ABSTRACT: Sustainability has become a significant subject in most economies, causing many leading organizations to focus on product recovery and reverse logistics. However, work with quantitative models is still rare. This research is focused on inventory control in the remanufacturing process.

In this paper, the product demand can be met by remanufactured and new items to minimize the use of raw materials. In this regard, the basic assumption is that the remanufactured products are as good as new products, and the recurring items from customers can be remanufactured at a fixed rate. In this study, a method has been developed to model the reverse supply chain variables. An approach is used for the calculation of optimum level for the new items and the optimum level of the returned items for remanufacturing together. The major objective is to minimize the waste and gain the competitive advantage of the cost of conversion. From among the advantages of using the method, we may refer to the simultaneous calculation of optimized level of newly produced items, and optimized level of returning items for remanufacturing with the aim of minimizing wastes, and achieving competitive advantage of transformation cost. From the model analysis, it can be concluded that the cost of ordering new products, the capacity of the remanufacturing process, the final product demand, the maintenance costs of returning items inventory for remanufacturing, and the maintenance cost of new product inventory for should be at a low level.

Keywords: Inventory Control, Inventory Model, Reverse Supply Chain, Remanufacturing

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1. Introduction

The concept of Reverse supply chain, which is the management or return flow due to product recovery, product return, or overstock, is not new (Jayant et al, 2012). The reuse of products has been previously applied, essentially for the economic
benefits of reusing the component and materials instead of its disposal (Fleischmann et al., 1997). Also, technology advances have reduced the life cycles for many products. Product demand may grow rapidly at first and then decreases a few months later due to the importance of new products. Inventory control under a short life cycle of a product is not easy. Due to an increase in many problems such as large safety stock, high costs of obsolescence, and high forecasting errors, it is essential to consider the constantly varying demand and its uncertainty when making inventory management policy (Hsueh, 2011). In addition to economic motivations and technology improvements, manufacturers require producing products which are easy for disassembly, reuse and remanufacturing owing to the law of environmental protection. On the other hand, the number of customers supporting environmental protection by delivering their used goods to collection points and remanufacturing centers is increasing (Jayant et al, 2012). Moreover, government pressure has contributed to the increasing motivation for global environmental warning and green supply chain management principles (Sheu and Chen, 2012).

Reverse logistics has been implemented long before the term was initially adopted, such as military, customer service policy, and the automotive aftermarket (Rogers et al., 1999). The rise of e-commerce and the positive vision of environmental impact has elevated the formal use of reverse logistics (Bei et al., 2005). So, in today’s business environment, due to most common reasons such as product returns and incorrect delivery of product and damaged products, reserve logistic is inevitable (Fleischmann et al., 1997).

Moreover, it can be of higher importance in one industry over another due to production costs and the nature of remanufacturing of the products (Bazan et al., 2015b). In this way, some products can receive values along the reverse chain by mean of recycling or remanufacturing processes (Silva Filho et al., 2013). The remanufacturing process can be briefly described as (Quanwu et al., 2016):

First, products collected from customers (at the end of life) and disassembled into their constituent components. Next, the parts are remanufactured to exact specifications to ensure that they provide the same quality as new ones. Last, remanufactured components are assembled, tested and made ready for sale as the remanufactured product.

The concern for managing product return flow (Bazan et al., 2015a), justified the growing interest in development of inventory control models to study and analyze reverse supply chains (van der Laan et al., 2006; Behret et al., 2009). Also, another difficulty of managing reverse operations is the need to effectively coordinate manufacturing and remanufacturing activities (Inderfurth et al., 2001). This trend in the industry has led several researchers to investigate such problems and develop appropriate models to account for cases where remanufacturing of used products is an additional choice for the reserve logistics (Nenes et al., 2010). The present study focuses on the deterministic approach of optimizing the reverse logistics inventory model, to analyze the mathematics complication in the associated processes. The paper is organized as follows: a literature review is presented, followed by a review of mathematical models and a discussion summarizing what has been achieved and what needs to be done with the emphasis on environmental concerns. The paper concludes with findings and future research directions.

2. Related Works

Research on reverse supply chain has been growing since the sixties and research on strategies and models on reverse logistics can be seen in the publications in and after the Eighties (Jayant et al, 2012). There are numerous studies that analyze inventory models with return flow in Reverse supply chain. These models can be classified into two groups including deterministic models (e.g. Richter and Dobos, 2004; Teunter et al., 2009; Zanoni et al., 2012; Feng et al., 2014; Özceylan et al., 2014) and stochastic models (e.g., Fleischmann et al., 2002; Behret and Korugan, 2009; Zolfagharia and Haughton, 2012; van Donselaar and Broekmeulen, 2013). In this section, we aim to make contributions to the stochastic and deterministic review modeling literature.

The root of these models is the work of Schrady (1967) (Fleischmann et al., 1997) which is based on the classical economic order quantity (EOQ) model. He analyzed an inventory situation with constant demand and return rates when production and recovery are instantaneous (Brojeswar et al., 2013). The work of Schrady has been developed extensively as presented in the literature (Fleischmann et al., 1997).

Richter’s model, a direct extension of the model of Schrady, shows that for low disposal rates, the repair setup cost has no effect on the total lot size. For large waste disposal rates, the setup cost of manufacturing has no effect (Richter, 1996b). This assumption by Richter state that some items may be disposed of as waste (Richter, 1996a). Richter and Dobos developed the
work of Richter (1996a, 1996b) to discuss a model in which the disposal option of the returned items is allowed and all the recycling batches follow the production batches (Dobos and Richter, 2003).

Similar to Richter, Teunter extended the work of Schrady by assuming that unit holding costs for newly manufactured and remanufactured items are different, and considered more than one production and repair cycles (Teunter, 2001). He considered stochastic demand and return rates and assumed no lead time. Discounted costs were also considered to make the model more realistic (Teunter, 2002). Widyadana and Wee also developed an integrated solution procedure for each of the two policies of Teunter using algebraic approaches (Widyadana & wee, 2010).

Konstantaras and Papachristos developed the model of Teunter by allowing for the complete back ordering of demand (Konstantaras and Papachristos 2006) and improved his work by developing an exact solution that leads to the optimal number of manufacturing and remanufacturing for certain parameter classes (Konstantaras, Papachristos, 2008). However, they consider only the restricted classes of (1, R) and (P, 1) policies (Konstantaras and Papachristos 2006).

Koh et al. (2002) studied a model allowing the recovery rate to be both smaller and larger than the demand rate. Working within two classes of policies, namely the class of policies with one procurement lot and a variable number of recovery lots and the class of policies with one recovery lot and a variable number of procurement lots, they derived optimal lot-sizing formulas (Koh et al., 2002). Choi et al also presented a joint EOQ and EPQ model for an inventory control problem with a reserve supply chain, in which the demand can be satisfied by purchasing brand-new products and remanufacturing used products (Choi et al., 2007).

El Saadany and Jaber also extended the work of Richter (1996a, 1996b) to account for setup changeover costs when switching between productions and remanufacturing runs (El Saadany and Jaber, 2008). They suggested that the return flow of used items depends on the purchase price and the accepted quality level of the returned items (El Saadany and Jaber, 2010). Van der Laan et al. (1996) considered several inventory control strategies with remanufacturing and disposal (van der Laan et al., 1996) by assuming that the demand rate and the return rate are independent (Hsueh, 2011). Also Push and pull strategies are considered in the inventory model (van der Laan and Teunter, 2006) to coordinate production, remanufacturing, and disposal operations. Sun et al. in their study investigated a manufacturing and remanufacturing inventory system with return rate dependent on the product demand. Also, a three stage stochastic dynamic programming is developed to explore an optimal inventory policy (Sun et al., 2013).

Another work developed by the Poles is a production and inventory system for remanufacturing using a System Dynamics simulation modeling approach. The research findings reveal efficiency in the remanufacturing process with higher remanufacturing capacity if the quantity of returning items and the remanufacturing lead time are increased and decreased respectively (Poles, 2013). A stochastic linear quadratic Gaussian (LQG) model with constraints has been formulated by Silva Filho and Salviano to provide optimal annual plans for manufacturing, remanufacturing and disposal variables. Assuming the demand as stationary and normally distributed, an associated equivalent deterministic problem is introduced. So, optimal inventory-production scenarios can be created by a variety of parameters such as the return rate of used products, delay of return, or even both (Silva Filho & Salviano, 2013).

Zolfagharinia et al., developed a new inventory control model for joint purchasing and remanufacturing in a reverse supply chain. Also, they designed a new hybrid solution method, a meta-heuristic algorithm seeking for a near-optimum solution with

<table>
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<th>Contents</th>
<th>Literature</th>
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<td>Modeling of uncertainty</td>
<td>Pati et al. (2008) (Shih, 2001; Krikke et al., 2003), (Fleischmann et al., 2001), (Shih, 2001), refrigerator (Krikke et al., 2003), (Schultmann et al., 2003), LPG-tanks (le Blanc et al., 2004), (Salema et al., 2006), (Salema et al., 2007), (Pati et al., 2008), carpet recycling (Realff et al., 2000, 2004), Krikke et al. (2003), Du and Evans (2008) (Srivistava, 2008), (Lee and Dong, 2008).</td>
</tr>
</tbody>
</table>

Table 1. Literature Based on Various Issues of RL Design (Jayant et al, 2012)
an embedded simulation model that approximates the objective value of each generated solution (Zolfagharinia et al., 2014). In table 1 papers related to the parameters investigation on reverse logistics has been listed.

3. Describing the Proposed Model

As shown in figure (1), two operational cycles are working simultaneously. Remanufacturing and repairable process, consisting of items from direct production, and remanufacturing. Soon after collection of a certain value of returning products (R), remanufacturing begins. It is supposed that remanufactured items are as good as new items, and could be used as an appropriate alternative for products from direct production. Again, when certain uncertainties exist in the rate of collecting returned items for remanufacturing, safety stock is suggested to be used as a shock absorber during the fluctuations of rates of returning items.

From mathematical calculations, we are aiming at calculating the optimized value of direct manufacturing (Q) and Remanufacturing (R) to begin remanufacturing and safety stock cycles.

4. Variables and Parameters of Proposed Model

Table (2) shown the variables used for the model:

<table>
<thead>
<tr>
<th>variables</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>r</td>
<td>Average rate of collecting returned items from customers per time unit (unit/time);</td>
</tr>
<tr>
<td>P_R</td>
<td>Remanufacturing Capacity (unit/time);</td>
</tr>
<tr>
<td>D</td>
<td>Average demand for final products(time/unit);</td>
</tr>
<tr>
<td>C_R</td>
<td>Set up cost of remanufacturing process (Rls./set up);</td>
</tr>
<tr>
<td>C_O</td>
<td>Cost of ordering new items(Rls./order);</td>
</tr>
<tr>
<td>C_Hr</td>
<td>Maintenance cost of returned items for remanufacturing(Rls./unit/time);</td>
</tr>
<tr>
<td>C_H</td>
<td>Direct products maintenance cost(Rls. unit/time);</td>
</tr>
<tr>
<td>T</td>
<td>Cycle time;</td>
</tr>
<tr>
<td>T_r</td>
<td>The time after which remanufacturing begins;</td>
</tr>
<tr>
<td>P</td>
<td>Inventory level of remanufactured items;</td>
</tr>
<tr>
<td>Q</td>
<td>Value of order for new items;</td>
</tr>
<tr>
<td>S_r</td>
<td>Value of safety stock in remanufacturing for taking care of variation of rate of collecting returned items;</td>
</tr>
<tr>
<td>Z</td>
<td>The number of standard deviations for a specified service level, which can be extracted directly from the normal distribution table;</td>
</tr>
<tr>
<td>σ_{L}</td>
<td>The standard deviation during the lead time period;</td>
</tr>
<tr>
<td>d_{t}</td>
<td>The standard deviation for each day during the lead time period;</td>
</tr>
</tbody>
</table>

Figure 1. Estimating market demand through remanufactured and new items modeling

Table 2. Proposed model variables and parameters
The proposed model is used for behavioral analysis and finding the optimized value of $R$, $Q$, $S_i$.

In order to formalize the model, following hypotheses are considered:

- a) Cost parameters are specified and will remain fixed;
- b) Purchase and remanufacturing time intervals are fixed;
- c) Collection rate is lower than remanufacturing rate;
- d) Demand rate is higher than collection rate.

### 4.1 Mathematical Formulation

Cost of remanufactured items includes set up and maintenance costs. So, total cost of remanufacturing is:

$$TC = C_R + R \times C_{HR} / 2 \times T$$ \hspace{1cm} (1)

Cost of repairable items includes ordering cost and cost of maintaining for triangles $\alpha$ and $\beta$ (figure 1).

$$C_o = \text{ordering cost} \hspace{1cm} (2)$$

The cost of maintaining for triangle $\alpha$ in figure (1) equals:

$$= \frac{D \times C_H}{2} \left\{ T_r - \frac{P_R - D}{D} (T-T) \right\}^2$$ \hspace{1cm} (3)

The cost of maintaining for triangle $\beta$ in figure (1) equals:

$$= \frac{P_R (P_R - D) C_H}{2D} (T-T)^2$$ \hspace{1cm} (4)

It also could be shown that:

$$= P_R \times \frac{R}{r (P_R - r)}$$ \hspace{1cm} (5)

And,

$$T_c = \frac{R}{r}$$ \hspace{1cm} (6)

Using equations (1) to (6), we have total cost as:

$$TC = \frac{r (P_R - r) (C_R + C_o)}{P_R \times R} + \frac{C_{HR}}{2} R + \frac{C_H}{2D (P_R - r)} \left\{ \frac{r (P_R - D) + P_R (D - r)}{r} \right\} R$$ \hspace{1cm} (7)

Since equation (7) is a function of $R$, we may have optimized value of $R$ through distinguishing total cost ($TC$) according to $R$, and then making it equal to zero. Therefore, the optimized value of $R$ would be:

$$R = \sqrt{\frac{r (P_R - r) (C_R + C_o)}{P_R \times A_1}}$$ \hspace{1cm} (8)

Where:
\[ A_1 = \frac{C_{HR}}{2} + \frac{C_H}{2D(P_a - r)} \left\{ \frac{P_a \times (D - r)^2}{r} + r(P_a - D) \right\} \]

Also, \( Q \) would be calculated via following equation:

\[ Q = \frac{P_a \times (D - r)}{r(P_a - r)} R \tag{9} \]

### 4.2 Safety Stock

Due to the variation of the demand rate, it is so important to consider the stock out condition in the direct manufacturing. The model considering the safety stock is shown in figure (1) with higher demand in the triangle \( AOB \). Due to higher demand, the zero stock condition take place at \( X \) instead of \( B \) (figure 1). Therefore to deal with this situation, a safety stock is essential.

The Safety stock can be computed via concept of service level. The service level is adjusted for very high values (more than 90%), which means that there is one such situation in every 100 cases, and items could be supplied from stock at least 90 times.

The Safety stock can be specified as: \( Z \sigma_L \)

\( \sigma_L \) can be calculated as:

\[ \sigma_L = \sqrt{\sum_{i=1}^{\sigma_L} (\sigma_{d_i})^2} \]

### 4.3 Numerical Example

In order to validate the model, a sample problem has been considered with following data:

- \( r = 150 \text{ unit/month} \)
- \( P_a = 350 \text{ unit/month} \)
- \( D = 250 \text{ unit/month} \)
- \( C_o = 15 \text{ Rls.} \)
- \( C_R = 25Rls. \)
- \( C_{HR} = 2\text{Rls.} \)
- \( C_{Rt} = 3\text{Rls.} \)
- \( \sigma_{d_i} = 5 \text{ unit} \)

Level of services which have to be maintained is 95%.

Purchase cost of new items = 8Rls.; and,

The purchase cost of collecting returned items for remanufacturing = 4Rls./unit.

Using equations (8), (9), and (5), following values could be obtained, respectively:

\[ R \approx 37, \; Q = 55, \; T = 0.555 \text{ month} = 16.65 \text{ day} \approx 17 \text{ day} \]

Safety stock also would be obtained as:

\[ Z\sigma_T = 1.65 \times \sqrt{\sum_{i=1}^{\sigma_T} (\sigma_{d_i})^2} = 1.65 \times \sqrt{17 \times 25} \approx 34 \]

Therefore, total cost of each cycle is:
\( TC = \) Total cost of inventory without safety stock, as calculated through equation (7) + cost of safety stock + cost of purchasing new items + cost of purchasing returned items for remanufacturing.

Now,
\( TC \) without safety stock = 109.55Rls.

Cost of safety stock = \( 8 \times 34 = 272 \) Rls.

Cost of purchasing new items without safety stock = \( 8 \times 55 = 440 \) Rls.

Cost of purchasing returned items = 222Rls.

Total cost = 109.55 + 272 + 440 + 222 = 1043.55 which is considered 1044 Rls.

5. Data Analysis and Discussion

To analyze the model and its inventory system, a developed mathematical model was used to find the relationship between various parameters in various situations. Hence, parameters will be selected and their relationships become clear. Applied values in the analysis are standard values used in ordinary production units. A brief interpretation of behavior is also presented to examine validity of the model.

As it is demonstrated, figure (2) shows value of \( C_o \) against big values of \( R \) and \( Q \). It is clear that in general, the values of \( R \) and \( Q \) are uniformly increased through the increase of \( C_o \), and the difference between \( R \) and \( Q \) shows that in bigger values of \( C_o \), the relative increase of \( Q \) is higher.

![Figure 2. Effect of order cost (C_o)](image)

Changes (shown in figure (3)) have considerable effect on the values of \( R \) and \( Q \). First, both items are increased. After \( C_s \) reaching to almost 20, \( Q \) will be increased with higher rate. However, this increase is of little effect on the difference between \( R \) and \( Q \); so, it could be mentioned that both of these values are changing in linear form through \( C_s \) changes.

Affect from \( P_r \) changes in figure (4) shows that through increase of \( P_r \), \( Q \) is decreased and \( R \) is increased.

Affect from \( D \) changes in figure (5) shows that in small values of \( D \), \( R \) and \( Q \) are close to each other. Through the increase of \( D \), \( Q \) would be increased and \( R \) will be decreased with fixed rates.
Figure 3. Effect of remanufacturing setup cost ($C_R$)

Figure 4. Effect of Capacity of remanufacturing ($P_R$)

Figure 5. Effect of demand rate ($D$)
Affect from $C_{HR}$ changing (figure 6) shows that $R$ and $Q$ will be decreased with fixed rate, through increase of $C_{HR}$. Also, the $R$ value is relatively smaller than $Q$.

![Figure 6. Effect of maintenance cost of returned items for remanufacturing ($CH_p$)](image)

From above analysis, it is clear that parameters $C_o, P, D,$ and $C_{HR}$ have to be kept small, to the extent possible. This helps the balance to be maintained between $R$ and $Q$; so that, both values will be kept close to each other. We are aiming at using returned items to make remanufacturing equal to the products from direct production. From the model analysis, it can be concluded that the cost of ordering new products, the capacity of the remanufacturing process, the final product demand, the maintenance costs of returning items inventory for remanufacturing, and the maintenance cost of new product inventory for should be looking at a low level.

5. Conclusion

Since the reverse supply chain is the most important factor in modern production, input resources to the production system may be put forward with less expense, in the form of returned items for remanufacturing. Through deduction of input resource costs, profitability of manufacturing systems will be increased. In the paper, a model of inventory system analysis has been presented, in which demand could be calculated through remanufactured and newly produced items. In such a system, returned items are used as raw materials, leading to a minimum level of raw material usage. The interpretive approach of modeling used in the model shows that reverse logistic variables are regularly found in reverse supply chain. From among the advantages of using the method, we may refer to the simultaneous calculation of optimized level of newly produced items, and optimized level of returning items for remanufacturing with the aim of minimizing wastes, and achieving competitive advantage of transformation cost. Moreover, the company may use the same production line for a longer period of time. Other studies in respect of the results from this research may be performed as follows: researchers may add other aspects of reverse supply chain such as recycling; rate of collection and/or rate of demand could be considered as random variables; and, final suggestion is that, a probable model would be discovered and developed in terms of reverse supply chain with remanufacturing.

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