Geometric Modeling of Parallel Robot and Simulation of 3-RRR Manipulator in Virtual Environment

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ABSTRACT: *This paper introduces a novel presentation of mathematics related to robotics studies for reasons of simplicity and extension, we start with a complete geometric modeling to determine the essential points for the design and simulation of our architecture, a choice of 3-RRR delta robot with a pure translation is proposed in this paper, passing briefly through singularities; results of our analytical study are showed in virtual reality environment, where we used several mathematical software , the design software CAD and VRML, the user can interact with application in outside of laboratory by using Matlab interface GUI.*

Keywords: Delta Robot, Spatial 3DOF, 3-RRR, Direct Geometric Model, Inverse Kinematic Model, Singularity Analysis, VRML, Matlab

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1. Introduction

The use of the parallel robot in the industry does not cease to grow; it has led many researchers to propose new architectures, methods of modeling and solutions to problems of performance and reliabilities of parallel robot.

This architecture is based on the principle of connecting the end-effector to its base by more than one kinematic chain to form a closed loop. The mechanism for three degrees of freedom has been the main subject of several architectures according design, movement and kind of actuators, we find the Linapod (or linear delta Zobel, the Orthoglide the Cartesian mechanism of Wenger & Khalil [2], 3 -CRR KONG robot with cylindrical actuators; the Tricept of Neumman or his rival is the ABB IRB 340 Figure 1, and end 3-UPU TSAI.[1]

These architectures are designed for implementer instead robots series in applications and tasks that require great precision and best rigidity with a speed more than elevate a series of robots.

Recent research that uses the delta robot manipulator as have Studied several topics, the criteria of the performance of the workspace and stiffness are studied by [3][17], dynamic modeling and control of the trajectory and the neuro fuzzy adaptive by [6] and also the singularity analysis and the use of intermediate jacobians matrices [7].

The study of singularities and comparison of the workspace of the robot pure translation Delta was studied by [8].

Figure 1. FlexPicker IRB 340 [4]

2. Geometric Analysis of Delta Robot

The robot study in this paper is with pure translation; its architecture for a mobile platform or the end-effector with opportunity to set a manipulations tool or pliers to the displacement of objects or pick and place, a base that contains a three rotational actuators placed in defines locus their distance compared to base frame and $\varnothing_i$ angle designated orientation relative to the same frame, distance and the orientation angle between the actuators are assumed to be identical in magnitude.

The relationship between the base and the platform is make by three identical kinematic chains, each is composed of one actuator attached to the base, one arm and a parallelogram; all body will be fixed to the mobile platform The description of its legs and actuators are shown in figure 2.

In this paper, a delta robot was selected with geometric parameters that can be modified as appropriate and the robot model while keeping the same principle and the same method for modeling and the case study for the kinds of singularity, the selected parameters are shown in Table 1.

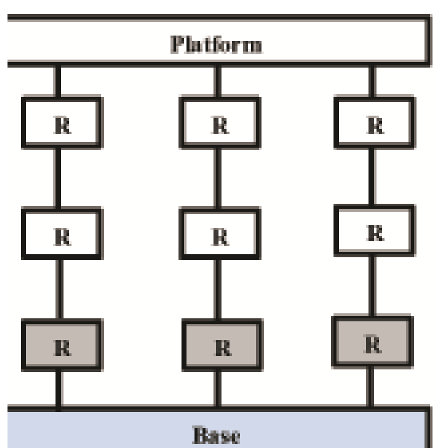

Figure 2. Blocs of principal body of 3-RRR parallel robot

Parameter	Value	Unit
Ra	67	mm
La	80	mm
Lb	160	mm
Rb	17	mm

Table.1. Geometric parameters of Delta Robot.[9]

2.1 Geometric model of the Delta robot

The geometric modeling of robots has been the subject of several methods and techniques; analytical method [11][16], based on the geometric computation that uses the properties of trigonometric functions and homogeneous transformation matrices between frames [10], iterative methods based on the convergence of a mathematical function to a solution of the given problem.

We define as the main frame of Delta robot, the axe system(o, x, y, z), where O is the center of base, and (A_i , x_i , y_i , z_i) relative frame to the main frame, both are define by the distance between centers frame and ϕ_i the rotational angle between (o, x) and (A_i , x_i)

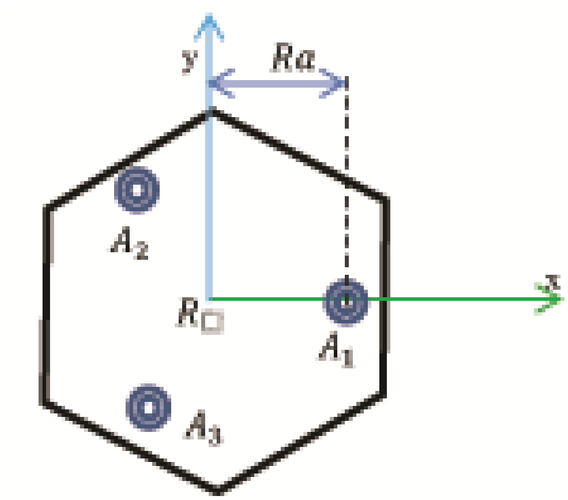

Figure 3. Position of actuator centers of delta robo

All frames representation, lengths of legs and notation is shown in figure 4.

Position of the end effector is given by vector $X = \begin{bmatrix} p_x \\ p_y \\ p_z \end{bmatrix}$

Joint vector for three articulations is $q = \begin{bmatrix} \theta_{11} \\ \theta_{12} \\ \theta_{13} \end{bmatrix}$

$R = (o, x, y, z)$ frame associated to Delta base.

$R_i = (A_i , x_i , y_i , z_i)$ frame associated to the i^{th} actuators.

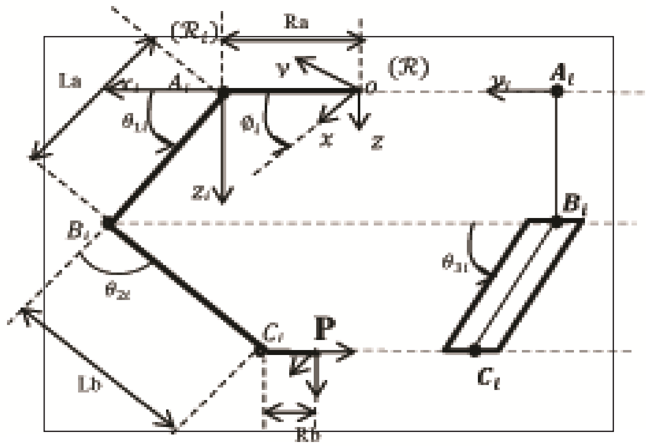

Figure 4. Schematic description of angles and frames of Delta robot

2.2 Inverse Geometric Model

The inverse geometric model is a description of the function which gives us the value of the joints to the Cartesian position vector, or the variation of the actuators in accordance with a given position of the end-effector.

$$\overrightarrow{OP} = \overrightarrow{OA_i} + \overrightarrow{A_iB_i} + \overrightarrow{B_iC_i} + \overrightarrow{C_iP} \qquad (1)$$

We will project this equation in the (R_i) frame, ϕ_i is the parameter of rotation matrix between frames (R) and (R_i), therefore the coordinates of end-effector is :

$$P_{R/Ri} = Rot\,(z, \phi_i)\,.\,P_R$$

And Rot (z, ϕ_i) i = 1, 2, 3 and $\phi_i = \dfrac{2\pi(i-1)}{3}$

We will obtain the following equations written as matrices:

$$\begin{bmatrix} p_x C\phi_i - p_y S\phi_i \\ p_x S\phi_i + p_y C\phi_i \\ p_z \end{bmatrix} = \begin{bmatrix} Ra \\ 0 \\ 0 \end{bmatrix} + La \begin{bmatrix} C\theta_{1i} \\ 0 \\ S\theta_{1i} \end{bmatrix} + Lb \begin{bmatrix} S\theta_{3i} C(\theta_{1i} + \theta_{2i}) \\ C\theta_{3i} \\ S\theta_{3i} S(\theta_{1i} + \theta_{2i}) \end{bmatrix} - \begin{bmatrix} Rb \\ 0 \\ 0 \end{bmatrix} \qquad (2)$$

Where: $C\theta_i = \cos(\theta_i)$

And $S\theta_i = \sin(\theta_i)$

We can obtain θ_{3i} is:

$$\theta_{3i} = \cos^{-1}\left(1/Lb\,(p_x\, S\,\phi_i + p_y\, C\phi_i)\right) \qquad (3)$$

After we can replace the value of θ_{3i} angle to obtain system of two equations with two unknowns and the problem become to solve the following equation:

$$(p_x C\phi_i - p_y S\phi_i - Ra + Rb - LaC\theta_{1i})^2 = (LbS\theta_{3i} - p_z - LaS\theta_{1i})^2 \qquad (4)$$

$$p_x C\phi_i - p_y S\phi_i - Ra + Rb - LaC\theta_{1i} = \pm(LbS\theta_{3i} - p_z - LaS\theta_{1i})$$

Case 1:

$$p_x C\phi_i - p_y S\phi_i - Ra + Rb - LaC\theta_{1i} = LbS\theta_{3i} - p_z - LaS\theta_{1i} \qquad (5)$$

Case 2:

$$p_x C\phi_i - p_y S\phi_i - Ra + Rb - LaC\theta_{1i} = -LbS\theta_{3i} + p_z + LaS\theta_{1i} \qquad (6)$$

From (3) we obtain:

$$(LbS\theta_{3i})^2 = Lb^2 - (p_x S\phi_i + p_y C\phi_i)^2$$

We replace in each case of (4) we obtain:

For case 1:

$$LbS\theta_{3i} = p_x C\phi_i - p_y S\phi_i - Ra + Rb - LaC\theta_{1i} + LaS\theta_{1i} + p_z$$

Therefore:

$$Lb^2 - (p_x S\phi_i + P_y C\phi_i)^2 = (p_x C\phi_i - P_y S\phi_i - R_a + R_b - L_a C\theta_{1i} + L_a S\theta_{1i} + p_z)^2 \qquad (7)$$

We make:

$$A = Lb^2 - (p_x S\phi_i + p_y C\phi_i)$$

$$B = p_x C\phi_i - p_y S\phi_i - Ra + Rb + p_z$$

The equation (5) becomes as follow:

$$A^{\frac{1}{2}} = B + La(S\theta_{1i} - C\theta_{1i})$$

$$S\theta_{1i} = \frac{1}{La}\left(\sqrt{A} - B\right) + C\theta_{1i}$$

This equation is as the second kind [14]

It will be resolved in the same way [11], the result is as follow:

$$\theta_{1i} = \cos^{-1}\left(\frac{1}{2}\left(\frac{\sqrt{A}-B}{La}\right) \pm \left(\left(\frac{\sqrt{A}-B}{La}\right)^2 - 2\right)\right) \qquad (8)$$

This equation gives us two solutions of θ_{1i} .

For programming this equation, two loops (For …) are used, the first gives us the actuator position *i*, and the second gives us the value of θ_{1i} .

3. Singularity Analysis of Delta Robot

The A_i point is the center of circle where its rayon is $A_iB_i = La$, therefore the velocity of B_i in the (R_i) frame can be written as follow:

$$V_{Bi} = \dot{\theta}_{1i} \wedge A_i B_i$$

Where:

θ_{1i} : Is the angular velocity of B_i point.

$\wedge$: denotes the vector product.

So the velocity of platform is the velocity of each point in this platform; therefore we can calculate the velocity of point C_i as the velocity of the end effector.

$$\dot{p}_{R_i} = \dot{\theta}_{1i} \wedge A_i B_i + \dot{\theta}_{2i} \wedge B_iC_i \tag{9}$$

We multiply (9) by B_iC_i we obtain:

$$B_iC_i \cdot \dot{p}_{Ri} = B_iC_i(\dot{\theta}_{1i} \wedge A_iB_i) + B_iC_i(\dot{\theta}_{2i} \wedge B_iC_i) \tag{10}$$

The mixed product of three vectors is the determinant of these vectors therefore: $B_i C_i (\dot{\theta}_{2i} \wedge B_iC_i) = 0$

And with according to the properties of the mixed product of three vectors we have:

$$B_iC_i(\dot{\theta}_{1i} \wedge A_iB_i) = \dot{\theta}_{1i} \wedge (A_iB_i \wedge B_iC_i)$$

Therefore (9) becomes as follow:

$$B_iC_i \cdot \dot{p}_{Ri} = \dot{\theta}_{1i} \cdot (A_iB_i \wedge B_iC_i) \tag{11}$$

We have

$$\dot{\theta}_{1i} = \begin{bmatrix} 0 \\ -\dot{\theta}_{1i} \\ 0 \end{bmatrix}; \qquad A_iB_i = La\begin{bmatrix} C\theta_{1i} \\ 0 \\ S\theta_{1i} \end{bmatrix}$$

$$B_iC_i = Lb\begin{bmatrix} S\theta_{3i}C(\theta_{1i} + \theta_{2i}) \\ C\theta_{3i} \\ S\theta_{3i}S(\theta_{1i} + \theta_{2i}) \end{bmatrix}$$

$$\dot{p}_{\mathcal{R}_i} = \begin{bmatrix} \dot{p}_x C\phi_i - \dot{p}_y S\phi_i \\ \dot{p}_x S\phi_i + \dot{p}_y C\phi_i \\ \dot{p}_z \end{bmatrix}$$

We will replace in (10) the vectors value we obtain the equation form as following:

$$J_p.\dot{p} = J_\theta.\dot{\theta_{1i}} \tag{12}$$

Therefore:

$$\dot{\theta_{1i}}.(A_iB_i \wedge B_iC_i) = \det(\theta_{1i}, A_iB_i, B_iC_i)$$

$$= \dot{\theta_{1i}}.\begin{bmatrix} LaC\theta_{1i} & LaS\theta_{1i} \\ LbS\theta_{3i}C(\theta_{1i}+\theta_{2i}) & S\theta_{3i}S(\theta_{1i}+\theta_{2i}) \end{bmatrix}$$

$$= LaLbS\theta_{3i}(S\theta_{1i}C(\theta_{1i}+\theta_{2i}) - C\theta_{1i}S(\theta_{1i}+\theta_{2i}))$$

And

$$S\theta_{1i}C(\theta_{1i}+\theta_{2i}) - C\theta_{1i}S(\theta_{1i}+\theta_{2i}) = S(\theta_{1i}+\theta_{2i}-\theta_{1i}) = S\theta_{2i}$$

Therefore:

$$\dot{\theta_{1i}}.(A_iB_i \wedge B_iC_i) = LaLbS\theta_{3i}\, S\theta_{2i}\dot{\theta_{1i}}$$

So

$$J_\theta = LaLb\begin{bmatrix} S\theta_{31}\, S\theta_{21} & 0 & 0 \\ 0 & S\theta_{32}\, S\theta_{22} & 0 \\ 0 & 0 & S\theta_{33}\, S\theta_{23} \end{bmatrix} \tag{13}$$

$$B_iC_i.\dot{p_{\mathcal{R}_i}} = B_iC_i^T.\dot{p_{\mathcal{R}_i}}$$

$$= Lb.[S\theta_{3i}C(\theta_{1i}+\theta_{2i}), C\theta_{3i}, S\theta_{3i}S(\theta_{1i}+\theta_{2i})]\begin{bmatrix} \dot{p_x}C\phi_i - \dot{p_y}S\phi_i \\ \dot{p_x}S\phi_i + \dot{p_y}C\phi_i \\ \dot{p_z} \end{bmatrix}$$

$$= (S\theta_{3i}C\phi_iC(\theta_{1i}+\theta_{2i}) + C\theta_{3i}S\phi_i)\dot{p_x}$$
$$+(-S\theta_{3i}C\phi_iC(\theta_{1i}+\theta_{2i}) + C\theta_{3i}C\phi_i)\dot{p_y}$$
$$+(S\theta_{3i}S(\theta_{1i}+\theta_{2i}))\dot{p_z}$$

Thus:

$$B_iC_i.\dot{p_{\mathcal{R}_i}} = J_{i1}\dot{p_x} + J_{i2}\dot{p_y} + J_{i3}\dot{p_z} \tag{14}$$

$$J_p = [J_{i1};\quad J_{i2};\quad J_{i3}] = \begin{bmatrix} J_{11} & J_{12} & J_{13} \\ J_{21} & J_{22} & J_{23} \\ J_{31} & J_{32} & J_{33} \end{bmatrix} \tag{15}$$

In [15] they define three kinds of singularities of parallel closed loop; we calculate each singular position and illustrated it by figures.

The first kind of singularity is when:
$\det(J_\theta) = 0$

Therefore:

$$S\theta_{31}\ S\theta_{21}\ \ S\theta_{32}\ S\theta_{22}\ S\theta_{33}\ S\theta_{23} = 0 \Rightarrow S\theta_{3i} S\theta_{2i} = 0$$

Thus:

$$S\theta_{3i} S\theta_{2i} = 0 \Rightarrow \begin{cases} \theta_{2i} = 0 \ \ or\ \pi \\ \quad OR \\ \theta_{3i} = 0 \ \ or\ \pi \end{cases} \quad (16)$$

θ_{2i} is the angle formed by A_i B_i and B_i C_i segments , so: The First solution of jacobian equation is when:

A_i, B_i, C_i and are aligned, e.g $\theta_{2i} = 0$, so it represents the boundaries of the workspace in a given direction and the max between base and platform of robot, this result is shown in figure 5.

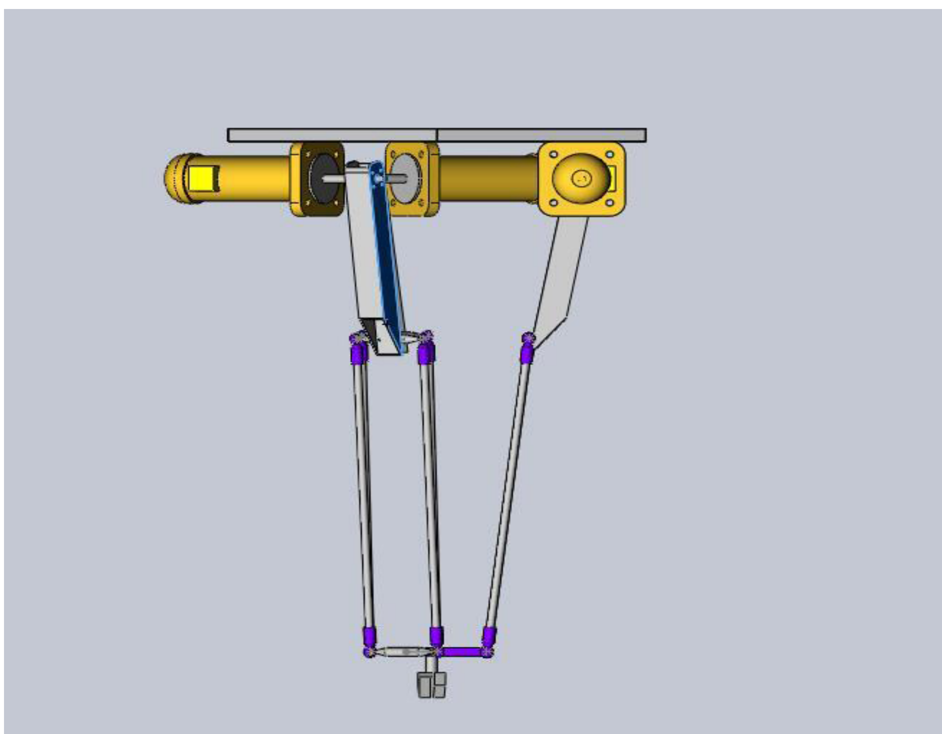

Figure 5. Singular positions of legs (when $\theta_{2i} = 0$, i =1...3)

The second solution of θ_{2i} in (11) is when B_i is located between A_i and C_i, this solution is mechanically impossible, due to the constraints of the base, figure 6 illustrate this concept.

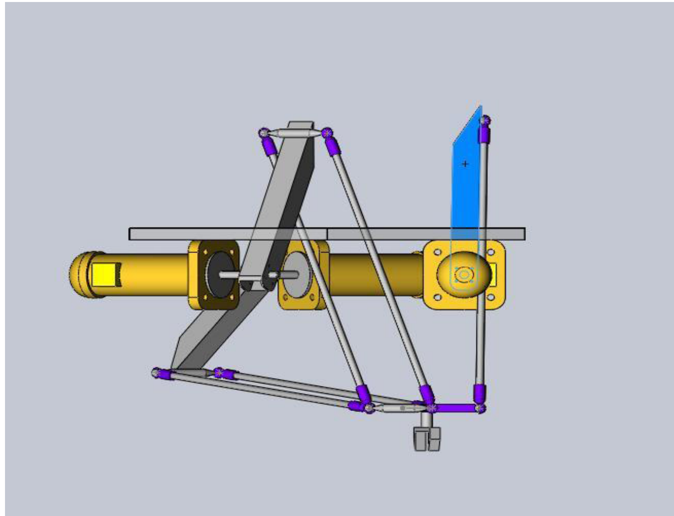

Figure 6. Singular positions of legs (when $\theta_{2i} = \pi$, i =1..3)

For θ_{3i} angle between parallelogram and plan parallel to the base of robot, its position legs influence is when the chain cannot generate a velocity in the certain direction. figure 7 show one singular position when $\theta_{3i} = 0$.

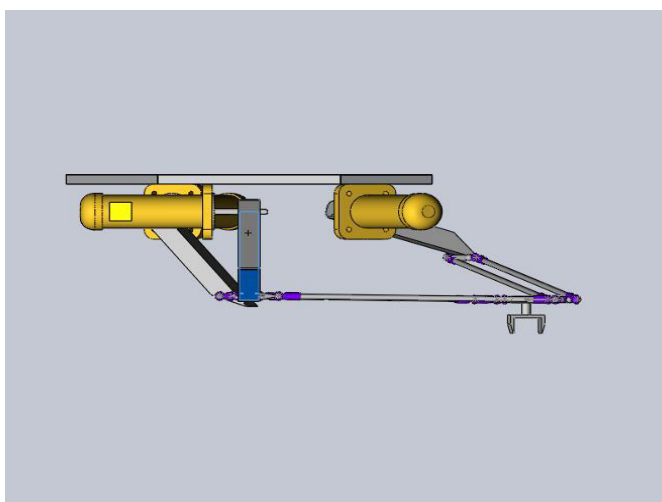

Figure 7. Singular positions of legs (when $\theta_{3i} = \pi$, i =1..3) Front view

4. Graphical User Interface for Control

This work is the beginning of a complete project using the parallel delta robot, which is later to establish the control laws and the generation of motion with different kinds of PID controller, for this, we started by designing a GUI that helps us visualize the delta robot in a virtual reality environment and we could using the Matlab software that can provide libraries of controls and digital computing. Figure 9.

For the control interface we have developed several functions to facilitate the programming and control, this interface including the resolution of the direct and inverse kinematics and singular positions of the delta robot figure 10.

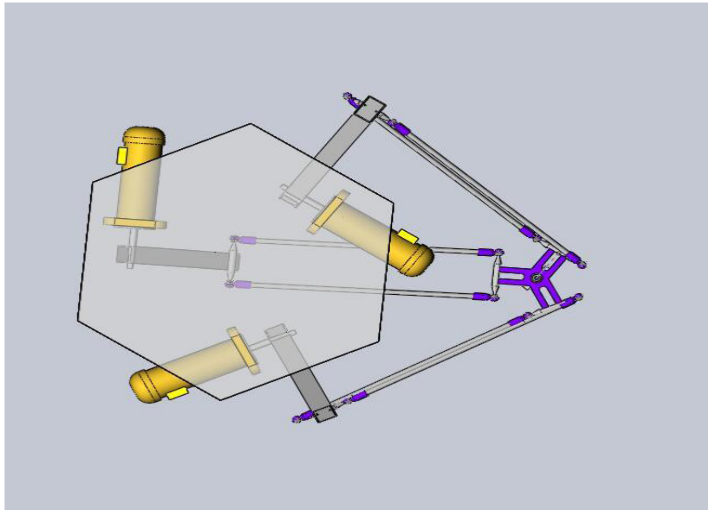

Figure 8. Singular positions of legs (when $\theta_{3i} = \pi$, $i = 1..3$)Top view

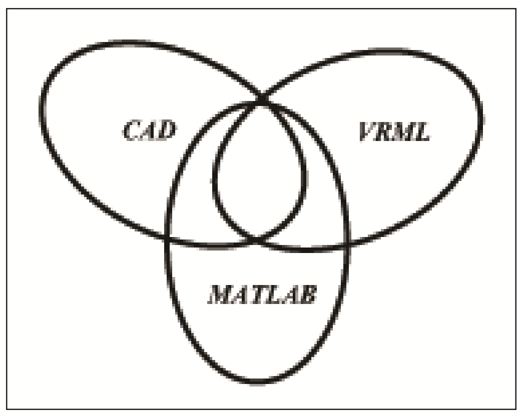

Figure 9. The interconnection of different tools

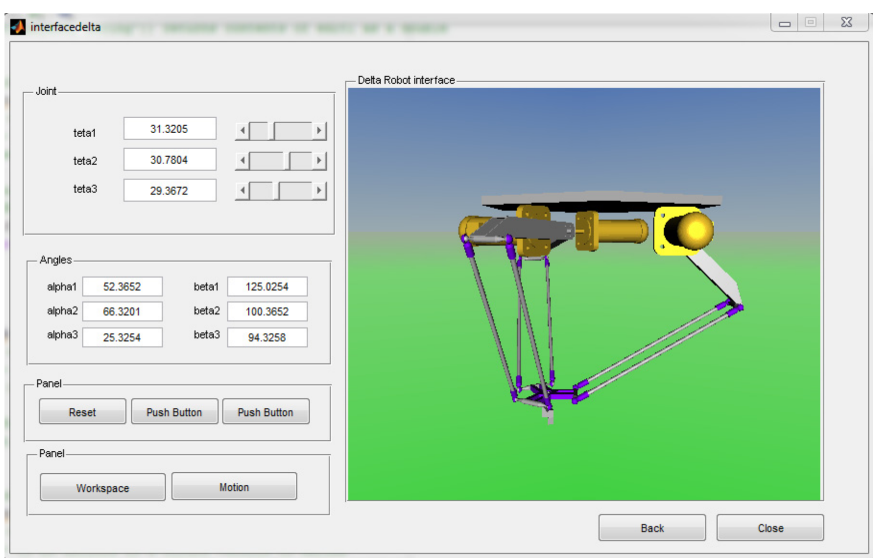

Figure 10. Graphical user interface for control

5. Conclusion

In this article we integrate with the efficiency delta robot with pure translation in a virtual reality environment.

Geometric Modeling is done with symbolic variables, or we can fit several robots whose concept is the delta and with different architectures, the inverse kinematic model is proposed (direct kinematic model is reliable but not introduced because of the limit of pages), the study of singular positions is made to determine breakeven with an illustration of cases in a graphical user interface.

References

[1] Merlet, J. P. (Ed.). (2001). Parallel robots (74). Springer Science & Business Media..

[2] Guegan, S., Khalil, W., Chablat, D., Wenger, P. (2007). Modélisation Dynamique d'un Robot Parall\ele\a 3-DDL: l'Orthoglide. arXiv preprint arXiv:0707.2185..

[3] Stan, S., Manic, M., Szep, C., Balan, R. (2011, May). Performance analysis of 3 DOF Delta parallel robot. *In Human System Interactions (HSI)*, 2011 4th International Conference on (215-220). IEEE..

[4] ABB Robotics: http://www.ABB.com

[5] GRABCAD. Récupéré sur http://grabcad.com/

[6] Asgari, M., Ardestani, M. A. (2013, February). Dynamics and control of a novel 3-DoF spatial parallel robot. In Robotics and Mechatronics (ICRoM), 2013 First RSI/ISM International Conference on (183-188). IEEE.I.

[7] López, M., Castillo, E., García, G., Bashir, A. (2006). Delta robot: inverse, direct, and intermediate Jacobians. Proceedings of the Institution of Mechanical Engineers, Part C: *Journal of Mechanical Engineering Science*, 220 (1) 103-109..

[8] Asgari, M., Ardestani, M. A., Asgari, M. (2014). Singularity analysis of a novel three translational DoF parallel robot considering kinematic asymmetric property. *In*: Robotics and Mechatronics (ICRoM), Second RSI/ISM International Conference on (113-118). IEEE..

[9] Clavel, R. (1994). Robots parallèles. Techniques de l'Ingénieur..

[10] Khalil, W., Dombre, E. (1999). Modélisation, identification et commande des robots. Hermès science publ.

[11] Bouzgou, Kamel., Zoubir, Ahmed-Foitih. (2014). Geometric modeling and singularity of 6 DOF Fanuc 200IC robot. Innovative Computing Technology (INTECH), Fourth International Conference on IEEE,.

[12] Bouzgou, Kamel., Hanifi Elhachemi Amar, Reda., Ahmed-Foitih, Zoubir (2015). Development of HM interface for generating Motion of the 6 dof Fanuc 200iC Robot in a Virtual Reality. International Conference on Artificial Intelligence. 6.

[13] Bouzgou, Kamel., Zoubir Ahmed-Foitih, and Usto Mb, Oran-Algeria (2014). Singularity Analysis and Illustration of Inverse Kinematic Solutions of 6 DOF Fanuc 200IC Robot in Virtual Environment. *Journal of Intelligent Computing 5*(3) 91-105.

[14] Khalil, W., Dombre, E. (1999). Modélisation, identification et commande des robots. Hermès science publ.

[15] Gosselin, C., Angeles, J. (1990). Singularity analysis of closed-loop kinematic chains. *IEEE Transactions on Robotics and Automation*, 6 (3) 281-290.

[16] Bouzgou, Kamel., Hanifi Elhachemi Amar, Reda., Ahmed-Foitih, Zoubir . Virtual reality simulation and singularity analysis of 3-RRR translational parallel robot." Innovative Computing Technology (INTECH), 2015 Fifth International Conference on. IEEE, 2015.

[17] Bouzgou, Kamel., Ahmed-Foitih, Zoubir (2015). Workspace Analysis and Geometric Modeling of 6 DOF Fanuc 200IC Robot. Procedia-Social and Behavioral Sciences 182. 703-709.