Automated Linear Voltage Regulator Design With a New Sliding Mode Controller Topology With Non-linear Disturbances

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ABSTRACT: Sliding mode controller may be used as a nonlinear controller that alters the dynamic behavior using an interrupting control signal to drive the system along a cross-section of the system’s normal behavior. In this paper, a new topology of sliding mode controller is designed for a linear voltage regulator operating under nonlinear conditions.

Control schematic diagrams and units developed in this paper for the gain and band width of the controller are obtained for voltage variation, model in accuracies and robustness in the load, to maintain a dynamic performance comparable to that obtained with current control techniques. The chattering effect of the controller is also taken into consideration.

The results and performance of the proposed controller is verified using automated software including Simplorer 7 and Matlab (Simulink).

Keywords: Linear Voltage Regulator, Sliding Mode Controller, Automated PID Controller, Chattering Effect, PWM

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1. Introduction

Sliding mode control is a special method of uninterrupted control that forces the system to follow a sliding surface after it is deviated from its normal behavior. To keep control on the nonlinear sliding mode controller, Lyaponov stability method is applied [1].

A DC to DC regulator converts directly from DC to DC and like the power transformer, it is used to step the voltage of the load down and at the same time to step up the current of the DC voltage source. Because of their ability to continuously produce a variable DC voltage, voltage regulator plays a big role in low and high power industrial and control technology [2].

A chopper down voltage regulator is a variable structure system (VSS) since it is commonly used in low power applications such as powering microprocessors and in communication systems such as for Ultra low power wireless applications. They are used to chopper down dc voltages and at the same time to step up the currents to levels required by microprocessors and communication systems. Other applications include Smart phones, tablets, digital cameras, navigation systems and digital medical equipment, in which different integrated circuits that typically require several independent supply voltages are integrated, each usually driven by different voltage than that supplied by the battery.
Sliding mode controller for Variable Structure Systems (VSS) utilizes the intrinsic variable structure nature of voltage regulators. Specifically, the regulator main switches are triggered depending on the momentary values of the state variables in such a way so as to force the system trajectory to move along a precise selected surface on the phase space known as the sliding surface which results in a very robust control system [3-5].

A new topology of sliding mode controller for a linear voltage regulator is addressed in this paper. It is shown that feasible results concerning the robustness of the load and supply voltage fluctuations can be obtained, while maintaining a dynamic response at least as compared to default current control techniques [6].

Certain drawbacks and sophistications, like current restrictions, constant switching frequency and load voltage steady-state error annulment are also discussed.

Consequently, the high harmonic content in the output voltage and the load current are reduced which results in little demands on the smoothing filter. The power produced at the output of the regulator is also increased which makes the control smooth and easy. It also increases the chopping frequency of the main switches of the regulator and by this way the size of the filters used at the input and at the output of the regulator becomes smaller and more economical [7-8].

2. Basic chopper down linear voltage regulator

The circuit diagram of the basic chopper down voltage regulator is shown in Figure 1. In this circuit $V_1$ is the voltage applied at the input of the regulator. This voltage is chopped down by a BJT transistor ($Q$) to produce a steady dc at the output with some ripple superimposed. The control circuit (not shown in the Figure) causes switch $Q$ to switch on and off at a pre-determined frequency.

During the on-state of $Q$, the source voltage $V_1$ is applied to the input of the $LC$ filters causing the inductor current $I_1$ to increase. During the off-state of $Q$, the energy stored in the inductor $L$ maintains current flow to the load circulating through the freewheel diode $D$ and the input of the $LC$ filter is maintained at a low negative voltage about 1 volt (drop of diode $D$). Current $I_1$ decreases to its original steady state value and the cycle repeats [9-10]. The voltage and current simulation waveforms at various points of the filter are shown in Figure 2 [9-11].

![Figure 1. Basic chopper down voltage regulator](image)

Mathematically, the output voltage across the filter is assumed to be $V_1$ during $t_{on}$ and zero during $t_{off}$. The voltage across the inductor $L$ is in general $v_L = L \frac{di}{dt}$. With reference to Figure 2, the inductor current rises exponentially from its minima to its
maxima in time $t_{on}$. Hence:

$$V_i = R_1 i_1 + L_1 \frac{di_1}{dt} + v_o \Rightarrow L_1 \frac{di_1}{dt} = V_i - R_1 i_1 - v_o$$

$$i_1 = i_o + i_c + i_v + \frac{C}{\frac{dv}{dt}}$$

$$v_o = L_o \frac{di}{dt} + R_o \frac{dv}{dt} = L_o \frac{di_1}{dt} - L_o C \frac{d^2 v}{dt^2} + R_o i_1 - R_o C \frac{dv}{dt}$$

From which:

$$L_1 \frac{di_1}{dt} = V_i - (R_1 + R_o) i_1 - L_o \frac{di_1}{dt} + R_o C \frac{dv}{dt} + L_o C \frac{d^2 v}{dt^2}$$

Neglecting the load inductance, $L_o$, replacing $R_o + R$ by $R_e$ and rearranging Equations 2, yields:

$$\frac{di}{dt} = \frac{1}{L_1} v_i - \frac{1}{L_1} \left( \frac{R_o R_e}{R_e} \right) i_1 - \frac{1}{L_1} \frac{R_o}{R_e} v_o$$

$$\frac{dv}{dt} = \frac{R_o R_e}{R_e C} \frac{1}{R_o} \frac{1}{v}$$

Again, during the off period of the main switch, $Q$, the diode voltage is zero, current $i_1$ flows through $L_1$, $C$ and $D$, thus maintaining a direct current through the inductance while keeping the voltage across $C$ constant. The inductor current falls exponentially from its maximum to its minimum in time $t_{off}$, and therefore:

$$0 = R_1 i_1 + L_1 \frac{di_1}{dt} + v_o \Rightarrow \frac{di_1}{dt} = -\frac{R_1}{L_1} i_1 - \frac{1}{L_1} v_o$$

$$C \frac{dv}{dt} = i_1 - \frac{v_o}{R_o}$$

$$v_o = R_o i + v_c = R_x \frac{dv}{dt}$$

Solving, one gets:

$$\frac{di_1}{dt} = -\frac{1}{L_1} \left( \frac{R_o R_e}{R_e} \right) i_1 - \frac{1}{L_1} R_o v_o$$

$$\frac{dv}{dt} = \frac{1}{C} \frac{1}{R_e} i_1 - \frac{1}{CR_e} v_o$$

The above relations are rather important for designing a chopper down voltage regulator. The significance of these relations is explained in later sections but it is readily seen that the current ripple at the load is minimized by inductor $L$ and therefore, the input current may be uninterrupted for very low values of this inductance. Consequently input filter is necessary to reduce output ripple. The output voltage and current is unidirectional [12-15].

Ripple voltage can be set at any arbitrary value by proper design of the filter components.

**Output voltage $v_o$ can be regulated by:**

- Varying $t_{on}$ respective $t_{off}$, and keeping the total time period $T$ constant
• Keeping $t_{off}$ constant and varying the frequency
• Or altering both

Generally, the switching frequency is kept constant and $t_{on}$ is varied to regulate energy during $T$ in each cycle. The value of $C$ must be chosen so that the reactance $X_C$ is several times smaller than the load resistance $R_o$ at the switching frequency.
Consequently, the energy accumulated in the capacitor $C$ is larger than that stored in $L$. If the time constant $(R_1 + R) C$ is very large compared to the total time period $T$, the voltage at the output $v_o$ is considered to be fixed [15].

The current simulation waveforms through $Q$, $L_1$ and $D$ are shown in Figure 2. The average current through $L_1$ is constant and is equal to the current through the load, but the current in $L_1$ ramps upwards linearly during $t_{on}$ and ramps down linearly during $t_{off}$ when the current flows through $D$.

### 3. State space model basic structure

The voltage regulator usually behaves as a nonlinear system while it is in the on regime of its main switch, and it behaves as another nonlinear system while its main switch is off and finally as yet a new nonlinear system while the current through its inductor passes through zero [4].

Therefore, to build the state space model for such type of systems, the energy storage elements represented by the inductor currents and the capacitor voltages are used as physical state variables to form space vector equations while the regulator is driven by a dc voltage source that forms its input state space vector.

Cuk state space averaging model is used to convert nonlinear system into a linear time invariant system using the state space model of each nonlinear system individually as a starting point. Three models may then be obtained whereas the model averaged from these models during the switching period of the regulator is derived. This new averaged model is linear, time invariant with a new duty cycle [16]. Thus,

\[
\begin{bmatrix}
\frac{di_1}{dt} \\
\frac{dv_c}{dt}
\end{bmatrix} = \begin{bmatrix}
\frac{1}{L_1} \left( R_1 + \frac{R_o R_c}{R_e} \right) - \frac{1}{L_1 R_e} \\
\frac{1}{C} - \frac{1}{C R_e}
\end{bmatrix} \begin{bmatrix}
i_1 \\
v_c
\end{bmatrix} + \begin{bmatrix}
\frac{1}{L_1} \left( \frac{R R_c}{R_e L_1} \right) \\
0 - \frac{R_e}{R_c}
\end{bmatrix} \begin{bmatrix}
i_g \\
v_1
\end{bmatrix}
\]

\[
v_o = \frac{R_1 R_c}{R_c} \frac{R_o}{R_e} \begin{bmatrix}
i_1 \\
v_c
\end{bmatrix} + \begin{bmatrix}
0 - \frac{R_o R_c}{R_c} \\
0 \frac{R_e}{R_c}
\end{bmatrix} \begin{bmatrix}
i_g \\
v_1
\end{bmatrix}
\]

Figure 2. Voltage and current simulation waveforms at various points of chopper down regulator
Where $i_g$ shown in Figure 3 is a dc current injected at the output of the regulator as a dc control signal required to determine the output impedance of the regulator. So, the specific model for the regulator may be achieved by setting all values of $V_1$ in Equation 6 to zero, therefore:

\[
\begin{bmatrix}
\frac{di_1}{dt} \\
\frac{dv_c}{dt}
\end{bmatrix} = 
\begin{bmatrix}
-\frac{1}{L_1} \left( R_{1_e} + \frac{RR_c}{R_e} \right) & -\frac{R}{R+R_cL} \\
-\frac{R_c}{(R+R_cC)} & -\frac{1}{(R+R_cC)}
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2
\end{bmatrix} + 
\begin{bmatrix}
\frac{1}{L_1} \left( \frac{RR_c}{R+R_cL} \right) \\
0 - \frac{R_c}{(R+R_cC)}
\end{bmatrix}
\begin{bmatrix}
V_1 \\
i_g
\end{bmatrix}
\] (7)

\[
V_o = \frac{RR_c}{R+R_c} \frac{R}{R+R_c} \begin{bmatrix}
-x_1 \\
x_2
\end{bmatrix} + \begin{bmatrix}
0 \\
0
\end{bmatrix}
\begin{bmatrix}
V_1 \\
i_g
\end{bmatrix}
\] (8)

\[
\begin{bmatrix}
\frac{dx}{dt} \\
\frac{dy}{dt}
\end{bmatrix} = 
\begin{bmatrix}
A_1 & B_1 \\
C_1 & E_1
\end{bmatrix}
\begin{bmatrix}
x \\
y
\end{bmatrix}
\] (9)

3.1 Cuk model is a state space averaging model

It describes the regulator behavior under quasi-stationary conditions. As it has been said, the regulator behaves as one nonlinear time variant system when its main switch is on, and as another completely different nonlinear time variant system when it is off [16].

Theoretically, the nonlinear system represented by a set of first-order differential equations is converted into a linear one by transforming its mathematical model into a set of state space equations [10]. Thus, by using Equation 3 and Equation 5 the following state space model may be obtained for the on-state during which the main switch is on:

\[
\frac{dX}{dt} = A_1 X + B_1 U
\]

\[
Y = C_1 X + E_1 U
\]

Where $X$ is the state space vector of all state variables of the voltage regulator described above. The off-state, during which the regulator main switch $Q$ is off, is described as follows:

\[
\frac{dX}{dt} = A_2 X + B_2 U
\]

\[
Y = C_2 X + E_2 U
\]

Equations 8 and 9 can be converted into a consolidated form using matrix symbolic representation. For this system, physical variables that represent the circuit’s energy storage elements were chosen as the two state variables.

The substitution of Equations 8 and 9 into Equation 6 and Equation 7 yields to Equation 10 and Equations 11, respectively:

\[
\begin{bmatrix}
\frac{dx_1}{dt} \\
\frac{dx_2}{dt}
\end{bmatrix} = 
\begin{bmatrix}
-\frac{1}{L_1} \left( R_{1_e} + \frac{RR_c}{R_e} \right) & -\frac{R}{R+R_cL} \\
\frac{1}{C} - \frac{R_c}{CR_e} & -\frac{1}{CR_e}
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2
\end{bmatrix} + 
\begin{bmatrix}
\frac{1}{L_1} \left( \frac{RR_c}{R+R_cL} \right) \\
0 - \frac{R_c}{R_e C}
\end{bmatrix}
\begin{bmatrix}
V_1 \\
i_g
\end{bmatrix}
\] (10)

\[
V_o = \frac{RR_c}{R+R_c} \frac{R}{R+R_c} \begin{bmatrix}
-x_1 \\
x_2
\end{bmatrix} + \begin{bmatrix}
0 \\
0
\end{bmatrix}
\begin{bmatrix}
V_1 \\
i_g
\end{bmatrix}
\] (11)
Figure 3. Voltage regulator operating in on and off regime topology

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2
\end{bmatrix} = \begin{bmatrix}
-\frac{1}{L_1} \left( R_1 + \frac{R_1 R_0}{R_e} \right) - \frac{1}{L_1} \frac{R_1}{R_e} \\
\frac{1}{C} - \frac{R_0}{C R_e} - \frac{1}{C R_e}
\end{bmatrix} \begin{bmatrix}
x_1 \\
x_2
\end{bmatrix} + \begin{bmatrix}
0 \\
0
\end{bmatrix} \frac{R_0 R_e}{R_e L_1} + \begin{bmatrix}
0 \\
0
\end{bmatrix} \frac{R_0 R_e}{R_e C}
\]

\[ (11) \]

\[
v_o = \begin{bmatrix}
\frac{R_o}{R_e} \\
\frac{R_o}{R_e} \\
\end{bmatrix} \begin{bmatrix}
x_1 \\
x_2
\end{bmatrix} + \begin{bmatrix}
0 \\
0
\end{bmatrix} \frac{R_o R_e}{R_e}
\]

cuk state space model states that if a low frequency unidirectional non-harmonic signal exists in the dynamic system, the quasi-stationary behavior of such system may be obtained by using a single, weighted sum of both modes:

\[
\frac{dX}{dt} = A_2 X + B_2 U + (A_1 - A_2)XD + (B_1 - B_2) UD
\]

\[ (12) \]

\[
Y = C_2 X + E_2 U + (C_1 - C_2) XD + (E_1 - E_2) UD
\]
Equation 12 describes the time varying system, where the duty ratio $D$ is another input signal characterized as an input vector. In general, the matrices $A_1$, $A_2$, $B_1$, $B_2$, $C_1$, $C_2$, $E_1$ and $E_2$ are defined as follows:

\[
A_1 = \begin{bmatrix}
\left(-\frac{R_1 + \frac{R}{R_c}}{R_c}\right) & -\frac{R}{R_c} \\
\frac{R_c}{R_c} & -\frac{1}{R_c}
\end{bmatrix}, \quad B_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}
\]

\[
C_1 = \begin{bmatrix}
\frac{-R}{R_c} & \frac{R}{R_c} \\
1 & 0
\end{bmatrix}, \quad E_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}
\]

\[
A_2 = \begin{bmatrix}
\left(-\frac{R_1 + \frac{R}{R_c}}{R_c}\right) & -\frac{R}{R_c} \\
\frac{R_c}{R_c} & -\frac{1}{R_c}
\end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}
\]

\[
C_2 = \begin{bmatrix}
\frac{-R}{R_c} & \frac{R}{R_c} \\
0 & 0
\end{bmatrix}, \quad E_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}
\]

Under steady state conditions and with a small ac variation in the signals of the system, the state space variables and the operating point of the system may be divided into two components, dc and ac [1, 16]. The two components of the system, dc and ac, may be separated and the derivative of the dc component is zero, therefore:

\[
0 = AX + BU
\]

\[
Y = CX + EU
\]

Thus

\[
X = -A^{-1} BU
\]

\[
Y = (-CA^{-1}B + E)U
\]

Common state variables in switching-mode regulating system are the current through inductor, $L_1$ versus the voltage across the capacitor, $C$.

4. Design of a sliding mode controller

It has been said that sliding mode controller alters the dynamic behavior of a linear system using an interrupting control signal to drive the trajectories of the system along a chopper down surface and maintain it on this surface once it has been reached. The local attractivity of this controller is to introduce Lyaponov stability method discussed in this section [1]:

Having a non-linear system exploiting the inherent nonlinear structure nature of a chopper down voltage regulator described by the following equations:

\[
k\dot{x} + x = f(x) = y
\]

It is better to express the states of the system in terms of the actuating error signal $v_e$ and its derivative with respect to time
\[ K v_e + \frac{d v_e}{dt} = k x + \dot{x} = 0 \]  

(18)

rather than the output load current and voltage. Hence:

Where \( v_e = v_c - v_{\text{ref}} \), \( v_c = v \) is the capacitive voltage which is equal to the regulator output voltage, \( v_o \), under the condition of neglected capacitive resistance, \( R \) and \( v_{\text{ref}} \) is the reference voltage. And let the quasi-stationary state of this nonlinear system occurs at the equilibrium point \( f(x_{eq}) = 0 \), where \( f \) is a differentiable function at least in the region \( H \) in the neighborhood of \( x_{eq} \).

The main target required for designing the sliding mode controller is to find \( f(x) \), a function in \( x \), which satisfies the following items:

* \( f(x) \) is continuously defined for all \( x \in D \).
* \( f(x) \) has its minimum value at \( x_{eq} \).
* \( f(x) \) does not dramatically change its value and stays within the given limit in \( H \).

Hence, to achieve the equilibrium state of the system at the origin, it is required to introduce a sliding surface as a switching trajectory for the system as shown in Figure 4 and a lyapunov function as follows:

\[ \sigma(x) = a_1 x_1 + a_2 x_2 = P^T x \]

(19)

\[ f(x) = \frac{\sigma^2(x)}{2} \]

Where \( P^T = (\alpha_1, \alpha_2) \) is a vector of sliding surface coefficients \( \alpha_1 \) and \( \alpha_2 \). This function is obviously positive everywhere except at the origin.

That is, in steady state the difference between the voltage at the output and the desired voltage should be:

\[ x_1 = v_e \]

(20)

\[ x_2 = \dot{x}_1 = k \left( \frac{v_o}{L_1} - \frac{\int \dot{V}_1 - \frac{v_o}{L_1} dt}{L_1} \right) \]

Figure 4. Phase plane trajectory of a voltage regulator
Where $\dot{u}$ is a varying input gain having a value of 1 or 0 in accordance with the operating mode of the regulator. For the voltage regulator, sliding mode control is attained by triggering the main switch on when $f(x) < 0$ (below the sliding line) and off when $f(x) > 0$ (above the sliding line), as shown in Figure 5.

Thus, it is good to define a control law that adopts a switching function such as $\dot{u} = 0$ when $\sigma(x) > 0$ and $\dot{u} = 1$ when $\sigma(x) < 0$:

$$\dot{u} = \frac{1}{2(1+\text{sign } \sigma)}$$

or

$$\dot{u} = -3x_1 - 1.4 \text{sign } \sigma$$

(21)

As soon as the system hits the sliding surface, then $\sigma(x) = 0$, the dynamic behavior of the system is represented as follows:

$$\sigma(x) = a_1 x_1 + a_2 x_2 = a_1 \dot{x}_2 + a_2 x_1 = 0$$

(22)

From which one can get:

$$x_2(t) = e^{-\frac{a_2}{a_1}} x_2(0)$$

$$x_1(t) = -\frac{a_2}{a_1} e^{-\frac{a_2}{a_1}} x_2(0)$$

(23)

It is evident from Equation 23 that this system obeys Routh’s stability criterion and as long as its sliding coefficients have the same sign, it will go exponentially to zero value, $x(0)$.

Concerning the voltage regulator, the state variables are chosen to be the current through inductor $L_1$ and the capacitive voltage. De facto, the control law defined here ensures that the sliding surface and the state plane of the corresponding substructures are directed in the orientation of the sliding line within the given limit in $H$ [16].

Figure 5 shows that the perfect representation and structure of the state plane with the sliding surface of the voltage regulator has a physical limitation due to the commutation properties of the freewheel diode in the regulator. Consequently, when the switch is off, the energy stored in the inductor forces the current to flow in one direction with a nonnegative value.

Therefore, when the inductor current $i_1$ falls down to zero value, it may become interrupted if the charging and discharging cycle of the capacitor does not repeat itself again and again. This situation corresponds to the interruption mode of the voltage regulator and introduces a certain limitation to the definition, selection and thus the convergence region of state variable. The limits of this region may be derived as follows:
Figure 6. Closed loop implementation of PID controller for voltage regulator

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Figure 7a. Simulation waveforms of Voltage regulator signals with PID controller and purely resistive load
Equation 24 represents a straight line with negative slope equal to \(-1/RC\) and crossing point \((-V_{\text{ref}}, 0)\), as it is illustrated in Figure 5.

5. Simulation of a voltage regulator with PID controller

In this section a simulation of PID control method used to control switch mode voltage regulator is given, and the reasons for preferring the sliding mode control (SMC) to be used in this paper is explained. There are many control methods to control
voltage regulators. A good understanding of the P, I, and D control actions, in both analytical and physical terms, is therefore important to know how a dynamic compensation can change the system performance.

Derivative control action, when used with a proportional controller which produces an offset to a system without an integrator in its transfer function, introduces a controller with high sensitivity. The derivative component of the PID controller has an effect on the regulator performance at high frequencies and is used to improve the phase margin since it reacts to the rate of variation in the following error signal and it reduces the magnitude of this error before it reaches high values [11]. $v_{ref}$

The regulator performance can be further improved by adding an integral component which helps removing the offset or steady state error by producing a high gain at low frequency [1, 11].

Figure 6 shows a simple schematic diagram for a general voltage regulator topology with implemented PID controller. A gate drive circuit (driver) is used to provide isolation for the transistor switch.
The pulse width modulation is given as a comparison between a high frequency saw tooth signal and the error signal of the system. The saw tooth signal is increasing linearly from 0 to 1 with the error signal with a period of $T$. When it is greater than the error signal, a control signal is generated to turn the main switch on, otherwise, it is off, and the diode is on. A generator is used to generate the reset signal. The output signal of this generator is equal to 1 if its input signal is positive and equal to 0 if its input signal is negative [10-12].

To prove the originality of previously mentioned analytical procedure, a numerical example is given. In this example, the following parameters of the voltage regulator are used: $L_1 = 10\, mH$, $R_1 = 0.05\, \Omega$, $R = 0.1\, \Omega$, $C = 100\, \mu F$ with a purely resistive load, $R_o = 2\, \Omega$ or with a highly inductive load, $R_o = 2\, \Omega$, $L_o = 2\, mH$. The switching frequency, $f$, is 200Hz and the supply voltage, $V_1$, is 25v.

Figure 7a respective 7b shows the simulation waveforms of the circuit in the continuous mode of operation with purely resistive respective with highly inductive load.
Figure 8a shows a bode plot of a Voltage regulator with a unity gain feedback, while Figure 8b shows the regulator response when a PID compensator is added. The phase margin ($\phi_m$) has improved and the gain is higher with the PID compensator.

Figure 9 shows the dynamic performance of the regulator and response of its parameters to load variations. This illustration shows that the transient response to the load ruthless variations is less than 1ns. The overshoot is less than 5% and voltage at the output ripple is maintained less than 2%. From this Figure, it is confirmed that the voltage at the output has fast response to load variations.

Figure 10. The above plots illustrate the behavior of the regulator under zero initial conditions of its voltage at the output and current

c) Capacitive voltage and output current simulation waveforms

Figure 8a shows a bode plot of a Voltage regulator with a unity gain feedback, while Figure 8b shows the regulator response when a PID compensator is added. The phase margin ($\phi_m$) has improved and the gain is higher with the PID compensator.

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a) Phase plane trajectory states $x_1, x_2$

b) Capacitive voltage at the output and output current simulation waveforms

c) Phase plane of the voltage regulator

Figure 11. The above plots illustrate the behavior of the regulator for an initial condition of 100V for the capacitive voltage and zero initial value for the output current.

6. Simulation of a voltage regulator with a sliding mode controller

Sliding mode Control is now examined and verified using MATLAB and SIMPLORER environments. Despite the fact that the state plane trajectory started at a very large deviation from the steady state value (110, 110), Figure 10a shows that it smoothly moved toward the steady state equilibrium of the voltage regulator represented through the switching surface $x_1 + x_2 = 0$ until it reaches this equilibrium at $x = 0$. It is evident that the system reaches its quasi-stationary state after 30ms. Figure 10b, c show
Figure 12. The above plots illustrate the behavior of the regulator for the following initial conditions: \(-50\) A for the output current and zero initial value for the capacitive voltage

the simulation waveforms of the regulator signals (control law, sliding surface, voltage at the output and output current) at zero initial conditions which are obtained for the above mentioned control law.

However, the control law considered in this here has the drawback that the control signal causes a fluctuation in the system response. Thus, the switching between small positive and negative values in the switching surface due to small fluctuation in \(x_1\) and \(x_2\) causes the control signal to fluctuate along the envelope of the signal.
Figure 13. The above plots illustrate the behavior of the regulator for the following initial conditions: -50A for the output current and -100V for the capacitive voltage.

Figures 11a, b show the simulation done to the same controller with different initial conditions for the capacitive voltage and inductor current. As expected, regardless of the initial conditions of these quantities, the controller was able to force the response states, $x_1$ and $x_2$ to behave in the same manner as previously, and thus to force the state $x$ to be zero after some time.
Figure 14. Phase plane trajectory states $x_1, x_2$ with the simulation waveforms of the capacitive voltage at the output and output current, phase plane of the voltage regulator, control signal and sliding surface for a sinusoidal switching function of frequency 50Hz.
Figure 15. Phase plane trajectory states $x_1, x_2$ with the simulation waveforms of the capacitive voltage at the output and output current, phase plane of the voltage regulator, control signal and sliding surface for a sinusoidal switching function of frequency 2000Hz.
Although the capacitive voltage initial value was chosen to be -100v which is equal in magnitude but opposite in value to the supply voltage, it does not take a long time for the system and its output response to settle down and to move again along the phase plane to reach the sliding surface as it is required and as it is illustrated in Figure 11c.

The same stable performance is obtained even when the initial value of the inductor current is chosen to be twice its final value but negative as shown in Figure 12 or when these above mentioned initial conditions for the capacitive voltage and inductor current are considered simultaneously as shown in Figure 13.

In order to eliminate the fluctuation in the control signal shown above in the previous plots, the sign function in the control law is replaced simply by a sinusoidal function and its frequency is varied while the amplitude of it is kept constant at 1.4 just to see the cause and the effect on the dynamic performance of the regulator. Therefore, the frequency is changed from low values to high values and the plant is simulated and the phase plane trajectory and control signal are plotted as shown in Figures 14 and 15. These Figures show that the response of the system takes longer time to reach the equilibrium state and the control signal chattering and fluctuating problem increased with low frequency but as the frequency is increased to high values, it decreases significantly as compared to the previous cases.

7. Conclusion

A chopper down regulator with sliding mode control is simulated in this study. The simulation results show the propriety of the chopper down regulator model with sliding controller and the robustness of this system performance against variations in the load or input supply. Therefore, the system achieves a sturdy and robust voltage at the output beside the disturbances at the load and against the source voltage variations just to guarantee that the load is supplied with full stability. Robustness and good performance of the system is achieved even for large and fast supply and load variations.

The nonlinear structure of the chopper down regulator has been used for a simplified control scheme that ensures robustness, full stability of system parameters and signals and has the ability to provide the required performance by varying the sliding line/surface in the state space accordingly. Sliding mode control of different methods and their respective scores when applied to a voltage regulator have been verified through simulation.

In this paper, the innovation is that small signal variation has been designedly avoided resulting in the handling with the voltage regulator as a time variant system. A PID switching surface can be used to avoid the steady state error that is paid in favor of using sliding mode control.

Similarly, any type of voltage regulators such as the chopper down demonstrated in this paper could be used as a discrete time system having a switching duty cycle which is kept constant over a switching period. In such a case, a discrete sliding surface/line is designed and Lyapunov conditions will be different from those adopted here for the continuous time systems. Using the simulation, it is concluded that replacing the sign function to the sat function in the control law could minimize the chattering drawbacks of the control signal although the chattering is not completely eliminated.

References


