Light Robust Edge Preserving Smoothing using Self-quotient Referential ε-Filter

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ABSTRACT: *In this paper, we propose a light robust edge preserving smoothing filter named self-quotient referential ε-filter. Although many filters for edge preserving smoothing have been proposed, as it often uses local information, it is difficult to preserve edge information when the images are taken under light variation. To solve the problems, we pay attention to the feature of self-quotient filter, and combine self-quotient filter and ε-filter. Some experiments were done to show the effectiveness of the proposed method. Throughout the experiments, the validity of our method was verified.*

Keywords: Nonlinear Filter, Edge Preserving Smoothing, Face Beautification, ε-filter, Self-quotient Filter

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1. Introduction

Keeping our body young is our general desires in our life. Some people do everything in power to stay looking young. However, physical decay is unstoppable as the months passed. Wrinkles and age spots appearing on face of aged persons make them feel a bygone year, and may decrease their motivation to communicate with other people. As many people try to keep skin looking youthful, if we can reduce such wrinkle and age spots by face beautification system, we will have many applications for elderly people communication. For instance, users can take younger-looking photographs. They also can talk with each other through television-phone system with face beautification system. Aged persons can have a lot of motivation to communicate with other people if they can use such types of systems. Youthful face helps them to keep their young minds.

As wrinkles and age spots on our face are basically small cracks and noise on face, smoothing with filtering is a solution to realize face beautification.

Filtering is one of fundamental functions in image processing. For example, Gaussian filter computes a weighted average of pixel values in the neighborhood, in which, the weights decrease with distance from the neighborhood center. Although it can smooth the image noise including wrinkles and age spots on our face, edge is also blurred.

To handle these types of problems, there are many studies on edge-preserving smoothing [1]-[3]. Among them, ε-filter is a promising approach for face beautification among them because of its simple algorithm [4][5].

ε-filter does not require the signal and noise models in advance. As the calculation time of ε-filter is small, it has high affinity with real-time face beautification system. Implementation of ε-filter is also easy.

In the past, Arakawa et.al showed an extended ε-filter and its application to face beautification system [6]. However, as ε-filter including the extended version cannot handle photographs including light variation, the applications are limited.

This is because ε-filter utilizes local intensity to smooth the image, while preserving edge information. When photographs include light variation, they have not only high contrast area but also low contrast area. Therefore, ε-filter cannot handle low contrast area in the image and the area is blurred as the results. The same problems occur when we employ other nonlinear filters that utilize local intensity information. For example, bilateral filter is a typical example [7].

To solve the problems, Eisemann et.al., have proposed a method combining a flash image and a non-flash image. Their method refers to a flash image, and smooths the non-flash image based on the flash image information. Although it can handle photographs with light variation, it is necessary to prepare both a non-flash image and a flash image taken at the same location [8].

In our method, we pay attention to self-quotient filter. Self-quotient filter (SQF) aims to differentiate between extrinsic and intrinsic factors in imaging process, and extract the intrinsic factors from an image [9][10].

Self-quotient filter is an expanded filter of quotient image filter [11][12]. It related to some other studies such as spherical harmonic subspace [13]-[17] and image ratio [18]. By utilizing SQF output as the reference in ε-filter, we can distinguish the edge from the other parts not only in high contrast area but also in low contrast area, and therefore can preserve edge information not only in high contrast area but also in low contrast area. It is also not necessary to prepare another reference image. As the results, we can reduce wrinkles and age spots while preserving edge information even when the photographs with light variation.

The rests of this paper are organized as follows. In the next section, we describe the algorithm of the conventional ε-filter and summarize its problems. The algorithm and the features of the proposed method are also described in the next section.

In Sec. 3, experimental results are shown to clarify the effectiveness of our method. Calculation cost compared to other methods is also estimated. In sec.4, we give discussion and conclusions to summarize our method.

2. Proposed method

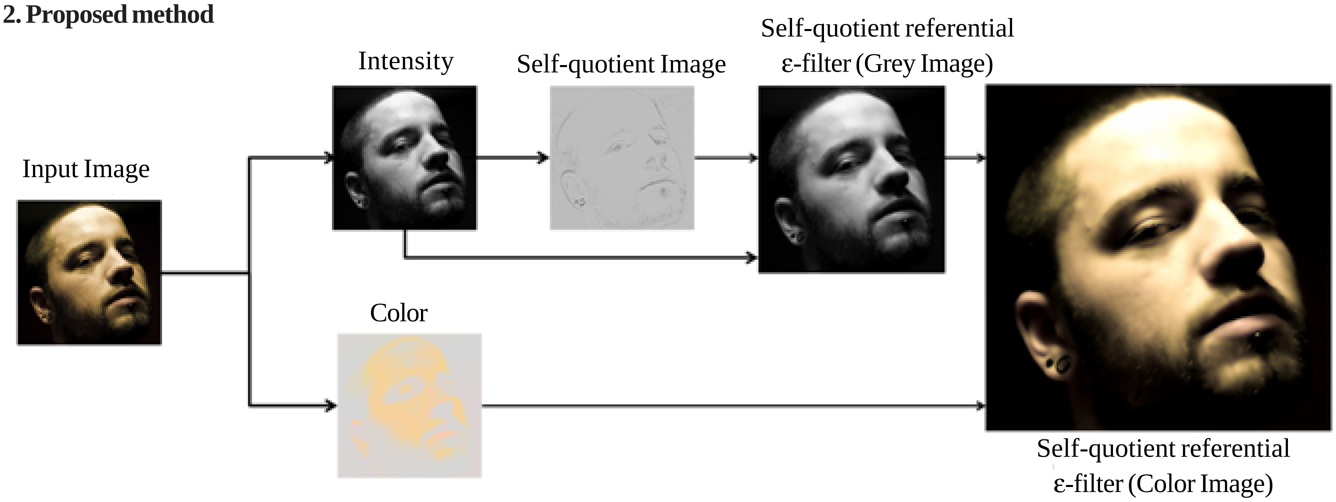


Figure 1. Process of the proposed method

Let us consider a color image $x(i_1, i_2) = (x_Y(i_1, i_2), x_{Cb}(i_1, i_2), x_{Cr}(i_1, i_2))$ in *YCrCb* space. $x_Y(i_1, i_2)$ is the intensity of the image. $x_{Cb}(i_1, i_2)$ and $x_{Cr}(i_1, i_2)$ are the values of *Cb* color space and *Cr* color space at position (i_1, i_2), respectively.

Figure 1 shows the procedure of our approach. As shown in Figure 1, the intensity and color components are firstly separated in our method. We then apply ε-filter to $x_Y(i_1, i_2)$, and the filter output $y_Y(i_1, i_2)$ is obtained as follows:

$$y_Y(i_1, i_2) = x_Y(i_1, i_2) + \sum_{j_1=-M}^{M} \sum_{j_2=-N}^{N} a(j_1, j_2) F(x_Y(i_1+j_1, i_2+j_2) - x_Y(i_1, j_2)) \quad (1)$$

In Equation (1), $a\,(j_1, j_2)$ is the filter coefficient, and is constrained as follows:

$$\sum_{j_1=-M}^{M} \sum_{j_2=-N}^{N} a\,(j_1, j_2) = 1 \tag{2}$$

Equation (2) is to normalize the filter output and to ensure that direct current signals are preserved when it is filtered by ε-filter. The window size of ε-filter is $(2M+1)(2N+1)$. $F(.)$ in Equation (1) is a nonlinear function, and its absolute value is constrained as follows:

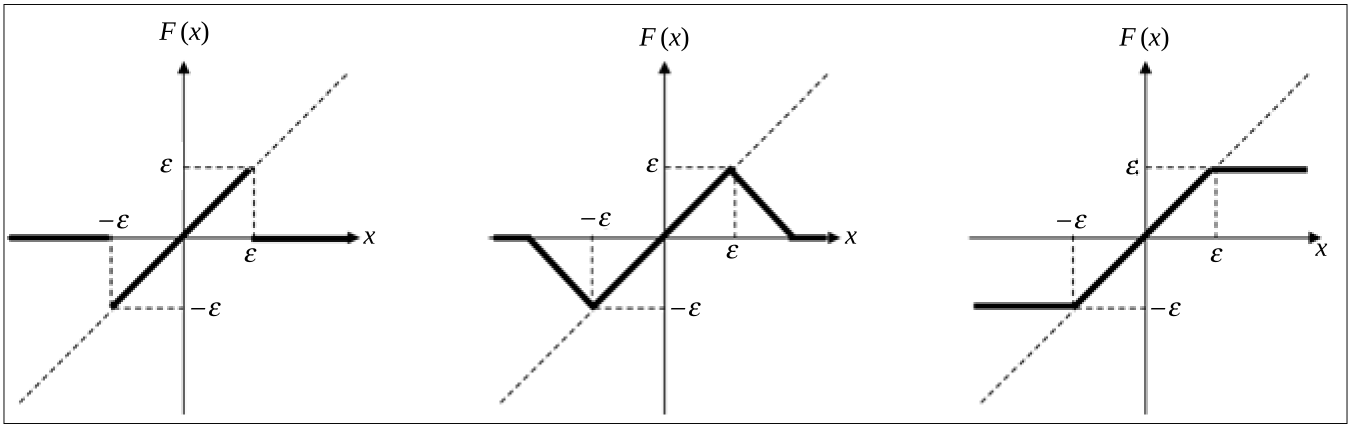


Figure 2. Examples of nonlinear function $F(x)$

(a) Original image

(b) Filter output of ε-filter

Figure 3. Examples that ε-filter does not work

$$|F\,(x)| \le \varepsilon : -\infty \le x \le \infty \tag{3}$$

where ε is the constant number.

Figure 2 shows the examples of nonlinear function $F(.)$. Although we can set various types of $F(.)$, the effects are similar. ε-filter requires only switching and linear operation especially when we set the nonlinear function $F(.)$ as the left one in Figure 2. Due to its feature, the calculation cost of ε-filter is small compared to bilateral filter.

To clarify this point, the difference between the input signal and the output signal is shown as follows:

$$\begin{aligned} &|\,y_Y\,(i_1, i_2) - x_Y\,(i_1, i_2)| \\ &= \sum_{j_1=-M}^{M} \sum_{j_2=-N}^{N} a(j_1, j_2)\, F\,(x_Y\,(i_1+j_1, i_2+j_2) - x_Y\,(i_1, j_2)) \\ &\le \sum_{j_1=-M}^{M} \sum_{j_2=-N}^{N} |\,a(j_1, j_2)\,|\,\varepsilon \le \varepsilon \end{aligned} \tag{4}$$

Therefore, ε-filter smooths small amplitude signal less than ε value, while large amplitude signal are preserved.

When ε value is set to a large value, ε-filter works as a simple linear low-pass filter. As shown in Equation (1), ε-filter uses the difference between the intensity of the processing pixel and that of the neighboring pixels to preserve the edge information. Therefore, ε-filer works well in high contrast area, while the edge is blurred in low contrast area.

To clarify this point, some examples of face images and the close-up image around right eye are shown to clarify the features. Figures 3 (a) and 3 (b) are the input image and the filter output of ε-filter, respectively. Figures 4 (a) and 4 (b) are the close-up image of the input image and the filter output of ε-filter, respectively.

a) Input image (b) ε-filter

Figure 4. Examples that ε-filter does not work. (Close up image around right eye)

Note that we lightened the image after filtering to clarify the problem. As shown in Figures 3 and 4, ε-filter can smooth the image while preserving the edge in high contrast area. (For example, ε-filter can preserve the edge around the left cheek.) However, it cannot preserve the edge in low contrast area. (For example, the image is blurred around a double eyelid in the filter output of the ε-filter.) This type of problem occurs not only in ε-filter but also when we employ the filter that uses the information of local intensity to preserve edge information. For instance, bilateral filter is a typical example. We aim to solve these types of problems.

A solution to solve this problem is to separate the filtered signal and the reference image, e.g., to use a flash image as reference to process the non-flash image.

When we take photographs without flash under dark condition, the contrast of the photograph becomes very low, and it is difficult to apply the filter that uses the information of local intensity like ε-filter and bilateral filter to these types of images. On the other hand, when we take photographs with flash, although the color distribution is not natural, the contrast of the photograph becomes high. Eisemann et.al., look to these features and uses the intensity of the flash image as reference for bilateral filter.

However, we do not have such referential images in usual cases. In normal case, as we only have an image, it is necessary to create the reference image from the image itself.

To create the referential image from an image taken under varying light condition, we apply self-quotient filter to $x_Y\,(i_1, i_2)$.

In self-quotient filter, Lambertian model is assumed regarding the object. Under the above assumption, $x_Y(i_1, i_2)$ can be described as follows:

$$x_Y(i_1, i_2) = \rho\,(i_1, i_2)\; l\,(i_1, i_2)^T s \tag{5}$$

where $\rho\,(i_1, i_2)$, $l\,(i_1, i_2)$ and s represent the surface reflectance in gray scale, the surface normal direction and the light source direction, respectively. T expresses the transpose.

$\rho\,(i_1, i_2)$ is constrained as follows:

$$0 \le \rho\,(i_1, i_2) \le 1 \tag{6}$$

The aim of self-quotient filter is to obtain the intrinsic property from an image including intrinsic property and the extrinsic factor. Of course, it is impossible to obtain $\rho\,(i_1, i_2)$ and $l\,(i_1, i_2)$ from a single image analytically. In self-quotient filter, we assume that the illumination of the original image and that of the smoothed image are approximately same. Under the above assumption, we can describe self-quotient filter as follows:

$$y_Y(i_1, i_2) = \frac{x_Y(i_1, i_2)}{G_\sigma(x_Y(i_1, i_2))} \tag{7}$$

where $G_s(x)$ shows two-dimensional Gaussian kernel defined as follows:

$$G_\sigma(x) = \frac{1}{2\pi\sigma^2} \exp\left(-\frac{\pi^2}{2\sigma^2}\right) \tag{8}$$

Throughout the procedure of Equation 7, we can extract the texture and edge not only in high contrast area but also in low contrast area because the original image is divided by the smoothed image. As results, we can obtain the image including edge information even when the photograph was taken under dark condition. The key idea of our method is to use self-quotient image as the reference signal in ε-filter. To realize our filter, a new nonlinear function $F(x, y)$ is defined as follows.

$$\{F(x, y) \mid F(x) \text{ subject to } |F(x)|\} \tag{8}$$

where

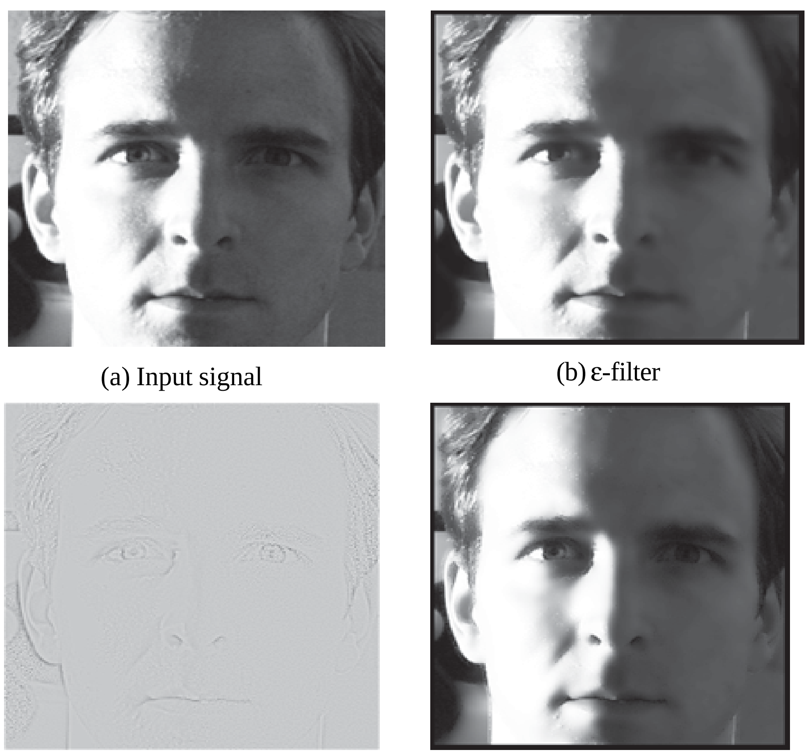
$$-\infty \le x, y \le \infty \tag{8}$$

In Equation 9, we check the value of y, and outputs $F(x)$ depending $|F(y)|$ value. Under the above definition, self-quotient referential ε-filter is designed as follows:

$$z_Y(i_1, i_2) = x_Y(i_1, i_2) + \sum_{j_1=-M}^{M} \sum_{j_2=-N}^{N} a(j_1, j_2) F(X, Y)$$

where we can respectively express X and Y as follows:

$$X = x_Y(i_1 + j_1, i_2 + j_2) - x_Y(i_1, i_2)$$



(a) Input signal

(b) ε-filter

(c) Self-quotient filter

(d) Self-quotient referential ε-filter

Figure 5. Filter outputs of ε-filter, self-quotient filter and self-quotient referential ε-filter. (We prepared the image from Yale image database. File name is y yaleB10_P00A+095E+00.pbm)

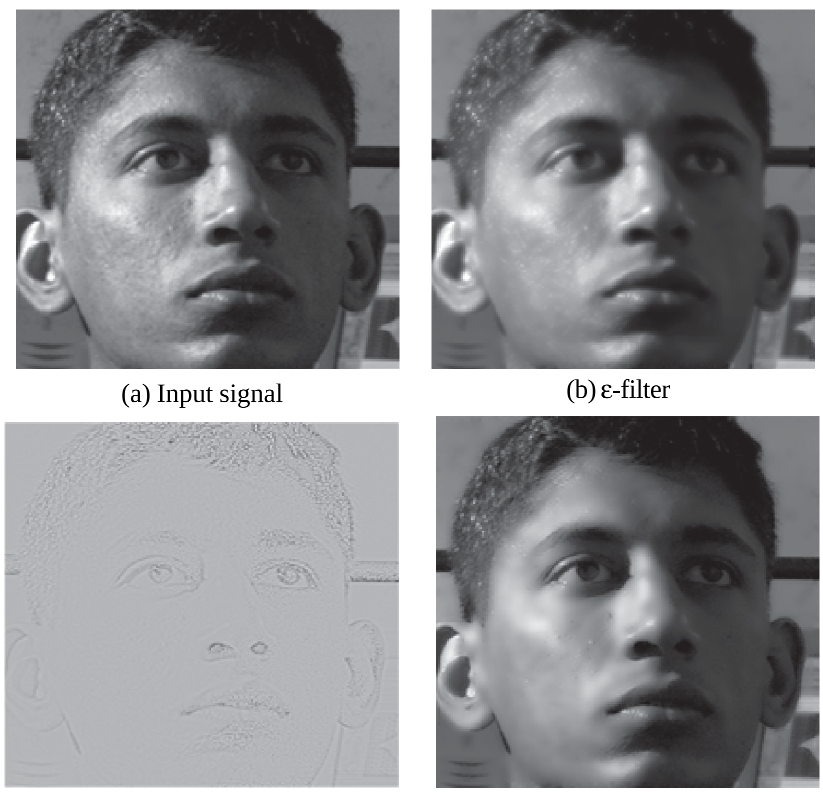
and

$$Y = y_Y(i_1 + j_1, i_2 + j_2) - y_Y(i_1, i_2) \tag{12}$$

(a) Input signal (b) ε-filter (c) Self-quotient referential ε-filter

Figure 6. Close up image around left eyes of original image, ε-filter, self-quotient filter and self-quotient referential ε-filter. (We prepared the image from Yale image database. File name is yaleB10_P00A+095E+00.pbm)

If X is used as Y in Equation 11, the reference signal becomes equal to the function output, that is, the filter corresponds to the normal ε-filter. In this meaning, self-quotient ε-filter is an expanded version of ε-filter. After obtaining $z_Y(i_1, i_2)$, the color information Cr and Cb are added to $z_Y(i_1, i_2)$, and convert it to RGB space. By using the filter output of self-quotient filter as reference, we can distinguish the edge from other area in low contrast area, and smooth the image while preserving the edge information not only around high contrast but also around low contrast. As we just use self-quotient filter addition to ε-filter, the



(a) Input signal (b) ε-filter

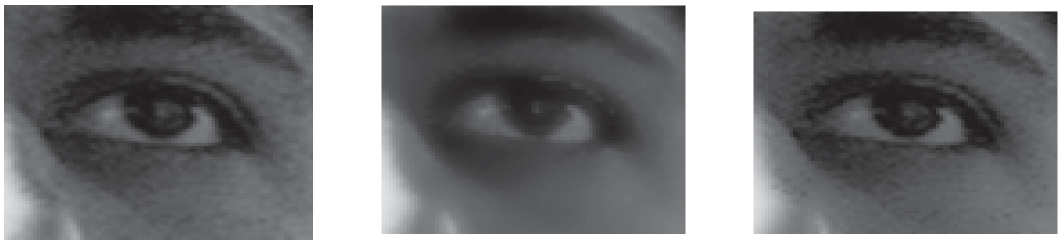
(c) Self-quotient filter (d) Self-quotient referential ε-filter

Figure 7. Filter outputs of ε-filter, self-quotient filter and self-quotient referential ε-filter. (We prepared the image from Yale image database. File name is yaleB12_P06A+085E+20.pbm)

method is still simple and the calculation cost is not so large. We show the validity of our method in the next section on the experiments.

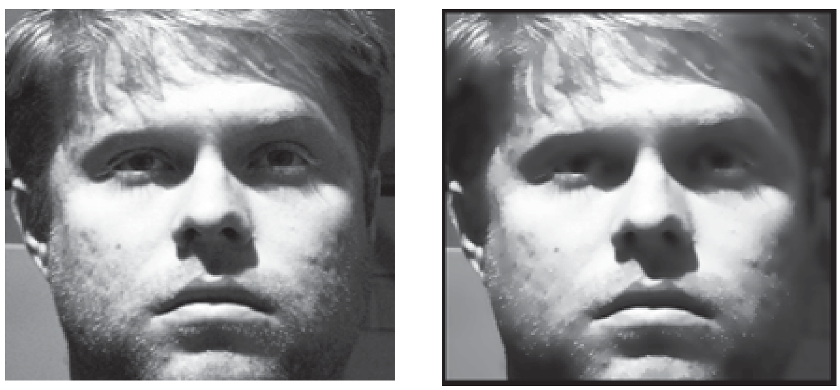
3. Experiments

3.1 Experiments on gray scale images in bad lighting condition

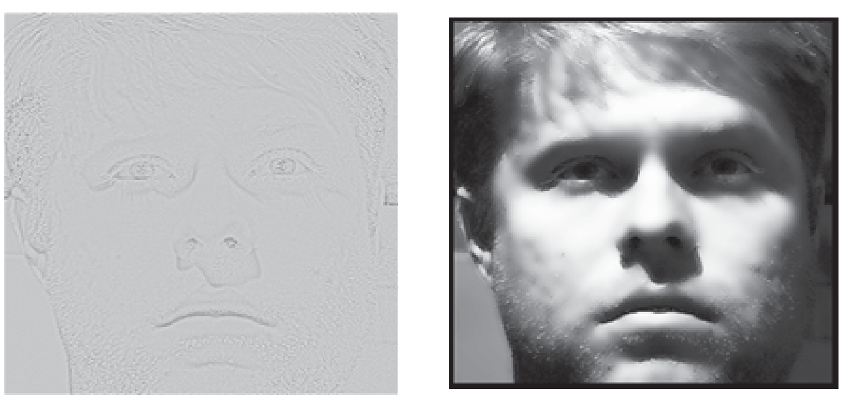


(a) Original image (b) ε-filter (c) Self-quotient referential ε-filter

Figure 8. Close up image around left eyes of original image, ε-filter, self-quotient filter and self-quotient referential ε-filter. (We prepared the image from Yale image database. File name is yaleB12_P06A+085E+20.pbm)



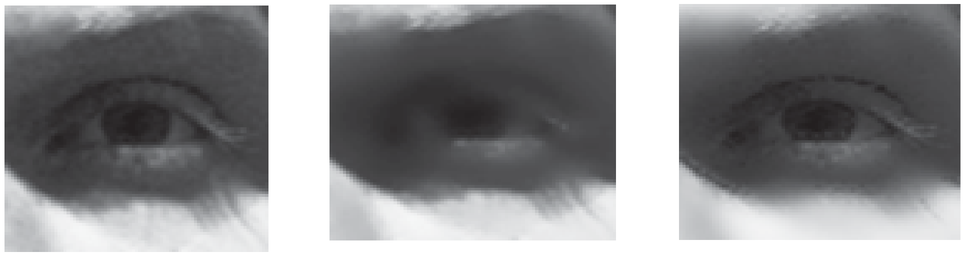
(a) Input signal (b) ε-filter



(c) Self-quotient filter (d) Self-quotient referential ε-filter

Figure 9. Filter outputs of ε-filter, self-quotient filter and self-quotient referential ε-filter. (We prepared the image from Yale image database. File name is yaleB01_P01A+035E+65.pbm)

In this section, we describe the experimental results on gray scale images. For program implementation, a laptop computer with an Intel Core i5-3210M 2.50GHz CPU was employed. The programs were implemented by MATLAB.



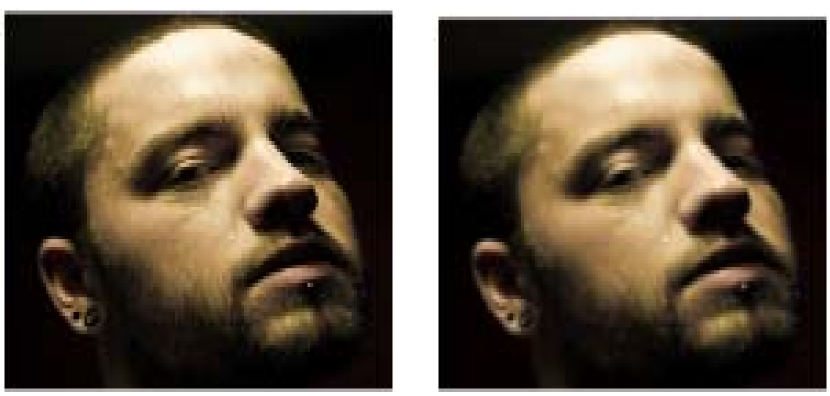
(a) Original image (b) ε-filter (c) Self-quotient referential ε-filter

Figure 10. Close up image around left eyes of original image, ε-filter, self-quotient filter and self-quotient referential ε-filter. (We prepared the image from Yale image database. File name is yaleB01_P01A+035E+65.pbm)

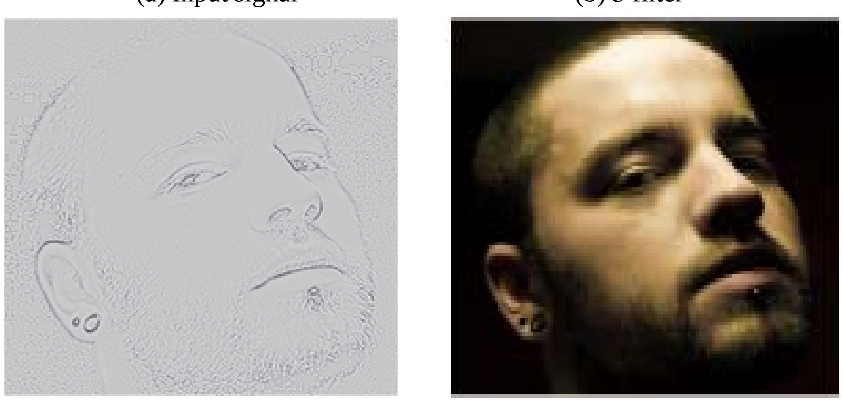
As the input images, we prepared some images from Yale image database [19]. Three samples are shown in this paper.

Figure 5 shows the example of the experiments. Figure 5(a) shows the original image. Figure 5(b) shows the filter output of ε-filter. Figure 5(c) shows the filter output of self-quotient filter. Figure 5(d) shows the filter output of self-quotient referential ε-filter.

As shown in Figure 5(b), when we employed a conventional ε-filter, although edge is preserved while smoothing in high contrast area, edge is blurred in low contrast area. On the other hand, the image is smoothed, while preserving edge information not only around high contrast area but also around low contrast when we employed our method as shown in Figure 5(d). Wrinkle and roughness on the face are smoothed. To clarify the effect of the self-quotient filter, we also show the filter output of self-quotient filter as shown in Figure 5(c). As shown in Figure 5(c), SQF can reduce extrinsic factors from the images, and extract the



(a) Input signal (b) ε-filter



c) Self-quotient filter (d) Self-quotient referential ε-filter

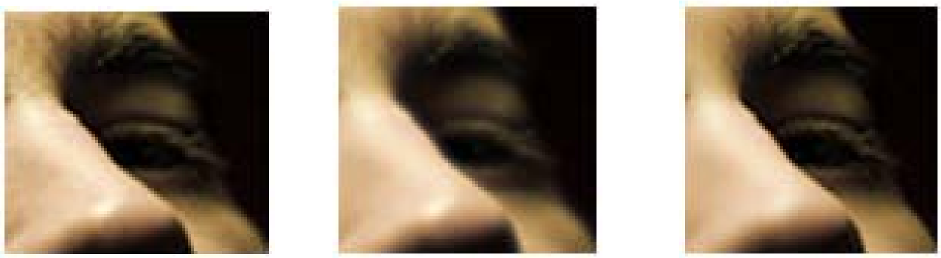
Figure 11. Filter outputs of ε-filter, self-quotient filter and self-quotient referential ε-filter. (We prepared the image from Internet. The image is cut from the image in http://www.flickr.com/photos/ch-ed/4255702269/lightbox/ (Photograph by Ch.Ed).)

feature of the face from the image clearly.

To clarify the difference between the ε-filter and the proposed filter, we also show the close-up image around the left eye as shown in Figure 6. Figure 6(a), 6(b) and 6(c) shows the close-up image of the input image, that of ε-filter and that of the proposed method. The edge around eye is blurred in the filter output of ε-filter as shown in Figure 6(b), while it is preserved in the filter output of the proposed filter as shown in Figure 6(c).

Figures 7 and 9 show other examples of the input images from Yale face image database (file name: yaleB12_P06A+085E+20.pgm and yaleB01_P01A+035E+65.pgm), respectively. Figures 8 and 10 show the close-up images around the left eye from the images (file name: yaleB12_P06A+085E+20.pgm and yaleB01_P01A+035E+65.pgm), respectively. As shown in Figures 6 to 9, similar results could be obtained regardless of the input images.

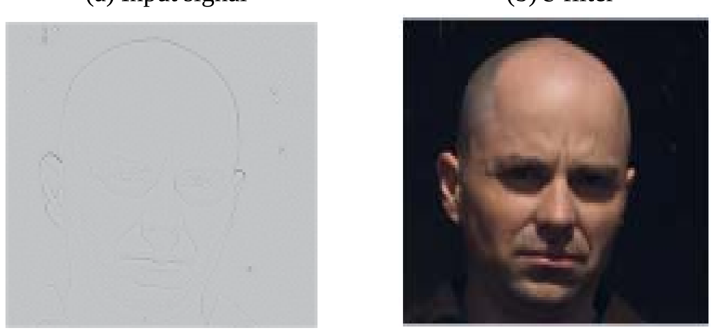
3.2 Experiments on color images



(a) Original image (b) ε-filter (c) Self-quotient referential ε-filter

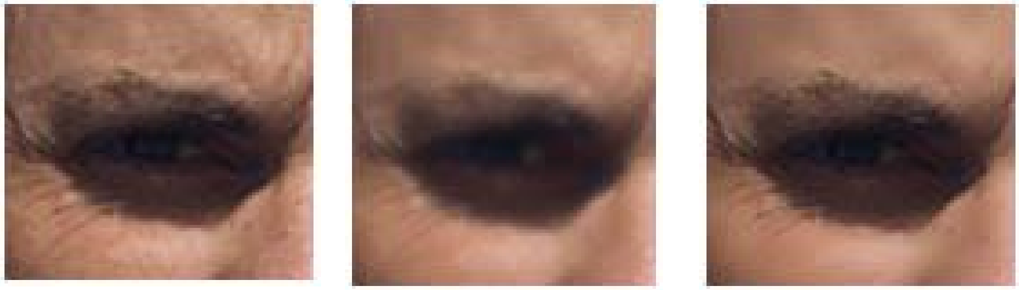
Figure 12. Close up image around left eyes of original image, ε-filter, self-quotient filter and self-quotient referential ε-filter. (We prepared the image from Internet. The image is cut from the image in http://www.flickr.com/photos/ch-ed/4255702269/lightbox/ (Photograph by Ch.Ed).)

(a) Input signal (b) ε-filter



(c) Self-quotient filter (d) Self-quotient referential ε-filter

Figure 13. Filter outputs of ε-filter, self-quotient filter and self-quotient referential ε-filter. (We prepared the image from Internet. The image is cut from the image in http://www.flickr.com/photos/jonwiley/2109674871/lightbox/ (Photograph by Jon Wiley))



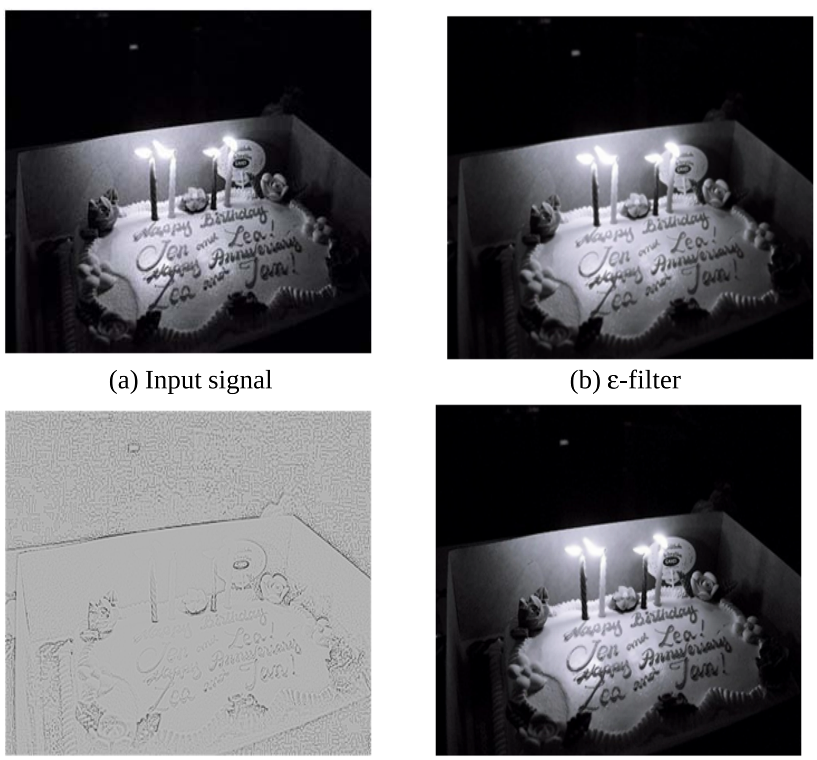
(a) Original image (b) ε-filter (c) Self-quotient referential ε-filter

Figure 14. Close up image around left eyes of original image, ε-filter, self-quotient filter and self-quotient referential ε-filter. (We prepared the image from Internet. The image is cut from the image in http://www.flickr.com/photos/jonwiley/2109674871/lightbox/ (Photograph by Jon Wiley).)

We also selected some color images from the Internet. The images are licensed with creative commons license, and the image reference is described in figure caption. The image size of the image is 256 pixels $\times$ 256 pixels.

Figure11(a) shows an example of the input image. Figures 11(b) and 11(d) show the filter outputs of ε-filter and the proposed method, respectively. As shown in Figure 11(b), when we employed a conventional ε-filter, although edge is preserved while smoothing in high contrast area, edge is blurred in low contrast area the same as the previous experiment.

On the other hand, the image is smoothed, while preserving edge information not only around high contrast area but also around low contrast when we employed our method as shown in Figure11(d).



(a) Input signal (b) ε-filter

(c) Self-quotient filter (d) Self-quotient referential ε-filter

Figure 15. Filter outputs of ε-filter, self-quotient filter and self-quotient referential ε-filter. (We prepared the image from Internet. The image is cut from the image in http://www.flickr.com/photos/jeneawhat/4922682764/ (Photograph by Jenea Medina.))

To clarify the effect of the self-quotient filter, we also show the filter output of self-quotient filter as shown in Figure11(c). As shown in Figure11(c), self-quotient filter can reduce extrinsic factors from the images, and extract the feature of the face from the image clearly.

To clarify the difference between the ε-filter and the proposed filter, we also show the close-up image around the left eye as shown in Figure12. As shown in Figure12, a double eyelid is blurred in the filter output of the ε-filter, while it is preserved in the filter output of the proposed filter.

Figure 13 shows another example of the input image from the Internet. The image size of the image is 256 pixels × 256 pixels. Figure 14 shows the close up images around the right eye from the image.

As shown in Figures 13 and 14, ε-filter also cannot preserve edge information in low contrast area, while our method can preserve edge can preserve edge information. On the other hand, wrinkle and roughness on the face are smoothed while keeping a double eyelid in our method.

3.3 Experiments on other images other than facial images

(a) Original image (b) ε-filter (c) Self-quotient referential ε-filter

Figure 16. Close up image around left area of original image, ε-filter, self-quotient filter and self-quotient referential ε-filter. (We prepared the image from Internet. The image is cut from the image in http://www.flickr.com/photos/jeneawhat/4922682764/ (Photograph by Jenea Medina).)

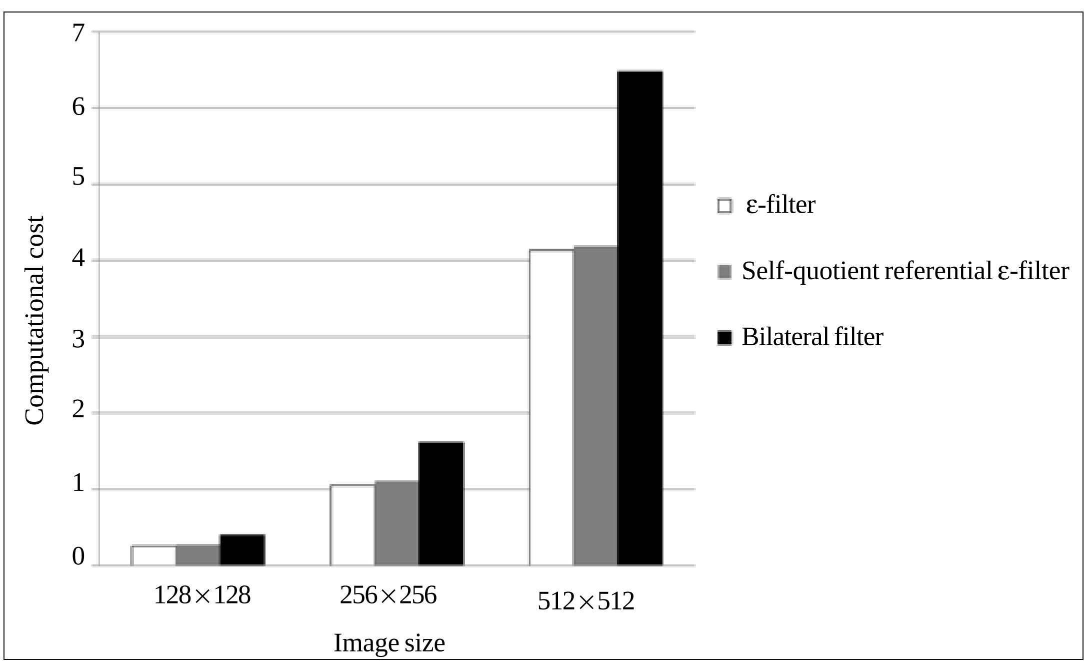


Figure 17. Calculation cost on ε-filter, bilateral filter and proposed method

We also conducted other experiments on the image other than facial images. This is to show that the method is used not only for face beautification but also for smoothing of other photographs. The image is from the Internet, and some parts are cut. The image size is 400 pixels × 300 pixels. Figure15(a) shows an example of the input image. Figures 15(b) and 15(d) show the filter outputs of ε-filter and the proposed method, respectively. Figure 16 shows the close-up image of them. As shown in Figures15 and 16, the surface on the cake in the original image is relatively rough because it is taken under low lighting condition. By using the proposed method, the surface is smoothed while preserving edge.

3.4 Experiments on computational cost

We also conducted the experiments to compare computational cost with other methods. We prepared three different sizes of images (128 pixels × 128 pixels, 256 pixels × 256 pixels and 512 pixels × 512 pixels) for testing.

For comparison, we also checked the calculation cost of ε-filter and bilateral filter. We used a computer with an Intel Core i5-3210M 2.50GHz CPU. The program was implemented by MATLAB. Figure 17 shows the calculation cost depending on the window size concerning ε-filter, self-quotient ε-filter and bilateral filter. In the experiments, the window sizes of the filters were set to 3 pixels × 3 pixels. As shown in Figure 17, the calculation cost of self-quotient ε-filter is rarely different from that of ε-filter, and is smaller than that of bilateral filter.

4. Discussions and Conclusion

In this paper, we described light robust edge-preserving smoothing using self-quotient referential ε-filter. Although many studies on edge-preserving smoothing have been done in the past, it is difficult to handle the photograph with light variation when we employ the typical edge-preserving methods using local intensity such as ε-filter and bilateral filter. On the contrary, our method can handle the image with light variation. The image is smoothed while the edge is preserved not only in high contrast area but also in low contrast area by using our method. Our method only requires an image unlike the method combining a flash image and non-flash image as it uses the self-quotient image as the reference in ε-filter. The calculation cost is small compared to bilateral filter.

We think that these features are suitable for real-time system, and the proposed method has high affinity with the existing multimedia system such as television phone system, TV programs and streaming movies. As the method only utilizes the image itself, we will also be able to apply it to the photographs and movies taken in the past.

Based on the prospects of the above consideration, we think that real-time implementation is important targets for future study. We would like to conduct long-term experiments using our system in users' daily life after developing real-time system. Theoretical analyses of self-quotient referential ε-filter should also be studied.

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