Inverse Fuzzy Modeling for the Cancellation of Nonlinearity in Unknown Hammerstein Model

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ABSTRACT: *For the ease of controlling of an unknown Hammerstein model, it is usually desired to cancel out the effects of static nonlinearities. This requires the proper cancellation of all the nonlinearities or otherwise it may either create control problems or can result in the optimization complexity. The paper discusses a novel approach towards the designing of fuzzy inverse model controller that can effectively cancel out the effect of static nonlinearities in unknown Hammerstein models. The controller is used in conjunction with Generalized Predictive controller for controlling an unknown Hammerstein model. Simple single layer convex optimization is found sufficient to generate a converged optimized solution. Simulation results are presented to show the effective performance of the proposed controller.*

Keywords: Fuzzy inverse model controller, Generalized Predictive controller, static nonlinearity, Hammerstein model, Optimization.

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1. Introduction

Hammerstein models are the famous structures to represent variety of practical systems such as water heater, distillation column, and ball and beam system etc. Different approaches have been proposed for controlling these models. These approaches usually deal with the cancellation of static nonlinearity present within the Hammerstein model [6], [1], [13],[14] and [15]. In spite of being a nice approach, it sometimes may lead to risky results, either in the form of bad control or the requirement of increasing optimization layers [1]. Fuzzy models which are well known as universal approximators are, on the other hand, a nice approach to identify the static nonlinearity. Babuska has proposed a simple approach to develop fuzzy inverse model using TS zeroth order fuzzy models (as shown in Fig 1a) [12]. These inverse models can be used for nonlinearity cancellation, however for the unknown systems; the conventional implementation of fuzzy inverse model as proposed by Babuska [12] may not be quite accurate. This is because the definition of output fuzzy sets on the basis of

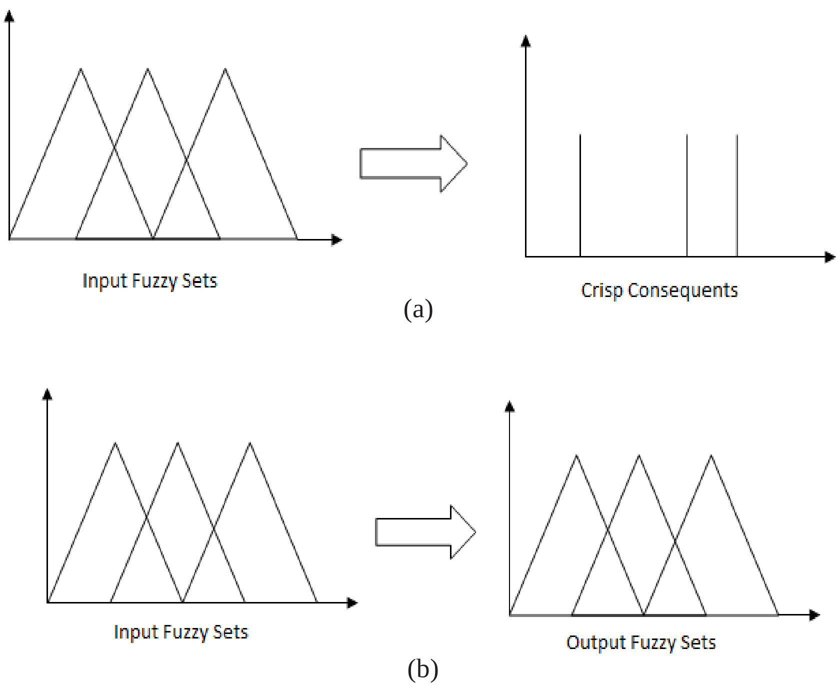

Figure 1 (a): TS structure of Fuzzy Model bMamdani Structure of Fuzzy Model.

are more favorable than Mamdani models because TS models are mathematically less complex due to their crisp consequents rather than output fuzzy sets [3]. .

This paper introduces Fuzzy inverse model of TS structure rather than that of Mamdani structure as proposed by Babuska. Moreover the problem of exact cancellation of static nonlinearity is resolved by implementing feedback error learning concept in the fuzzy inverse model. This approach will ensure that if the crisp output generated by inverse fuzzy model is not exact then feedback error learning implemented with then fuzzy inverse model will rectify the solution to be accurate. In section II, conventional approaches of static nonlinearity cancellation will be presented. Section III will then detail the proposed approach of fuzzy inverse model controller. In section IV, the combination of fuzzy inverse model with the GPC will be presented, the simulation results will then be shown in section V. Finally Section VI will conclude the work.

2. Conventional Approach of Fuzzy Inverse Model

Various approaches have been adopted for compensating the effect of nonlinearities. These include conventional nonlinear inverse strategy [Corradini and Orlando, 2002; Tao and Kokotovic, 1993; Nordin and Gutman, 2002; Chow and Clarke, 1994] and the newly emerging Fuzzy Inverse Modeling. The detailed study of fuzzy inverse models is presented by Babuska [12]. They have discussed the two types of Fuzzy model structure Mamdani and Takagi Sugeno and the corresponding inverse modeling. In this section the inverse fuzzy model structure from 0th order TS fuzzy model will be described in detail.

The discussion is regarding the inverse fuzzy model that is developed from 0th order TS fuzzy model. The partially reduced inverted model of singleton fuzzy model formulated as:

$$R: if \;\; x \; is \;\; A_j(x_i) \;\; then \;\; y = p_j^0 \qquad (1)$$

When the original fuzzy model is first order TS type, still the singleton fuzzy model results by using partial input reduction. This can occur if *zn* = *xi* and xi is not an element of *q*

$$p_j^{0} = \sum_{k=1}^{m}(\sum_{j=1}^{N_R}(\prod_{l=1 l\neq i}^{n} A_{L}, J(z_l).p_j^{l}).qk \tag{2}$$

The rules of the inverse fuzzy model have the following general form:

$$R: if\ x is\ A_j(x_i)\ then\ y = p_j^{0} \tag{3}$$

Where Aj represents the antecedent membership functions of the inverse model defined on the domain of the output of the forward fuzzy model. These antecedent sets are shown in Figure 2 where *Pj(y) = j(y) and pj = p0jfor1 ≤j ≤NR.*

$$A_{.,j}(y) = \frac{y - p_{j-1}^{0}}{p^{0}_{j} - p_{j-1}^{0}} \qquad for\ p_{j-1}^{0} \le y < p^{0}_{j} \tag{4}$$

$$A_{.,j}(y) = \frac{p_{j+1}^{0} - y}{p_{j+1}^{0} - p^{0}_{j}} \qquad for\ p_{j}^{0} \le y < p_{j+1}^{0} \tag{5}$$

$$A_{.,j}(y) = 0 \qquad otherwise \tag{6}$$

An interpolation method must be applied to obtain aj , the centers of the consequent fuzzy sets of inverse model because the consequents for inverse model, p0j, are singletons. This interpolation is carried out by the fuzzy sets Aj shown. If the desired reference $r(k + 1)$ is within the range of the reached states from the actual state $x(k)$, i.e. $p^0_1 \le r(k+1) \le p^0_{NR}$ then the serial connection of the inverted and the forward fuzzy model produces an identity mapping. The whole method is shown in Figure 3. It is clear that the inversion of reference $r(k+1)$ is given by one and only one point. If desired output is unable to be reached from the current state in one time

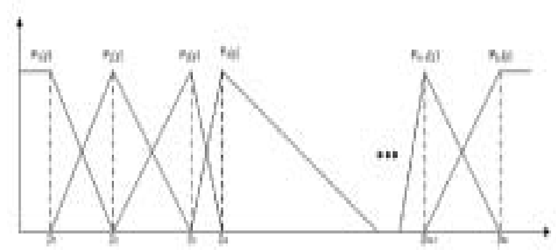

Figure 2. Fuzzy sets developed from crisp outputs

step, i.e. $r(k+1) < p^0_1$ or $r(k+1) > p^0_{NR,}$ the control action is still able to generate mapping with minimal error. When $r(k + 1) > p^0_{NR}$, $u^{NR}_{p0}(r(k + 1)) = 1$, and the control input is $a(k) = core(A_{NR})$. The degree of fulfillment of control action is given by:

$$u_{Aj}(r(k+1)) = 1 \tag{7}$$

which yields the model output

$$y(k+1) = p^{0}_{NR} \tag{8}$$

As $p^0_{Nr} > p^0_j$ when $1 \le j \le N_{R-1}$ the difference $|r(k+1) - y(k+1)|$

is the minimum possible. Parallel for r < p01, the control action $a = core(A_1)$ yields the output

$$y(k+1) = p^0_1 \quad (9)$$

This gives the best control action, since $p^0_1 < p^0_j$, when 2jNR. The discussion shows that Mamdani inference is basically used to generate the inverse TS model using the fuzzy sets developed from the singleton consequents of forward model. These fuzzy sets are used as the antecedents of Mamdani model and the fuzzy sets of the inputs of forward model are used as the consequents of Mamdani model.

3. Proposed Structure of Inverse Fuzzy Model

Fuzzy model requires fuzzy input sets and fuzzy output sets in case of Mamdani structure or crisp output in case of TS structure. So does the fuzzy inverse model. The concept of fuzzy inverse model is depicted in Figure 3. It is shown here that if the inverse model is accurately designed then the input is retrieved back. Unfortunately this is impossible practically because of various reasons. These include disturbances, noise, improper and inefficient designing of inverse model. Furthermore for unknown systems we cannot be sure of the system behavior between two points. That is if the designer chose triangular membership function. Input data is defined just before and after the core, then according to the designer selection, the system in between these two inputs will behave corresponding to the core of fuzzy set however practically there may not be such a curve. On the other hand, in case of TS structure, wrong crisp outputs may be defined which might be different from the actual plants behavior. For nonlinearity cancellation, we therefore need some improvement in the conventional approach. If prediction error learning is incorporated with the model, then we can easily train the inverse fuzzy model to depict the unknown nonlinearity accurately. Here in this work we have used this concept with the fuzzy inverse model. The general structure of the scheme is presented in Figure 5.

$$u = ú + \xi e \quad (10)$$

where:

ú is the output of inverse model controller

ξ is the error gain

e is the error between virtual control action generated by GPC [11], [15] and the virtual control action to be applied to the linear dynamics of Hammerstein model hence $e = v - v'$

In Figure 5 we see that there have to be three inputs for the adaptive inverse fuzzy model of the static nonlinearity of an unknown Hammerstein model:

- The virtual control action (v), obtained from GPC (will be explained in section IV)
- Parameters of static nonlinearity identified using Fuzzy Hammerstein model of the system [1] and
- The desired actual action to be produced by the inverse fuzzy model.

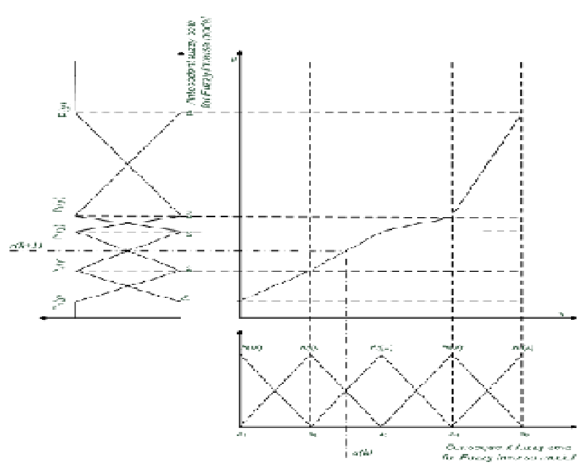

Figure 3. Inversion of SISO Fuzzy inverse model

The third signal is actually important. If there is no feedback then the designer has to actually estimate the output for the case of unknown systems. For known systems, the task is much simpler because there the designer has only to specify the desired input. However for unknown systems, there are many chances of error in estimation which may lead to inefficient cancellation of nonlinearity. When feedback error learning is incorporated then the fuzzy inverse model actually learns from the unknown systems dynamics about its required actual control action. This strategy helps to regenerate the actual control action efficiently.

4. Gpc Based On Fuzzy Inverse Model

Hammerstein models are nonlinear systems that contain static nonlinearities which are connected in series with the linear dynamics. The basic structure of Hammerstein model is shown in Figure 6. These models represent different practical nonlinear systems (as discussed in I). The proposed controller is actually combined with linear GPC algorithm [2], [7]The overall scheme is now discussed. The inverse fuzzy model inverts the virtual control action [7] produced by the GPC algorithm for unknown Hammerstein model. Furthermore we have assumed that the parameters of the unknown model (nonlinear parameters corresponding to static nonlinearity and linear parameters corresponding to linear dynamics) have been identified using Fuzzy Hammerstein model [7]. The general block diagram of overall control scheme is presented in Figure 7.

GPC algorithm is provided with identified parameters of linear dynamics only. Therefore the algorithm will generate the control action for linear dynamics (i.e. virtual control action). However the Hammerstein model comprises of the static nonlinearity as well. To get rid of this nonlinearity efficiently and effectively, feedback error learning inverse model controller is used.

By using feedback, the inverse model controller is learnt effectively regarding the actual control action. This is done by following the scheme in Figure 5. The incorporation of feedback error learning, fuzzy inverse model controller helps in reducing the unknown Hammerstein model to an approximate linear dynamics which can easily be controlled by linear GPC algorithm.

5. Simulation Results

The inverse model controller is designed for three systems *A*, *B* and *C*.

- For system A: Hammerstein model is assumed to have logarithmic static nonlinearity

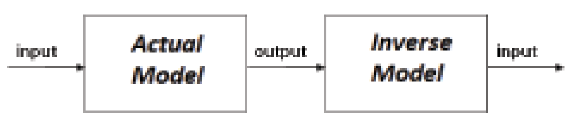

Figure 4. The concept of Inverse Fuzzy Model

- For system B: Hammerstein model is assumed to have exponential static nonlinearity
- For system C: Hammerstein model is assumed to have squared static nonlinearity

For all the three systems, delayed linear dynamics is associated with different time constants for the three systems. The Hammerstein model is identified as Fuzzy Hammerstein model using constrained Recursive least squares algorithm. Generalized Predictive controller is implemented to generate virtual control action to be applied to inverse model controller.

The inverse model controller is developed with six input fuzzy sets. All these sets have triangular shapes to define the dynamics. The tuning parameters are:

- The supports and the core of input fuzzy sets.

• The control horizon in GPC

• The prediction horizon in GPC The control weighting factor in GPC

• The error gain in feedback error learning

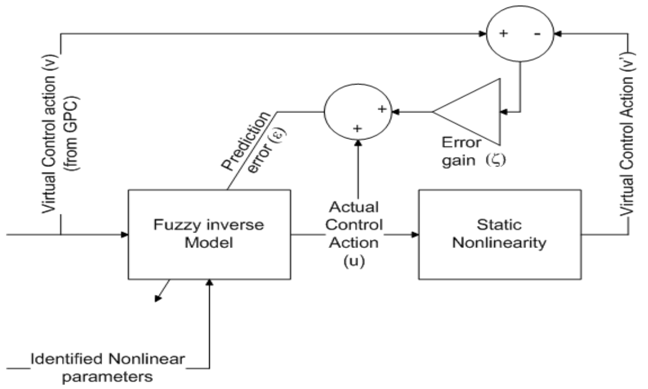

Figure 5. The general structure of inverse model controller

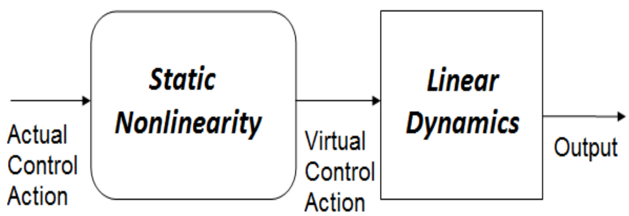

Figure 6. Structure of Hammerstein model

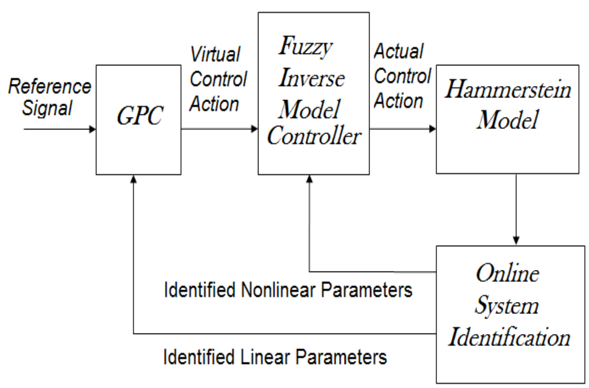

Figure 7. Placement of Proposed inverse model controller in overall control scheme

However the scope of this paper is limited to the nonlinearity cancellation by the inverse fuzzy model, therefore the focused tuning parameters here are just two. These are:

• Control weighting factor

• Feedback error gain

below along with the overall control performance.

System A comprises of logarithmic static nonlinearity with delayed linear dynamics (with τ = 120s). System B contains exponential static nonlinearity with the linear dynamics (τ = 50s). Finally system C describes a Hammerstein model cotaining squared static nonlinearity with the linear dynamics (with τ = 60s). The overall Hammerstein models of Systems A, B and C are shown in (11), (12) and (13).

$$\frac{y}{u}=\frac{1}{3.43}in(30u=1)^{e-10s}\left(\frac{1}{120s+1}\right) \tag{11}$$

$$\frac{y}{u}=u^{*}e(0.9u)e^{-10s}\left(\frac{1}{50s+1}\right) \tag{12}$$

$$\frac{y}{u}=(u^{2}+0.5u)e^{-10s}\left(\frac{1}{60s+1}\right) \tag{13}$$

The control performance of Fuzzy adaptive inverse model controller for system A is shown in Figure 8, for system B is shown in Figure 10 and for system C is shown in Figure 12. The virtual control action (shown in green color) is produced by GPC algorithm. The actual control action (shown in blue color) is produced after the cancellation of static nonlinearity by the fuzzy inverse model controller. It is observed that the actual control action is efficiently following the virtual control action which ensures that the static nonlinearity of the Hammerstein model has been effectively cancelled out by the proposed inverse fuzzy model controller. Figure 9, 11 and 13 show the root mean square errors in response following by fuzzy inverse model controller (i.e the error between the corresponding virtual control actions, produced by GPC algorithm, and the actual control action) for different values of feedback error gain (shown in Figure 8, 10 and 12). Table I shows the tuned parameter values for the three Hammerstein models.

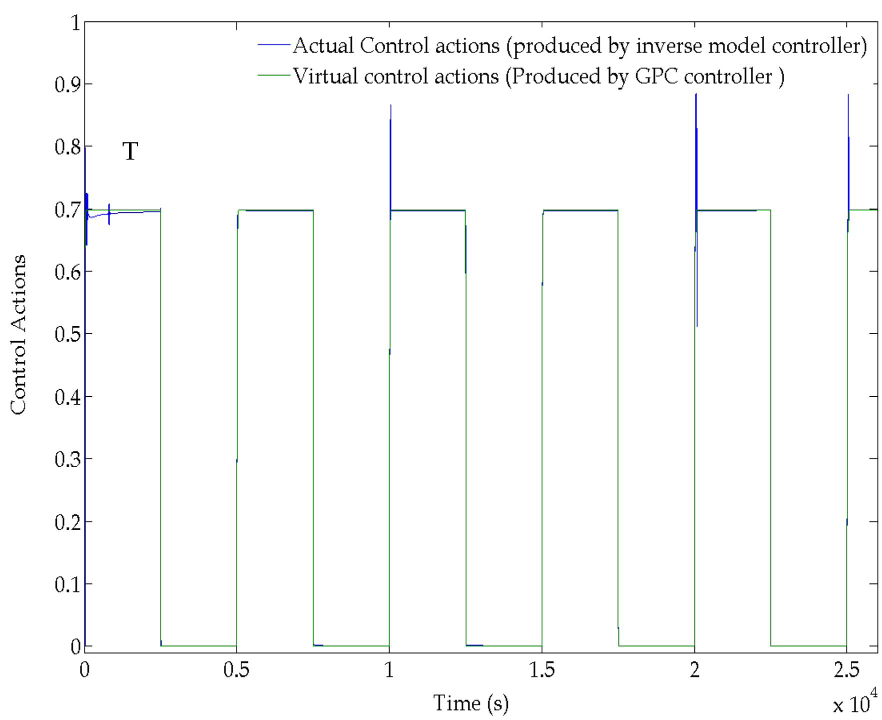

Figure 8. Performance of Inverse controller for system A

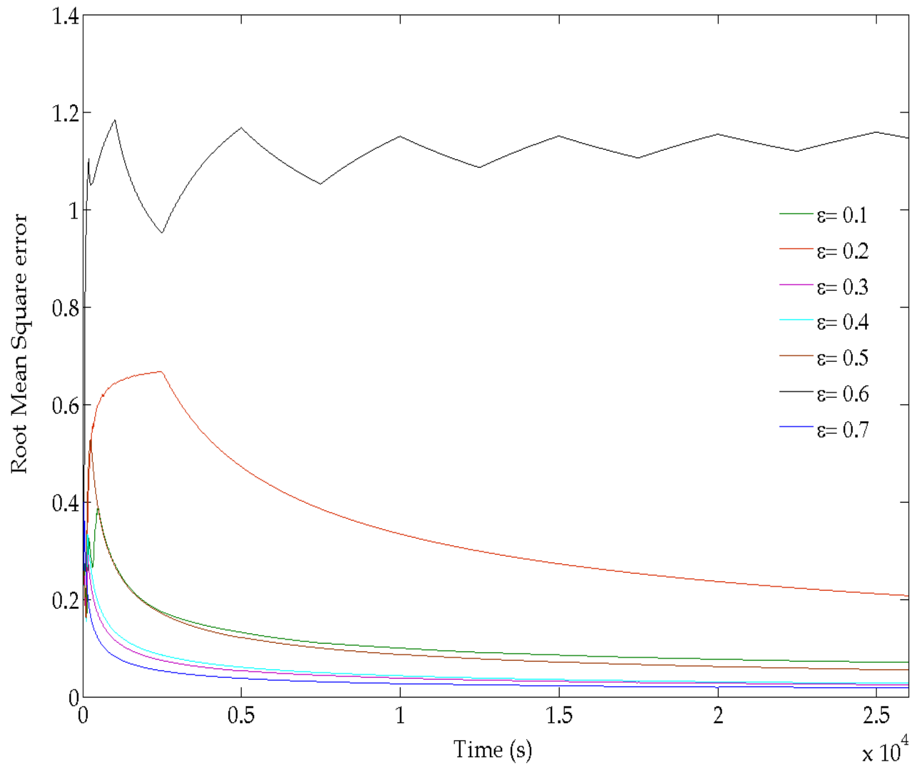

Figure 9. Root Mean Square Error plot against system A

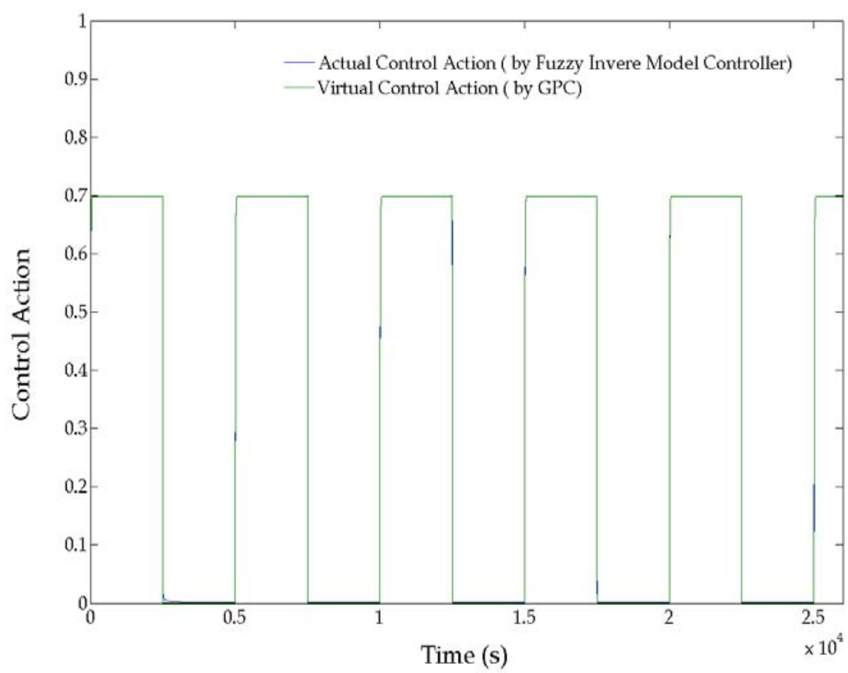

Figure 10. Performance of inverse model controller for system B

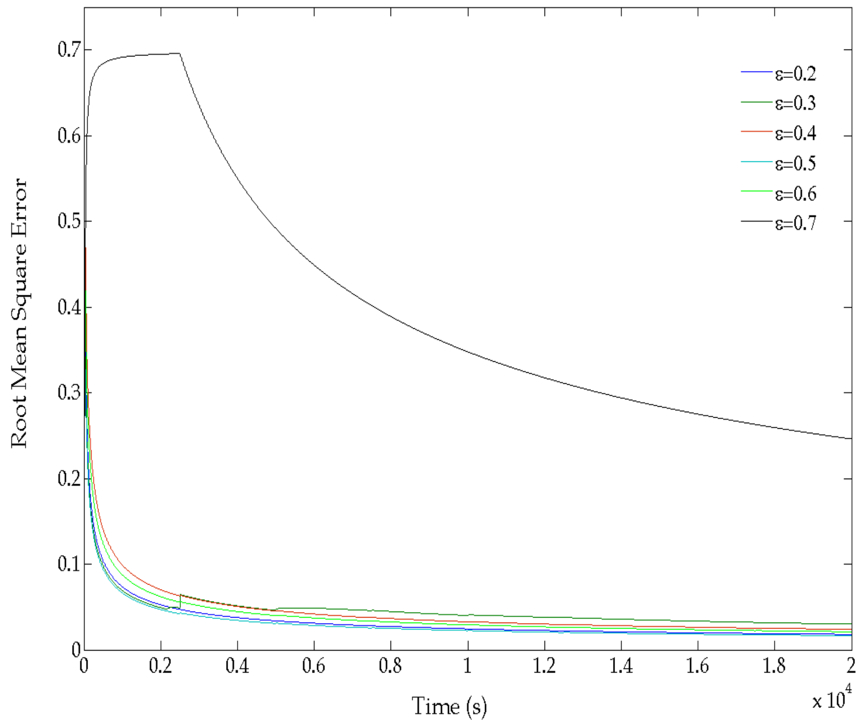

Figure 11. Root Mean Square Error plot against system B

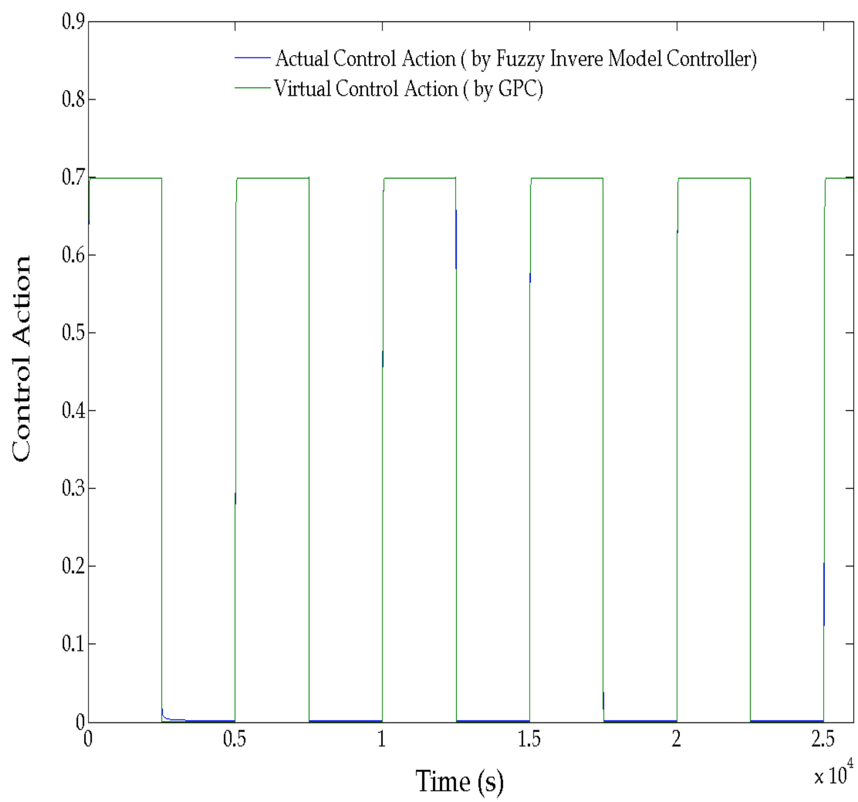

Figure 12. Performance of inverse model controller for system C

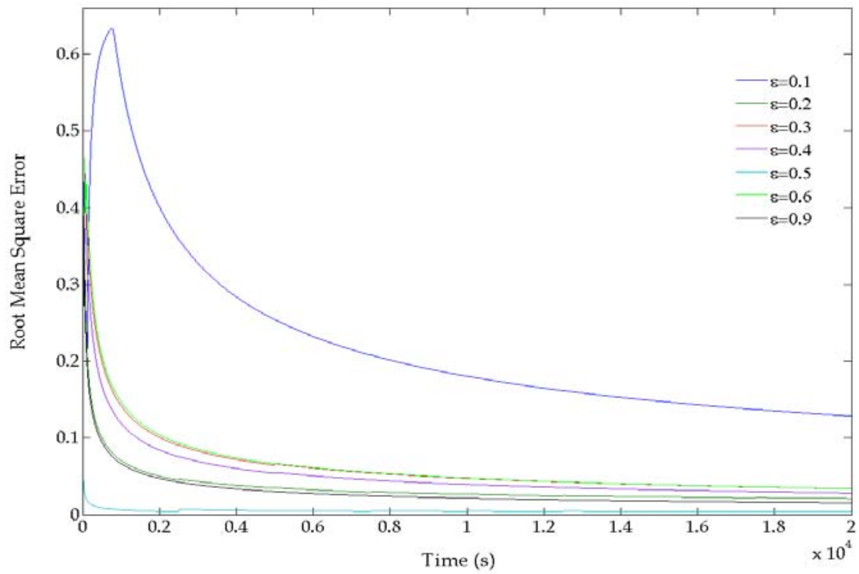

Figure 13. Root Mean Square Error plot against system C

6. Conclusion

This paper discusses a novel approach of adaptive inverse fuzzy model with TS model structure and feed back error learning. The controller adaptively and effectively cancels out the effect of static nonlinearities from Hammerstein model. The controller has been tested with different Hammerstein models and performs well in all the cases.

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