A Bi-Objective Mathematical Model using the Software GAMS

Mohammad Rahimi, El-Ghazali Talbi
INRIA Laboratory, CRISTAL-CNRS
59650 Lille
France
Mohammad.Rahimi@inria.fr
El-ghazali.Talbi@inria.fr

ABSTRACT: *In this work we introduced a bi-objective mathematical model using the GAMS software. Besides we calculate the cost associated with the impact of service level on the optimization. Through this current exercise we introduced a new bi-objective mathematical model to determine an optimized maintenance-routing policy, simultaneously. Basically the first objective function minimizes the total costs due to traveling and a delay in start time of a Preventive Maintenance (PM)/ Corrective Maintenance (CM) operation. The next intended function considers the service level which is measured based on waiting times before beginning of the CM operations. In the proposed model, we consider time windows in repairing the machines and skill-based technician assignment in performing PM/CM operations. Thus the new framework is modeled as a mixed-integer linear program and is solved by using the software GAMS.*

Keywords: Preventive and Unforeseen Maintenance, Vehicle Routing Problem, Scheduling, Service Level, Multi-objective Mathematical Model

Received: 29 April 2018, Revised 8 June 2018, Accepted 14 June 2018

1. Introduction

Regularly planned and scheduled maintenance is a critical requirement to reduce the occurrence of an unforeseen failure and keeping the equipment running at peak efficiency. Maintenance scheduling becomes complex when the machines are geographically distributed. In this case, in addition to assigning the maintenance operations to technicians, it is needed to find the best set of routes for technicians' visits. In fact, it is necessary to study the maintenance and the routing scheduling decisions simultaneously. Such a joint decision problem is known as the maintenance-routing problems.

In the literature there are various studies which investigate combination of maintenance and routing problem [1]–[5]. In the most of these studies, authors have two initial assumptions:

• The replacement would be done immediately, if an unforeseen failure occurs for the machines. In fact the authors do not consider waiting time for performing a CM operations. While considering the waiting time is important especially where the machines are geographically distributed and the number of technicians and machines are limited.

• The scheduling is predefined and authors try to assign the technicians to machines considering skill of technician, time windows and etc. An unforeseen failure causes changes of the maintenance scheduling. In this case, maintenance scheduling and routing should be done simultaneously.

To the best of our knowledge, few studies attempted to investigate the simultaneous maintenance scheduling and vehicle routing problem and consider two described assumptions. López-Santana et al. [6] combine maintenance and routing problems to schedule maintenance operations for a set of geographically distributed machines and plan to assign a set of technicians to perform preventive maintenance at the customer sites. The authors use a distribution function for taking into account failures of machines as an uncertain parameter. In this study, they use two-step iterative approach to solve the model which causes minimizing the total maintenance and routing cost, waiting time at each customer and failure probability.

In this study, we propose a new framework to model and to establish the trade-off between the service level (measured based on waiting times before beginning of the CM operations) and different maintenance costs by taking into account the presented issues.

2. Problem Description

In this section, a bi-objective mathematical model is proposed to determine optimized routing-maintenance policy. In this model, first objective function minimizes the total costs due to traveling, delay in start time of a Preventive Maintenance (PM)/ Corrective Maintenance (CM) operation at customer while second objective function attempts to minimize the waiting times before beginning of the CM operations.

In this study, we consider a system with geographically distributed customers, where each customer has one machine that should be visited and repaired by technician in different cycles. The PM operations are scheduled with a certain frequency to reduce the occurrence of unforeseen failure in the long term. Regarding the previous experiences, the time of unforeseen failure occurrence is known for each machine at each customer, but its repairing can be postponed until defined period. The time interval between occurrence of unforeseen failure and its repairing named waiting time. The set of technicians, who need to visit the set of machines to perform the PM/CM operations to prevent the system failure. The technician are different in duration time of doing a PM/CM operation which causes different in salary. A central depot is concerned as the point of departure and final destination. Since each technician should travel to perform PM/CM operation at the customer location, the distance between each two customer is defined. The main aim of this study consist of determining a joint routing-maintenance policy for all machines taking into account making a balance between the waiting time and total cost of system. The optimized maintenance policy determines in which periods the PM and CM operations should be performed at each customer. The optimized routing policy determines that which technician is assigned to which customers and in which sequence should visit and perform PM and CM operations at each period.

The detailed conditions of system are summarized as follows:

• The time required to perform a maintenance operation depends on the skill of the assigned technician.

• More skilled technicians receive more salary.

• All technicians are able to perform any PM/CM operation.

• The technicians start in the central depot in the beginning of each period and should return to the central depot by the end of the period.

• Each machine should be repaired by only one technician at each period. It means if the machine should be repaired in the specific period, only one technician should be assigned to the machine.

• The PM operation should be performed on all the machines at the first period.

• If no unforeseen failure occurs on the machine at planning horizon, the PM operations will be performed regarding the defined frequency. The frequency is defined regarding planning horizon and the duration of the interval between two consecutive PM operations.

• In the case of unforeseen failure occurrence on the machine, no predictive maintenance can be scheduled and performed before performing CM operation. In this case, CM operation should be scheduled to assign a technician on the machine until maximum L period. Moreover, next PM operation will be scheduled and performed after λ period.

• After performing a CM operation, the machine returns to the good condition and no unforeseen failure occurs until the next repairing that will be a PM operation in λ period. It means two unforeseen failure cannot occur consequently.

• The time required to perform a CM operation is longer than the time required to perform a PM operation on each machine.

• The CM cost is larger than the PM cost.

• The machines impose time windows on the system which means the technician should start maintenance operation before the latest possible start time. In cases where this time windows is not respected, a delay penalty applies if the technician starts after the latest allowed time.

• The travel time between two customers depends on the speed of the vehicle in the rout at each period.

2.1 Mathematical Formulation

The following notations are used in the proposed model.

Sets
M set of customers, index for customers $(1,2,\ldots, m)$
M' set of customers and central depot, $(0,1,2,\ldots, m+1)$
K index for technicians $(1,2,\ldots, k)$
t, t', t'' index for period $(1,2,\ldots, T)$
Parameters
c_k one unit time cost of a PM/CM operation by technician k
pm_k time required to perform a PM operation by technician k
cm_k time required to perform a CM operation by technician k
λ duration of the interval between two consecutive PM operations
L allowed duration to repair occurred unforeseen failure
z_{it} a binary parameter which determines occurrence of unforeseen failure in customer i at period t
t_{ij} traveling time between customer i and j
r transportation cost per unit time
$[a_i, b_i]$ earliest and latest possible start time of a PM/CM operation at customer i

p_i penalty cost of one unit time delay due to start time of a PM/CM operation at customer i after latest possible arrival time	
G a large value number	
Variables	
x_{ijkt}	1 if customer j is visited exactly after customer i by technician k at period t, otherwise 0
y_{it}	1 if PM operation is planned in customer *i* at period *t*, otherwise 0
$u_{it't}$	1 if a CM operation is planned in customer *i* at period t for the an occurred unforeseen failure at period *t'*, otherwise 0
$\beta_{itt'}$	1 if delay occurred in visiting customer *i* at period *t*, otherwise 0
μ_{ikt}	1 if customer i is visited by technician *k* at period t to perform a PM operation, otherwise 0
π_{ikt}	1 if customer i is visited by technician *k* at period t to perform a CM operation, otherwise 0
T_{ikt}	Start time of an operation by technician *k* in customer *I*, period *t*
d_{it}	Delay in start time of a PM/CM operation in customer *i* at period *t*

The mathematical model associated with the presented framework is provided in this section. Each equation in this model is detailed below.

$$Min \quad f_1 = \sum_{i,j,k,t} x_{ijkt}.t_{ij}.r + \sum_{i,t} d_{it}.p_i + \sum_{i,k,t} \mu_{ikt}.c_k.pm_k + \sum_{i,k,t} \pi_{ikt}.c_k.cm_k \tag{1}$$

$$Min \quad f_2 = \sum_{i,t,t'} \beta_{itt'} \tag{2}$$

S. t.

$$\sum_t y_{it} \le \left(\frac{|T|-1}{\lambda}\right) + 1 \qquad \forall i \in M \tag{3}$$

$$y_{it} \le 1 - z_{it} \qquad \forall i \in M, t \tag{4}$$

$$y_{i1} = 1 \qquad \forall i \in M \tag{5}$$

$$\sum_{t'=t+1}^{t+\lambda-1} y_{it'} \le 1 - (y_{it} + u_{it''t}) \qquad \forall i \in M, t, t'' \tag{6}$$

$$y_{it} \le y_{i(t+\lambda)} + \sum_{t'=t}^{t+\lambda} z_{it'} \qquad \forall i \in M, t \tag{7}$$

$$u_{it't} \le y_{i(t+\lambda)} + \sum_{t'=t+1}^{t+\lambda} z_{it'} \qquad \forall i \in M, t \tag{8}$$

$$\sum_{t=t'}^{t'+L} u_{it't} = z_{it'} \qquad \forall i \in M, t' \qquad (9)$$

$$\beta_{itt'} = (t'-t).u_{itt'} \qquad \forall i \in M, t, t' \qquad (10)$$

$$\sum_{t} u_{itt'} \le 1 \qquad \forall i \in M, t' \qquad (11)$$

$$y_{it} + u_{it't} \le 1 \qquad \forall i \in M, t, t' \qquad (12)$$

$$\sum_{k} \mu_{ikt} = y_{it} \qquad \forall i \in M, t \qquad (13)$$

$$\sum_{k} \pi_{ikt} = \sum_{t'} u_{it't} \qquad \forall i \in M, t \qquad (14)$$

$$\sum_{i \in M',\, i=0}^{|M'|-1} x_{ijkt} = \mu_{jkt} + \pi_{jkt} \qquad \forall j \in M, j \ne i, k, t \qquad (15)$$

$$\sum_{j \in M,\, j=1}^{|M|+1} x_{0jkt} \le 1 \qquad \forall j \in M', k, t \qquad (16)$$

$$\sum_{i \in M',\, i=0}^{|M'|-1} x_{ijkt} - \sum_{i \in M',\, i=2}^{|M'|} x_{jikt} = 0 \qquad \forall j \in M, k, t \qquad (17)$$

$$T_{ikt} + \mu_{ikt}.pm_k + \pi_{ikt}.cm_k + t_{ij} \le T_{jkt} + G.(1 - x_{ijkt}) \qquad \forall i, j \in M' \quad \forall k, t \qquad (18)$$

$$a_i.\sum_{j \in M',\, j=0}^{|M'|-1} x_{jikt} \le T_{ikt} \le b_i.\sum_{j \in M',\, j=0}^{|M'|-1} x_{jikt} + d_{it} \qquad \forall i \in M' \quad \forall k, t \qquad (19)$$

$$x_{ijkt}, y_{it}, u_{it't}, \beta_{it't}, \mu_{ikt}, \pi_{ikt} \in \{0,1\} \qquad \forall i, j \in M', k, t \qquad (20)$$

$$T_{ikt}, d_{it} \ge 0 \qquad \forall i \in M', k, t \qquad (21)$$

The first objective function (1) minimizes the total cost which consist of traveling cost between customers, penalty cost due to start time out of time windows and the wages of technicians for PM/CM operations. The second objective function (2) optimizes the customer satisfaction level by minimizing the waiting times until performing a CM operation in the case where an unforeseen failure occurs.

Constraint (3) checks number of PM operations on the machine of each customer should not be exceeded. Constraint (4) guarantees that if the unforeseen failure occurred, then the PM cannot be scheduled and performed for the same period. Equation (5) determines that at the first period, PM operation should be performed on the all the machines. Equation (6) guarantees that performing a CM operation return the machine to as good as new condition again and no PM operation is needed until next λ eriods. Constraint (7) ensures that when a PM operation is performed at the period t and no unforeseen failure occurs on the machine until the next λ periods, then a PM operation should be scheduled and performed at the period of $t + \lambda$. Constraint (8) checks that when a CM operation is performed, then a PM operation can be scheduled at the interval of λ periods or an unforeseen failure can be occurred until next λ periods. Equation (9) determines in which period a CM operation should be scheduled and performed to repair the occurred unforeseen maintenance. Moreover, this equation checks that CM operation should be scheduled in a way to assign a technician on the machine until maximum L period after the failure. Equation (10) calculates the waiting time until performing a CM operation in the case where an unforeseen failure occurs. Equation (11) ensures in case of unforeseen failure occurrence, the CM operation should be performed once. Constraint (12) guarantees that CM operation and PM operation cannot be scheduled and performed for the same period, simultaneously. Equations (13) and (14) determine that visiting the customer is related to a PM operation or a CM operation.

Equation (15) makes a connection between routing and maintenance variables. This equation checks when a PM/CM should be performed on the machine, a technician should be assigned to the machine.

Constraint (16) guarantees that only one technician should be assigned on the each machine at each period. Constraint (17) ensures that technician leave the current customer to the next one, after finish the PM/CM operation. Equation (18) determines the start time on the machine, which is calculated as the start time of the immediate previous customer, increased by the PM/CM operation time and the traveling time between the two customers. Equation (19) checks the time windows constraint and calculates the delay. Finally, (20) and (21) impose bounds on the variables.

3. Resolution Method

In this section we firstly introduce the instance generation method and solution procedure, briefly. Then, a numerical analysis is presented which derives managerial results.

Problem instances have been generated by a random generator. In this way, parameters of the problem are generated using random numbers by a discrete uniform distribution. Then, to solve the problem, we use the weighted sum method [7]. Under this method, the problem is solved by considering each objective function separately in both the maximization and the minimization for finding extreme points of each objective function. Then, a new single objective is considered that aims to minimize the weighted sum over the normalized and non-dimensional objective function.

In order to show the feasibility and applicability of proposed model, a small size problem is generated and it is solved based on generated instance problem. It is assumed that there are 6 customers ($m = 6$) where 3 periods are defined as duration of the interval between two consecutive PM operations ($\lambda = 3$) and 2 periods are considered as allowed duration to repair occurred unforeseen failure ($L = 2$) by using 3 types of technicians ($k = 3$) during 10 periods. To solve this problem, the “GAMS v22.2” optimization software using solver CPLEX v10.1 is used.

At the beginning, the problem is solved without considering the second objective. In this case the total cost is optimised. The results show that minimum value of total cost is 402 while waiting time in this situation is 14. In the next step, the first objective function is relaxed and model is solved by minimizing the second objective function. The obtained results shows when the minimum value of waiting time (second objective function) is 6, the value of total cost is 1,053. Table 1, shows the minimum and maximum value of objective functions.

Minimum value	**Maximum**	**Value**
First objective	402	1,053
Second objective	6	14

Table 1. Min. and Max. value of objective functions

The bi-objective model can be converted to a MILP model with one objective function using the equation (22).

$$f = \alpha \frac{f_1 - f_1^{\min}}{f_1^{\max} - f_1^{\min}} + \left(1 - \alpha\right) \frac{f_2 - f_2^{\min}}{f_2^{\max} - f_2^{\min}} \qquad (22)$$

In this equation, α presents the importance degree of each objective function and varies between 0 and 1.

Furthermore, the Objective Functions Value (OFV) by changing α value is introduced in Table 2.

According Table 2, the total cost from 1,053 to 402 causes increasing 57% in waiting time (from 6 to 14). It means the best value of waiting time can be reached by increasing 62% in total cost.

	α				
	0	**0.3**	**0.5**	**0.7**	**1**
First OFV	1,053	886	689	602	402
Second OFV	6	8	9	12	14
Run time (second)	157	166	147	133	158

Table 2. OFV against α

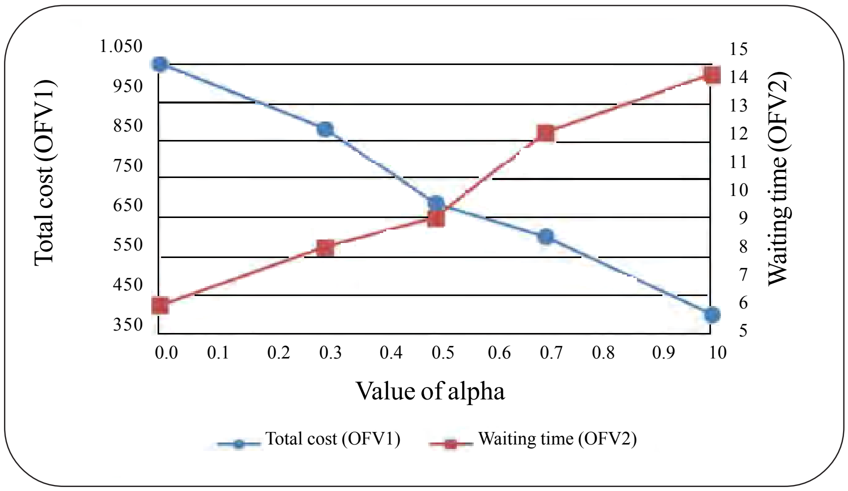

Figure 1. Variation of total cost against waiting time by changing value of α

The variation of objective functions value by changing of α value is presented in Figure 1. In this figure, X-axe shows value of total cost and waiting time while Y-axe presents different value of α. By this figure changes of total costs and waiting time is visualized against variation of α.

4. Conclusion

In this paper the integration of maintenance and routing problem is investigated by taking into account waiting time for performing a CM operation when unforeseen failure occurs. For this Purpose, a bi-objective mathematical model is proposed to find the optimized policy of maintenance and routing problems and make a trade-off between maintenance costs and service level which is measured by waiting time for performing a CM operation. In the proposed model the time windows is considered for starting maintenance operation on the machine by technicians. Moreover, the technician's skill regarding required time to perform a maintenance operation is considered. Our results for a small size instance show that to decrease by 57% of the waiting time, we have to increase the costs by 62%.

Our future research in this area includes the consideration of stochastic parameters and proposing an efficient solution approach.

5. Acknowledgments

This project is supported by The ELSAT2020 project which is cofinanced by the European Union with the European Regional Development Fund, the French state and the Hauts de France Region Council.

References

[1] Macedo, R., Benmansour, R., Artiba, A, Mladenovi, N., Uroševi, D. (2017). Scheduling preventive railway maintenance activi

ties with resource constraints, *Electronic Notes in Discrete Mathematics*, 58, p 215–222, (April).

[2] Blakeley, F., Argüello, B., Cao, B., Hall, W., Knolmajer, J. (2003). Optimizing Periodic Maintenance Operations for Schindler Elevator Corporation, Interfaces (Providence)., 33 (1), p. 67–79, (February).

[3] Kovacs, A. A., Parragh, S. N., Doerner, K. F., Hartl, R. F. (2012). Adaptive large neighborhood search for service technician routing and scheduling problems, *J. Sched.,* 15 (5), p 579–600, (October).

[4] Goel, A., Meisel, F. (2013). Workforce routing and scheduling for electricity network maintenance with downtime minimization, *European Journal of Operational Research*, 231 (1), p. 210–228, (November).

[5] Berrade, M. D., Scarf, P. A., Cavalcante, C. A. V., Dwight, R. A. (2013). Imperfect inspection and replacement of a system with a defective state: A cost and reliability analysis, *Reliability Engineering & System Safety*, 120, 80–87, (December).

[6] López-Santana, E., Akhavan-Tabatabaei, R., Dieulle, L., Labadie, N., Medaglia, A. L. (2016). On the combined maintenance and routing optimization problem, *Reliability Engineering & System Safety*, 145, p199–214, (January).

[7] Rahimi, M., Baboli, A., Rekik, Y. (2014). A bi-objective inventory routing problem by considering customer satisfaction level in context of perishable product, *In*: 2014 *IEEE Symposium on Computational Intelligence in Production and Logistics Systems* (CIPLS), 91– 97.