Robust Fault Detection for Discrete-time Periodic Systems using Model Matching Problem

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ABSTRACT: *This paper deals with the robust fault detection (FD) identification of discrete-time periodic systems using modelmatching problem. Discrete-time periodic system is first converted into linear time invariant (LTI) system using lifting technique and then conventional fault detection filter (FDF) problem is applied on such discrete LTI system. This problem is multi-objective or optimization problem in which effect of disturbance on residual is minimized and at the same time effect of fault on residual is maximized by solving some linear matrix inequalities (LMI). In the end, a design example is solved and simulation results are provided.*

Key words: Optimization Problem, Control Systems, Residual generation, lifting technique, discrete-time linear periodic system, model-matching problem, fault detection filter, optimization problem, H_∞ norm, l_2-gain, LMI.

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1. Introduction

Persistent development in technology demands for high performance and excellent product quality on one side and efficient cost economics and safe operation on the other side. To achieve such a blend of objectives at the same time requires safety and reliability. Model based fault diagnosis addresses such issues extensively by detecting faults and enhancing robustness against unknown inputs [1]. Fault is defined as any deviation of one or more system parameters from normal operating set points which results into abnormal system behavior. Fault is detected in the system by designing a residual generator and difference between plant output and post filter output is indicative of faults in the system and is called residual signal *r(k)*. Residual generator consists of a model redundancy and a post filter. Residual signal is zero in case of fault free system and non zero when a fault occurs in the system. Fig. 1 shows how residual is generated in software for indication of a fault [2].

$$r(k) = \begin{cases} 0 & \text{if fault} = 0 \\ \neq & \text{if fault} \neq 0 \end{cases}$$

Numerous state of the art algorithms are devised which indicate identification of faults. These algorithms fall into the category of software redundancy which is cost effective and a good alternative to hardware redundancy which is costly and requires maintenance [3]. Faults should be identified early otherwise their effects on the systems are disastrous and catastrophic.

Model redundancy block shown in Fig. 1 based on the idealized assumption that it depicts the faithful replica of actual system dynamics. In practice, it is generally not true due to unavoidable disturbance and noise. So in this unfortunate case, residual should be robust in nature which means that effect of disturbance should be as small as possible and at the same time, the effect of faults should be maximum on residual signal. This problem is referred to as optimization problem or multi-objective design.

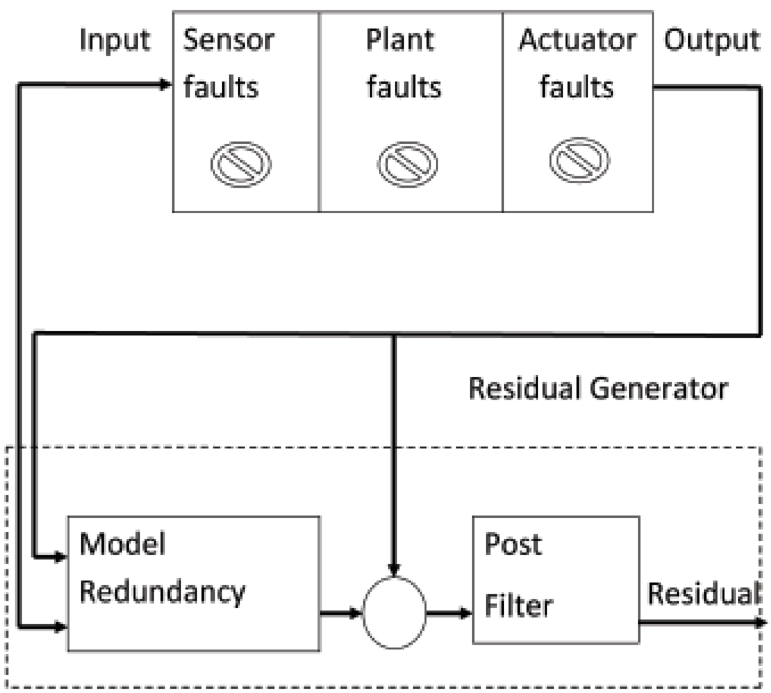

Figure 1. Residual Generation

Periodic systems are an important class of systems which repeat themselves after certain interval of time [4], [5] and they are represented in state space form as

$$x(k+1) = A(k)\,x(k) + B(k)\,u(k) + E_d\,d(k) + E_f\,f(k) \tag{1}$$
$$y(k) = C(k)\,x(k) + D(k)\,u(k) + F_d\,d(k) + F_f\,f(k) \tag{2}$$

where $x(k) \in R^n$ represents discrete time state space vector, $u(k) \in R^{n_u}$ is input vector, $y(k) \in R^m$ represents output vector, $d(k) \in R^{n_d}$ represents bounded disturbance vector and $f \in R^{n_f}$ represents fault vector. Fault vector consists of various type of faults like sensor faults, actuator faults and component faults. Each entry in $f = [f_a\, f_b\, f_c \dots f_{nf}]^T$ represents a specific type of fault. Fault can be of any nature, i.e, it may be instantaneous or slowly developing. All system matrices are periodically varying with period τ and are of appropriate order and satisfy the following property.

The motivation behind this paper is that periodic system theory is continuously developing because of its importance in various real life examples. Active vibration control of helicopters [5]–[8], Networked control systems [9], Mutirate sampled data systems [10], Satellite attitude control [11] are important examples of periodic systems.

$$[\ A(k+\tau)\ \ B(k+\tau)\ \ C(k+\tau)\ \ D(k+\tau)\ E_d(k+\tau)\ \ E_f(k+\tau)\ F_d(k+\tau)\ \ F_f(k+\tau)\ ]$$
$$= [\ A(k)\ B(k)\ C(k)\ D(k)\ E_d(k)\ E_f(k)\ F_d(k)\ F_f(k)\ ]$$

One approach to deal with fault detection of discrete-time periodic system is to directly design a periodic observer or periodic FDF [12], [13], [14] but these techniques are computationally difficult and cumbersome to implement. A relative easier procedure is to convert periodic system into time invariant representation by utilizing the well known isomorphism between these two system classes. The technique which accomplish the above goal is called lifting technique [15].

Multiple solutions to optimization problem and designing of robust residual generator have been developed during the last two or three decades which include parity space approach [16], unknown input observer [17], eigenstructure assignment technique [18], LMI techniques [19] and H_∞-norm optimization method [20].

In this paper, lifting technique is discussed which gives analytic expression for converting discrete time periodic system into LTI formulation in section 2. After that, select a stable weighting function matrix as the desired transfer function from fault to residual [21] and formulate FDF design problem as model-matching problem. By using H_∞ optimization technique in section 3, an LMI approach is developed to design FDF. Proofs of such LMIs are omitted due to time and space constraints. In section 4, a design problem is solved along with simulation results.

2. Preliminaries And Problem Formulation

2.1 Lifting Technique

Let $\Phi(\tau_\circ, \tau)$ be the state transition matrix [22] which, when multiplied with state at a time τ_1 gives the value of state at any ater time $\tau_\circ$. It is basically the solution of state space equation and is given by

$$\Phi(\tau_1,\tau_\circ) = \begin{cases} I & \text{if } \tau_1 = \tau_\circ \\ A(\tau_1 - 1)\, A(\tau_1 - 2) \ldots A(\tau_\circ) & \text{if } \tau_1 > \tau_\circ \end{cases}$$

The system described by (1) and (2) can be reformulated to linear time invariant system by applying lifting technique to the following form

$$\begin{aligned} x_\tau(k+1) &= A_\tau x_\tau(k) + B_\tau u_\tau(k) + E_{d,\tau} d_\tau(k) + E_{f,\tau} f_\tau(k) \\ y_\tau(k) &= C_\tau x_\tau(k) + D_\tau u_\tau(k) + F_{d,\tau} d_\tau(k) + F_{f,\tau} f_\tau(k) \end{aligned} \tag{3}$$

Constant valued matrices from (3) obtained by lifting technique are used in conventional fault detection techniques. [23], [24], [25], [1].

Proof: Let

$$\begin{aligned} x(\tau+1) &= A(\tau)\, x(\tau) + B(\tau)\, u(\tau) \\ y(\tau) &= C(\tau)\, x(\tau) + D(\tau)\, u(\tau) \end{aligned} \tag{4}$$

where τ is an integer between 0 and T - 1 denoting the initial time. Now

$$x(\tau + kT + 1) = A(\tau + kT)\, x(\tau + kT) + B(\tau + kT)\, u(\tau + kT) \tag{5}$$

As we know that $A(k)$ and $B(k)$ matrix are periodic so

$$A(\tau + kT) = A(\tau) \qquad B(\tau + kT) = B(\tau)$$

Substituting the values in (5) we have

$$x(\tau + kT + 1) = A(\tau) x(\tau + kT) + B(\tau) u(\tau + kT)$$

One step further we have

$$x(\tau + kT + 2) = A(\tau + 1)\, x(\tau + kT + 1)\, B(\tau + 1)\, u(\tau + kT + 1) \tag{6}$$

Putting the value of $x\,(\tau + kT + 1)$ in (6) we get

$$x(\tau + kT + 2) = A(\tau + 1)\; A(\tau)\, x\,(\tau + kT) + A(\tau + 1)\; B(\tau)\; u\; (\tau + kT) + B(\tau + 1)\, u\,(\tau + kT + 1)$$

On similar footsteps we got

$$x(\tau + kT + 3) = A(\tau + 2)\, A(\tau + 1)\, A(\tau)\, x\,(\tau + kT) + A(\tau + 2)\, A(\tau + 1)\, B(\tau)$$
$$u\;(\tau + kT) + A(\tau + 1)\, B(\tau + 1)\, u\,(\tau + kT + 1) + B(\tau + 2)\, u\,(\tau + kT + 2)$$

$$x(\tau + kT + 4) = A(\tau + 3)\, A(\tau + 2)\, A(\tau + 1)\, A(\tau)\, x\,(\tau + kT) + A(\tau + 3)\, A(\tau + 2)\, B(\tau)\, u\,(\tau + kT) +$$
$$A(\tau + 2)\, B(\tau + 2)\, u\,(\tau + kT + 1)\, A(\tau + 1)\, B(\tau + 1)\, u\,(\tau + kT + 2) + B(\tau + 3)\, u\,(\tau + kT + 3)$$

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Now generalized relation is

$$x(\tau + kT + T) = A(\tau + T - 1) \ldots\, A(\tau)\, x\,(\tau + kT) + A(\tau + T - 1) \ldots A(\tau)\, B(\tau)\, u\;(\tau + kT) + \ldots + B(\tau + T - 1)\, u\;(\tau + kT + T\text{-}1)$$

By looking at the above equation A_τ is equal to

$$\Phi\,(\tau + T, \tau\,) = A(\tau + T - 1) \ldots\ldots A(\tau) \qquad (7)$$

B_τ is in a vector form because of multiple terms in above equation and is equal to

$$B_\tau = [B_{\tau,1}\;\; B_{\tau,2}\;\; \ldots\ldots\;\; B_{\tau,T}\,] \qquad (8)$$

where

$$B_{\tau,i} = \Phi\,(\tau + T,\, \tau + i\,)\; B\,(\tau + i - 1)$$

On same proceedings C_τ and D_τ can be computed as

$$C_\tau = \begin{bmatrix} C\,(\tau) \\ C\,(\tau + 1)\, \Phi\,(\tau + T, \tau\,) \\ . \\ . \\ . \\ C\,(\tau + T, -1\,)\, \Phi\,(\tau + T - 1, \tau\,) \end{bmatrix} \qquad (9)$$

$$D_\tau = \begin{bmatrix} D_{\tau,1,1} & 0 & \ldots\ldots & 0 \\ D_{\tau,2,1} & D_{\tau,2,2} & \ldots\ldots & . \\ . & . & . & . \\ . & . & . & . \\ D_{\tau,T,1} & \ldots\ldots & D_{\tau,T,T-1} & D_{\tau,T,T} \end{bmatrix} \qquad (10)$$

where

$$D_{\tau,i,j} = \begin{cases} D\,(\tau + i - 1) & \text{if } i = j \\ C\,(\tau + i - 1)\, \Phi\,(\tau + i - 1,\, \tau + j\,) & \\ \times B\;\; (\tau + i - 1) & \text{if } i > j \end{cases}$$

where as *u(k)* and *y(k)* are lifted in the following form

$$\zeta_\tau(k) = \begin{bmatrix} \zeta(kT+\tau) \\ \zeta(kT+\tau+1) \\ \vdots \\ \zeta(kT+\tau+T-1) \end{bmatrix}$$

2.2 Designing of *FDF* as model-matching problem

The robust *FDF* in this paper is given by

$$\hat{x}(k+1) = (A - LC)\,\hat{x}(k) + (B - LC)\,u(k) + L\,y(k)$$
$$\hat{y}(k) = C\,\hat{x}(k) + Du(k)$$
$$r(k) = V^*(y(k) - \hat{y}(k))$$

where L is the observer gain matrix, V* is post filter and $r(k)$ is discrete-time residual signal. Denoting $e(k) = x(k) - \hat{x}(k)$ as the error dynamics, then we have

$$e(k+1) = (A - LC)\,e(k) + (E_f - LF_f)\,f(k) + (E_d - LF_d)\,d(k)$$
$$r(k) = V^*\,(Ce(k) + F_f\,f(k) + F_d d(k))$$

For designing of robust FDF, we have to find L and V^* such that the following two conditions are simultaneously met.

(a) The residual generator (12) is robust stable

(b) The effect of disturbance on residual is minimum and is sensitive to faults at the same time.

One way to design FDF is to find observer gain matrix L and post filter V* such that (a) is satisfied and

$$\| G_{rd} \|_\infty < \in, \quad \|Q_f - G_{rf}\|_\infty \to \min$$

for some constant $\in > 0$, where G_{rd} and G_{rf} are the transfer function matrices from disturbance to residual and fault to residual, respectively. $Q_f(z)$ represents weighting function matrix. The basic idea behind this formulation is to consider FDF design as a model-matching problem and then design FDF in such a way that effect of disturbance is reduced to $\in$ and at the same time the difference between fault and residual signal, which is weighted in terms of Q_f, is minimized.

Let the state space realization of $Q_f(z)$ be

$$x_f(k+1) = A_Q x_f(k) + B_Q\,f(k)$$

$$r_f(k) = C_Q x_f(k) + D_Q\,f(k)$$

Then we get

$$\begin{bmatrix} e(k+1) \\ x_f(k+1) \end{bmatrix} = \begin{bmatrix} A - L_f C & 0 \\ 0 & A_Q \end{bmatrix} \begin{bmatrix} e(k) \\ x_f(k) \end{bmatrix} + \begin{bmatrix} E_f - L_f F_f \\ B_Q \end{bmatrix} f(k) + \begin{bmatrix} E_f - L_f F_d \\ 0 \end{bmatrix} d(k)$$

$$r_{err}(k) = r(k) - r_f(k) \tag{14}$$

In control theory for linear time invariant discrete-time system, we know that H_∞-norm of transfer function matrix is not greater than $\in$ means that l_2 - gain $\leq \in$. This implies that

$$\| G_{rd} \|_\infty \leq \in \Leftrightarrow \sum_{k=0}^{\infty} r'_{err}(k)\, r_{err}(k) \leq \in^2 \sum_{k=0}^{\infty} d'(k)\, d(k) \tag{15}$$

$$\| Q_f - G_{rf} \|_\infty \leq \lambda \Leftrightarrow \sum_{k=0}^{\infty} r'_{err}(k)\, r_{err}(k) \leq \lambda^2 \sum_{k=0}^{\infty} f'(k)\, f(k) \tag{16}$$

The performance index from the viewpoint of l_2-gain thus formulated as

$$\sum_{k=0}^{\infty} r'_{err}(k)\, r_{err}(k) < \in^2 \sum_{k=0}^{\infty} d'(k)\, d(k) \tag{17}$$

$$\sum_{k=0}^{\infty} r'_{err}(k)\, r_{err}(k) < \lambda^2 \sum_{k=0}^{\infty} f'(k)\, f(k) \tag{18}$$

If L and V^* are chosen such that system (14) is robust stable and then (17) and (18) are also satisfied. Based on the above

$$\sum_{k=0}^{\infty} r'_{err}(k)\, r_{err}(k) < \in^2 \sum_{k=0}^{\infty} d'(k)\, d(k) + \lambda^2 \sum_{k=0}^{\infty} f'(k)\, f(k) \tag{19}$$

preliminary discussion, the problem to be solved can be formulated as follows:

1) Weighting function matrix selection $Q_f(z)$: To select weighting function matrix $Q_f(z)$, find suitable observer gain matrix L_f that given performance index μ which describes the sensitivity of residual signal to the faults, is minimized. Q_f is then selected as

$$x_f(k+1) = (A - L_f C)\, x_f(k) + (B_f - L_f F_f)\, f(k) \tag{20}$$

$$r_f(k) = C x_f(k) + F_f\, f(k) \tag{21}$$

2) FDF for discrete-time system: By selecting weighting function matrix $Q_f(z)$ and given a small quantity $\in > 0$, find observer gain matrix L and post filter V^* such that (14) and (20)

$$\min_{L, V^*} \rightarrow \lambda$$

In the next section, weighting function matrix will be selected by utilizing LMI approach and an observer gain matrix L_f thus obtained will be put in (21) to get the desired state space realization. Then based on the selection of $Q_f(z)$, an observer gain matrix L and post filter V^* will be selected which optimize the given performance index through LMI approach.

3. Main Results

In the following section, we design a robust residual fault detection filter. The residual obtained will have minimum influence of disturbance on it and at the same it is sensitive to faults. The advantage of designing a robust FDF is that it only responds to faults and not disturbances and hence chances of false alarm are reduced.

3.1 Selection of weighting function matrix $Q_f(z)$

For the selection of weighting function matrix, consider the following performance index

$$\sum_{k=0}^{\infty} r'_f(k)\; r_f(k) > \mu^2 \sum_{k=0}^{\infty} f'(k)\, f(k) \tag{22}$$

where $\mu > 0$. μ measures the impact of fault on residual. So higher the value of μ, higher the sensitivity of faults on residual.

Theorem 1: Consider stable Q_f given by (14). If $F_f \neq 0$, then

$$Q_f(z) = Q_{f1}(z)\, X(z)$$

where

$$Q_{f1}(z) = C\,(zI - A + L_f C)^{-1}\,(E_{f1} - L_f F_{f1}) + F_{f1}$$

where

$$F_f\, J^{-1} = [F_{f1}\,,\, F_{f2}\,,\, 0]\,,\; E_f\, J^{-1} = [E_{f1}\,,\, E_{f2}\,,\, E_{f3}]$$

$$\text{rank}\,[F_{f1}\,,\, F_{f2}] = \text{rank}\,(F_f)$$

where J is non-singular matrix and $rank(J) = 1$.

***Theorem 2*:** Given $\mu > 0$ and system (14). Then (22) is satisfied if there exists $P > 0$ and matrix X such that following LMIs hold

$$\begin{bmatrix} -P & PA - XC & PE_{f1} - X F_{f1} \\ * & -P - C'C & -C' F_{f1} \\ * & * & \alpha^2 I - F'_{f1} F_{f1} \end{bmatrix} < 0 \tag{23}$$

$$\begin{bmatrix} -P & PA - XC \\ * & -P \end{bmatrix} < 0 \tag{24}$$

holds, and moreover

$$L_f = P^{-1} X \tag{25}$$

Proof: Please refer to [20].

2.1 Designing of fault detection filter

After choosing stable weighting function matrix, FDF can be designed for system (1) by considering the following LMI

Lemma 1: Consider discrete-time LTI system

$$x\,(k+1) = Ax\,(k) + Bm\,(k)$$
$$s(k) = Cx(k) + Dm(k)$$

Given $\in > 0$, then there exists $Y > 0$ such that following LMI hold

$$\begin{bmatrix} -Y & YA & YB \\ * & -Y + C'C & C'D \\ * & * & \in^2 I + D'D \end{bmatrix} < 0 \tag{26}$$

then the system is stable and satisfies

$$\sum_{k=0}^{\infty} s'(k)\, s\,(k) < \in^2 \sum_{k=0}^{\infty} m'(k)\, m\,(k)$$

Theorem 3: For given $\lambda > 0$, $\in > 0$, stable $Q_f(z)$, then there exist $X_1 > 0$; $X_2 > 0$ such that following LMI holds

$$\begin{bmatrix} -X_1 & 0 & J_1 & 0 & J_2 & J_3 & 0 \\ * & -X_2 & 0 & -X_2 A_q & J_5 & 0 & 0 \\ * & * & -X_1 & 0 & 0 & 0 & C' V^{*\prime} \\ * & * & * & -X_2 & 0 & 0 & -C_q' \\ * & * & * & * & -I & 0 & J_4 \\ * & * & * & * & * & -I & J_6 \\ * & * & * & * & * & * & -I \end{bmatrix} < 0 \tag{27}$$

and

$$L = Y_1 X_1^{-1}$$

where

$$J_1 = X_1 A - Y_1 C;\ J_2 = (X_1 E_f - Y_1 D_f) \in^{-1}$$
$$J_3 = (X_1 E_f - Y_1 F_f)\, \lambda^{-1},\ J_4 = (V^* F_f - D_q)' \in^{-1}$$
$$J_5 = X_2 B_q \in^{-1},\ J_6 = (X_1 E_f - Y_1 D_f) \in^{-1}$$

Proof: Proof can be done by using Lemma 1, omitted.

In next section, a design problem is solved by first applying lifting technique to periodic systems to obtain linear time invariant discrete-time system. Then robust fault detection filter is obtained by solving LMIs (23), (24), and (27) in an iterative manner.

4. Design Example

Consider the periodic system (1) with period $\tau = 2$ of the following matrices

$$A(0) = \begin{bmatrix} 0 & 1 \\ -1 & 0.5 \end{bmatrix} \qquad B(0) = \begin{bmatrix} 0.5 & 0.5 \\ 2 & 1 \end{bmatrix}$$

$$B(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \qquad B(1) = \begin{bmatrix} 0.5 \\ 0 \end{bmatrix}$$

$$E_d(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \qquad E_d(1) = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$E_f(0) = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \qquad E_f(1) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$C(0) = [\,1 \quad 1\,] \qquad C(1) = [\,1 \quad 0\,]$$

$$D(0) = 0 \qquad D(1) = 0$$

$$F_d(0) = 1 \qquad F_d(1) = 2 \qquad F_0(0) = F_0(1) = 1$$

We have to design a residual generator for this periodic system. Applying lifting technique formulas from (7) to (10), we get

$$A_0 = \begin{bmatrix} -0.5 & 0.75 \\ -1 & 2.5 \end{bmatrix} \qquad B_0 = \begin{bmatrix} 1 & 0.5 \\ 3 & 0 \end{bmatrix}$$

$$E_{d,0} = \begin{bmatrix} 1 & -1 \\ 3 & 1 \end{bmatrix} \qquad E_{f,0} = \begin{bmatrix} 0.5 & 0 \\ 1 & 1 \end{bmatrix}$$

$$C_o = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \qquad D_o = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$$

$$F_{d,0} = \begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix} \qquad F_{f,0} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Given $\mu = 1$, weighting function matrix comes out to be

$$f(k+1) = \begin{bmatrix} -1.00 & 0.25 \\ -2.00 & 0.50 \end{bmatrix} x_f(k) + \begin{bmatrix} -0.31 & -0.08 \\ -0.65 & -0.19 \end{bmatrix} f(k)$$

$$r_f(k) = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} x_f(k) + \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} f(k)$$

By solving LMI (27), L and V* comes out to be

$$L = \begin{bmatrix} 0.24 & -1.05 \\ 0.09 & 0.90 \end{bmatrix} \qquad V^* = \begin{bmatrix} 0.0034 & -0.0002 \\ -0.3718 & 0.1167 \end{bmatrix}$$

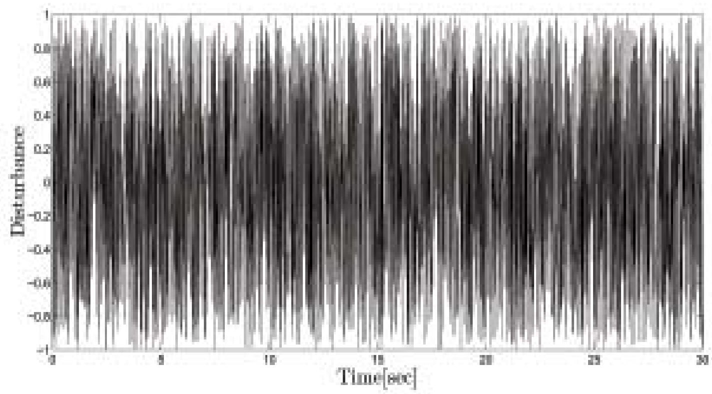

Figure 2. Disturbance

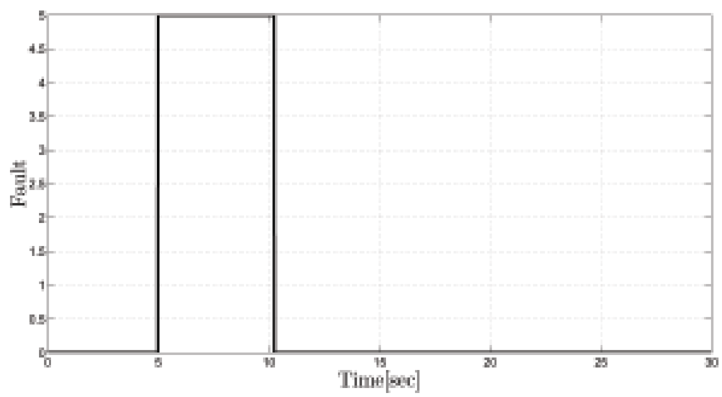

Figure 3. Fault Magnitude

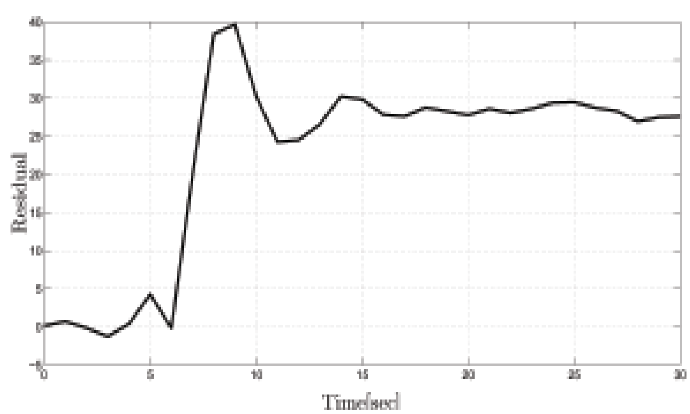

Figure 4. Residual Signal

As seen in Fig.3 fault appears at time $t = 5$ *sec* and residual responds to it as shown in Fig.4. The effect of disturbance on residual is minimum which is the desired objective and at the same time it is sensitive to fault. Noticeable change in residual is seen when fault appears in the system. So optimization is achieved which reduces the effect of disturbance and maximizes the influence of fault on residual simultaneously.

5. Conclusion

In this paper, robust fault detection filter is designed for discrete-time periodic system using an LMI approach. The main problem is sub-divided into following problems: Conversion of discrete-time periodic system to LTI system using lifting technique, selection of weighting function matrix, formulation of robust FDF as a model-matching problem and finally designing of FDF which is sensitive to faults and robust against disturbances by exploiting LMI technique. The results have been verified by the aid of a design example.

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