitudinal movement linear model has form:

$$\begin{bmatrix} \dot{u} \\ \dot{\alpha} \\ \dot{q} \\ \dot{\theta} \\ \dot{h} \end{bmatrix} = \begin{bmatrix} X_u & X_\alpha & 0 & -g \cos \theta^o & 0 \\ \frac{Z_u}{U^o} & \frac{Z_\alpha}{U^o} & 1 & -\frac{g \sin \theta^o}{U^o} & 0 \\ M_u^o & M_\alpha^o & M_q^o & -\frac{M_\alpha^o g \sin \theta^o}{U^o} & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & U^o & 0 \end{bmatrix} \begin{bmatrix} u \\ \alpha \\ q \\ \theta \\ h \end{bmatrix} + \begin{bmatrix} X_{\delta_m} & X_{\delta_r} \\ Z_{\delta_m} & Z_{\delta_r} \\ M_{\delta_m}^o & M_{\delta_r}^o \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \delta_m \\ \delta_r \end{bmatrix} \quad (8)$$

In equations (8) the state variables  $u$ ,  $\alpha$ ,  $q$ ,  $\theta$ ,  $h$  are the small declinations from their nominal values. The elements of the matrix depend on geometry, mass and aerodynamics features of the specific aircraft and can be calculated by the formulas given in [6], [7]. In the simulation are used data for aircraft Boeing 747-100 taken from [7].

#### 4. Automatic Aircraft Landing System Using GBAS

The concept of the guidance system for landing based on GBAS is presented on Fig.2. In short, DGPS station at the airport, through the GPS antennas (4 antennas) receiving signals from satellites on which determines its own position. Since, on the other hand, the exact position of the DGPS station is exactly known, calculates the error in position.

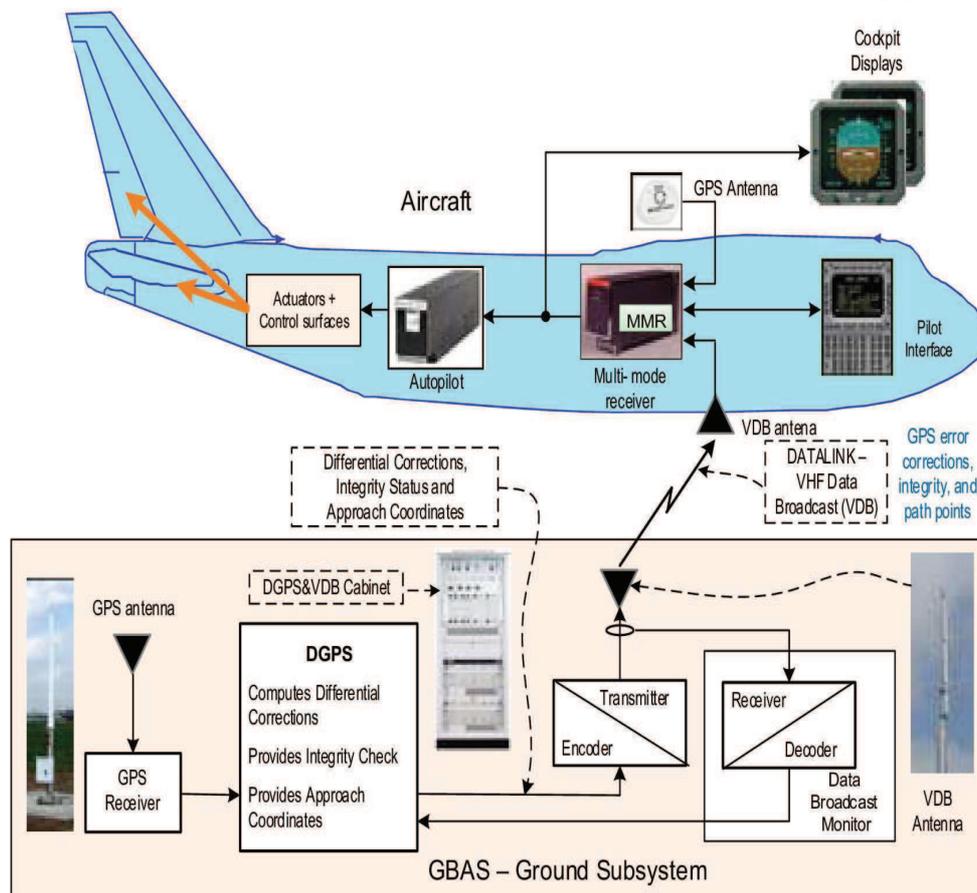


Figure 2. GPS based landing system using GBAS

The error in position is the same error that has a position of the aircraft defined on the GPS signal received in the receiver in the aircraft. Correction of this error is made on the basis of the calculated differential correction in DGPS station which is transmitted through VDB link multi - mode receiver of the aircraft. Multi - Mode Receiver - MMR receives signals from the GPS antenna on the aircraft, the pilot through the appropriate interface, the VDB antenna, and the ILS in the case when ILS is used on landing. The aircraft through VDB link gets information about the coordinates of the reference points for generating the reference trajectory for approach and landing.

The geometry of the landing in space is shown on Figure 3. In the case of ILS landing the receiver of aircraft measured errors, The guidance of aircraft using GPS, or GBAS, is used current coordinates of CG measured with GPS aircraft receiver which are corrected with error corrections obtained by VDB line. Corrected current coordinates are compared with the reference coordinates (trajectory) obtained on the basis of the reference points obtained from GBAS.

Figure 4 shows block diagram of the flight guidance and control system of the aircraft which is used for the development of the simulation model in MATLAB/ SIMULINK. The inner loop of stabilization and control mainly includes loops of stabilization and control with rotation in relation to the three axes - roll, pitch and yaw, and the loop for speed control of aircraft. The outer loop





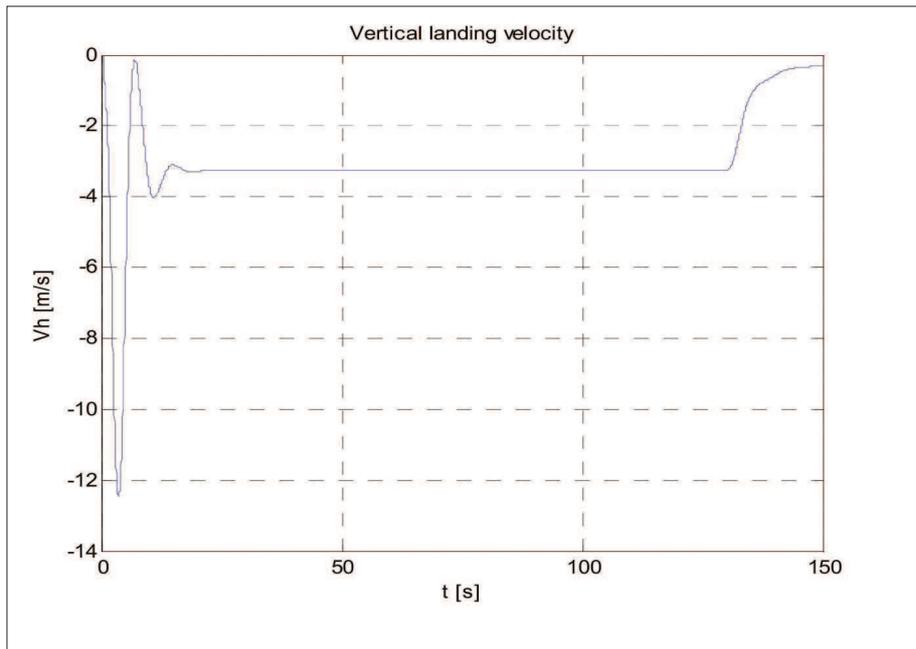


Figure 7. Vertical speed of landing

The next experiment concerns the guidance landing on curvilinear reference trajectory which at the beginning has a horizontal part, after a certain time, a parabolic curve is transformed into direction of angle of  $2.5^\circ$ , and ends with a “flare” manoeuver. Landing in this case is achieved using GPS and GBAS. Figure 8 shows tracking of curvilinear reference trajectory, and Figure 9 shows the vertical speed of the aircraft landing. Both diagrams are made in case of the effect of rear wind with average speed of 10 m/s and variance 2.

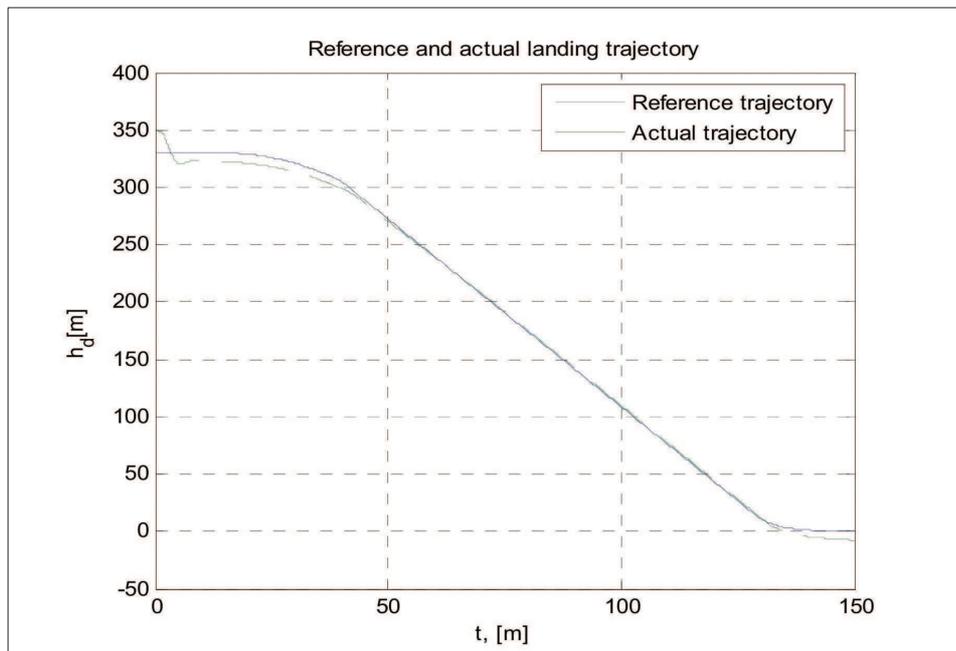


Figure 8. Tracking curvilinear reference trajectory

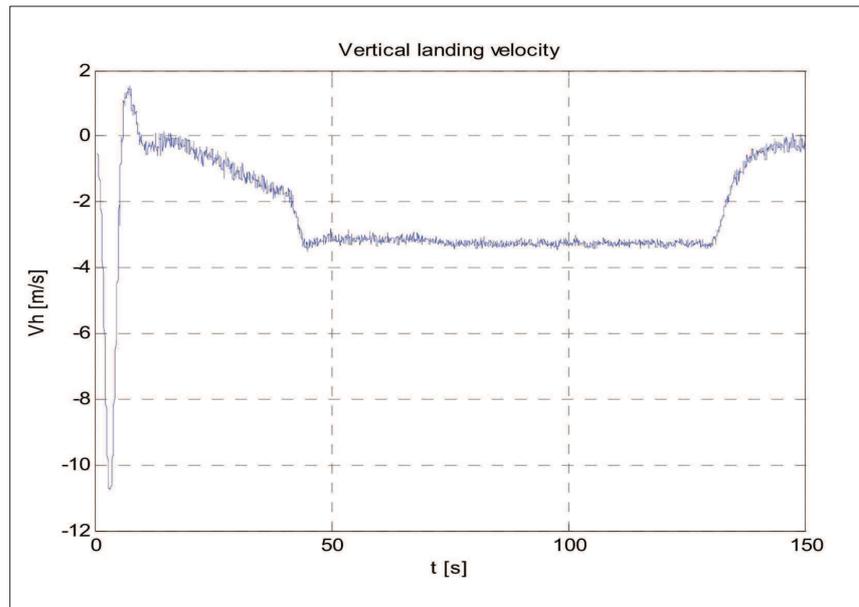


Figure 9. Vertical landing velocity in condition of wind

## 6. Conclusion

In this paper the system of automatic aircraft landing using GPS based technology is presented. The system GBAS (Ground Based Augmentation System) is composed of ground and aircraft subsystems. It uses signals from GPS for determining the position in 3D space, and has more advantages than existing ILS landing systems which in the near future should be fully replaced. These advantages are outlined in the paper. What especially is highlighted is the possibility of aircraft guidance in the phases of approach and landing along flexible curvilinear trajectories. This fact is suggesting the possibility of landing under larger angles and avoiding barriers in the area around the airport, reducing noise over populated areas etc. In terms of traffic, the GBAS is increasing the capacity of airports and supports a larger number of aircraft landings per unit time. With the development of GPS technology and increasing the accuracy of measuring the coordinates, and using differential GPS, which is at the base of GBAS, achieved accuracies that meet the prescribed requirements for landing. In this paper, aircraft models and the guidance system models of landing which are used for Matlab/SIMULINK simulation of the landing process are presented. The modelling approach and simulation is a good methodology for developing landing algorithms, evaluation of system performance and the effects of the action of various disorders such as wind and errors of measurement of sensors.

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