An Extension of PROMETHEE with Evidential Evaluations

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ABSTRACT: *We consider ranking problems where the actions are evaluated on a set of ordinal criteria and where the evaluations are imperfect and represented by basic belief assignments (BBAs). In this paper, a model inspired by PROMETHEE is proposed within this context. The notions of ascending and descending belief functions are used in order to compare the alternatives on each criterion. The proposed approach is also illustrated by a pedagogical example.*

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1. Introduction

Multicriteria decision aid (MCDA) [1] is a field that deals with problems involving multiple conflicting criteria. Generally, authors distinguish three problems: the choice, the ranking and the classification. A ranking problem consists in ordering a set of actions from the best to the worst by building partial or total preorders. Several methods have been developed to tackle this problem. Among others, one can cite: PROMETHEE [2], ELECTRE III [3], AHP [4], etc.

Within MCDA, the modeling phase requires the identification of different kinds of data such as the evaluations of the actions, the criteria weights, etc. In most cases, this assessment step cannot be perfectly achieved and therefore imperfect data should be considered. Evidence theory [5], also called belief functions theory or Dempster-Shafer theory, is one of the several mathematical models that have been used to tackle such problem. In this paper, we will be interested to PROMETHEE method where the actions are evaluated on a set of ordinal criteria and where the evaluations can be uncertain and imprecise. The concept of basic belief assignment (BBA) [5], which is the basic function representing imperfect data in evidence theory, will be used to represent imperfect evaluations. We will call them "*evidential evaluations*".

PROMETHEE method is an outranking approach which is based on pairwise comparisons between the actions. The comparison of the BBAs expressing the evaluations of the actions can be performed using the First Belief Dominance (FBD) [6] (a generalization of the first stochastic dominance [7]) or the RBBD approach (RBBD I and RBBD II) [8]. However, the FBD and RBBD I concepts can lead to incomparable BBAs. On the contrary, the RBBD II approach leads to comparisons without incomparabilities, but the results induced by this concept can be viewed as excessive. Both approaches are based on the notions of ascending and descending belief functions [6]. In this work, we will propose a model inspired by PROMETHEE which is based on these notions.

This paper is organized as follows: in Section II we introduce some concepts of belief functions theory. The proposed model is presented in Section III. Finally, an illustrative example is described in Section IV.

2. Evidence Theory: Some Concepts

Evidence theory, introduced by Arthur Dempster [9] and developed later by Glenn Shafer [5], is a generalization of the subjective probability theory. This model is a convenient framework for modeling imperfect information and for combining it.

2.1 Basic Functions

Let X be a finite set of mutually exclusive and exhaustive elements called the frame of discernment and let $2X$ be the powerset of X. A Basic Belief Assignment (BBA) [5], is a function m defined from 2^X to [0,1] such as $m(\emptyset) = 0$ and $\sum_{A \subseteq X} m(A) = 1$ $m(A)$ represents the belief mass committed exactly to proposition A. When $m(A) \neq 0$, A is called a focal element. When all the focal elements are singletons, m is a probability function. Moreover, a BBA can be represented equivalently by its related belief and plausibility functions [5] defined from 2^X to [0,1] respectively as follows:

$$Bel(A) = \sum_{\substack{B \subseteq A \\ A \neq \phi}} m(B) \tag{1}$$

$$Pl(A) = \sum_{A \cap B \neq \phi} m(B) \tag{2}$$

$Bel(A)$ is interpreted as the total belief associated to A whereas $Pl(A)$ is viewed as the amount of belief that could potentially be placed in A. These two functions are connected by the equation $Pl(A) = 1 - Bel(\bar{A})$ where $\bar{A}$ is the complement of A.

2.2 Ascending and descending belief functions

The ascending and descending belief functions are two notions that have been developed in the context of MCDA and which constitute the basis of FBD and RBBD concepts [6] [8]. These functions suppose that the frame of discernment $X = \{x_1, x_2, ..., x_r\}$ is composed by ordered elements defined such as $x_1 \prec x_2 \prec ... \prec x_r$. For all $k \in \{0, 1, ..., r\}$ let:

$$A_k = \begin{cases} \emptyset & \text{if } k = 0 \\ \{x_1, ..., x_r\} & \text{Otherwise} \end{cases} \tag{3}$$

And let $\vec{S}(X)$ denotes the set $\{A_1, A_2, ..., A_r\}$. Similarly, for all $l \in \{0, 1, ..., r\}$ such as $l = r - k$ let:

$$B_l = \begin{cases} \emptyset & \text{if } l = 0 \\ \{x_{r-l+1}, ..., x_r\} & \text{Otherwise} \end{cases}$$

And let $\overleftarrow{S}(X)$ denotes the set $\{B_1, B_2, ..., B_r\}$. k and l represent respectively the number of elements of the sets A_k and B_l.

Definition 1. [6] The ascending belief function , $\overrightarrow{Bel}$ induced by a BBA m, is a function $\overrightarrow{Bel} : \vec{S}(X) \rightarrow [0, 1]$ defined such as $\overrightarrow{Bel}$: $(A_k) = \sum_{C \subseteq A_k} m(C)$ for all $A_k \in \vec{S}(X)$.

Definition 2. [6] The descending belief function , $\overleftarrow{Bel}$, induced by a BBA m is a function $\overleftarrow{Bel} : \overleftarrow{S}(X) \rightarrow [0, 1]$ defined such as $\overleftarrow{Bel}$: $(B_l) = \sum_{C \subseteq B} m(C)$ for all $B_l \in \overleftarrow{S}(X)$.

$\overrightarrow{Bel}$ and $\overleftarrow{Bel}$ allow taking account implicitly the fact that $x_1 \prec x_2 \prec \ldots \prec x_r$. Indeed, the former represents the beliefs of the nested sets $A_1, A_2, \ldots, A_r$, i.e., $\{x_1\}, \{x_1, x_2\}, \ldots, \{x_1, \ldots, x_r\}$. Similarly, the latter represents the beliefs of the nested sets $B_1, B_2, \ldots, B_r$, i.e., the sets , $\{x_r\}, \{x_{r-1}, x_r\}, \ldots, \{x_1, \ldots, x_r\}$. Since x_1 and x_r are respectively the worst and the best elements of X, the more the values of $\overrightarrow{Bel}$ decrease and those of $\overleftarrow{Bel}$ increase, the better is the BBA m.

3. Proposed Approach

We consider ranking problems that can be represented by three elements:

- $A = \{a_1, a_2, \ldots, a_n\}$: the set of actions;
- $G = \{g_1, g_2, \ldots, g_n\}$: the set of ordinal criteria;
- $X = \{x_1, x_2, \ldots, x_n\}$: the assessment grades set used to evaluate the actions and defined such as $x_1 \prec x_2 \prec \ldots \prec x_r$ Without any loss of generality, we will consider that this set is the same for all the criteria.

In what follows, we will suppose that the evaluation of an action a_i on a given criterion g_h can be uncertain and imprecise and modeled by BBA denoted m_i^h .

In order to rank the alternatives, the proposed approach inspired by PROMETHEE proceeds in the following steps: the comparison based on the ascending and descending belief functions, the determination of the preference degrees and the ascending and descending rankings related to each comparison and finally the aggregation of these rankings.

3.1 Comparison

This step consists in comparing each pair of BBAs m_i^h and m_j^h representing the evaluations of actions a_i and a_j on each criterion g_h. These comparisons are established based on the notions of the ascending and descending belief functions defined above.

3.1.1 Comparison using the ascending belief function

This comparison requires at first computing $\overrightarrow{Bel}_i^h$ and $\overrightarrow{Bel}_j^h$ related respectively to m_i^h and m_j^h and then their sums. It allows deducing the following preference situations on g_h:

- a_i is preferred to a_j on g_h ($a_i \, P \, a_j$) if and only if

$$\sum_{A_k \in \vec{S}(X)} \overrightarrow{Bel}_i^h(A_k) < \sum_{A_k \in \vec{S}(X)} \overrightarrow{Bel}_j^h(A_k)$$

- a_j is preferred to a_i on g_h ($a_j \, P \, a_i$) if and only if

$$\sum_{A_k \in \vec{S}(X)} \overrightarrow{Bel}_i^h(A_k) < \sum_{A_k \in \vec{S}(X)} \overrightarrow{Bel}_j^h(A_k)$$

- a_i and a_j are indifferent on g_h ($a_i \, I \, a_j$) if and only if

$$\sum_{A_k \in \vec{S}(X)} \overrightarrow{Bel}_i^h(A_k) < \sum_{A_k \in \vec{S}(X)} \overrightarrow{Bel}_j^h(A_k)$$

3.1.2 Comparison using the descending belief function

This comparison is based on $\overleftarrow{Bel}_i^h$ and $\overleftarrow{Bel}_j^h$ and their related sums. As in the ascending case, three preference situations on g_h can be distinguished:

- a_i is preferred to a_j on g_h ($a_i \, P \, a_j$) if and only if

$$\sum_{B_l \in \overleftarrow{S}(X)} \overleftarrow{Bel}_i^h(B_l) < \sum_{B_l \in \overleftarrow{S}(X)} \overleftarrow{Bel}_j^h(B_l)$$

- a_j is preferred to a_i on g_h ($a_j \, P \, a_i$) if and only if

$$\sum_{B_l \in \overleftarrow{S}(X)} \overleftarrow{Bel}_i^h (B_l) < \sum_{B_l \in \overleftarrow{S}(X)} \overleftarrow{Bel}_j^h (B_l)$$

• a_i and a_j are indifferent on g_h $(a_i \ I \ a_j)$ if and only if

$$\sum_{B_l \in \overleftarrow{S}(X)} \overleftarrow{Bel}_i^h (B_l) = \sum_{B_l \in \overleftarrow{S}(X)} \overleftarrow{Bel}_j^h (B_l)$$

3.2 Calculation of the preference degrees

The preference function [2] is the distinctive feature of PROMETHEE. This function, denoted $F_h(a_i, a_j)$, quantifies the preference intensity of a_i to a_j on g_h. It varies between 0 and 1: the closer to 1, the greater the preference of a_i to a_j on g_h.

$F_h(a_i, a_j)$ can be one of the six types proposed by Brans and Vincke (usual, linear, Gaussian, etc) [2]. The usual type is the most used function when the criterion is ordinal. That is why we will consider it in our model. Formally, this type is defined as follows:

$$F_h(a_i, a_j) = \begin{cases} 1 & \text{if } a_i P \, a_j \text{ on } g_h \\ 0 & \text{otherwise} \end{cases} \quad (5)$$

3.2.1 Calculation of the preference degrees based on the comparison of the ascending belief function

In this case, the preference function is defined as follows:

$$F_h(a_i, a_j) = \begin{cases} 1 & \text{if } \sum_{A_k \in \vec{S}(X)} \overrightarrow{Bel}_i^h (A_k) < \sum_{A_k \in \vec{S}(X)} \overrightarrow{Bel}_j^h (A_k) \\ 0 & \text{otherwise} \end{cases} \quad (6)$$

Once all $F_h(a_i, a_j)$ are deduced on all the criteria, the preference degree (a_i, a_j) is computed. This degree represents the preference intensity of a_i over a_j on all the criteria. It is defined as follows:

$$\Pi(a_i, a_j) = \sum_{h=1}^{q} w_h F_h(a_i, a_j) \quad (7)$$

Where w_h is the weight of criterion g_h defined such as $w_h > 0$ and $\sum_h w_h = 1$. $\Pi(a_i, a_j)$ is a number between 0 and 1: the closer to one, the greater the global preference of a_i over a_j.

3.2.2 Calculation of the preference degrees based on the comparison of the descending belief function

In this case, the preference function is given by:

$$F_h(a_i, a_j) = \begin{cases} 1 & \text{if } \sum_{B_l \in \overleftarrow{S}(X)} \overleftarrow{Bel}_i^h (B_l) > \sum_{B_l \in \overleftarrow{S}(X)} \overleftarrow{Bel}_j^h (B_l) \\ 0 & \text{otherwise} \end{cases} \quad (8)$$

The preference degree is determined as above using formula (7).

Finally, it is worth mentioning that Ben Amor and Mareschal [10] have proposed an extension of PROMETHEE where the evaluations are imperfect and modeled by probability functions, BBAs, possibility distributions, etc.

They have developed a general framework in order to integrate these different types of imperfect information. The concept of stochastic dominance [11] has been used to compare the evaluations. However, this approach can lead to incomparable evaluations. To tackle this problem, the preference function has been defined as follows: $F_h(a_i, a_j) = 0.5$. This can solve the problem of incomparability but it is not well justified.

3.2.3 Ascending and descending PROMETHEE I and II rankings

PROMETHEE I is a partial ranking of the actions that is obtained on the basis of the following flows:

• The leaving flow $\phi^+(a_i)$:

$$\phi^+(a_i) = \frac{1}{n-1}\sum_{\substack{j=1 \\ j\neq i}}^{n} \prod(a_i, a_j) \tag{9}$$

• The entering flow $\phi^-(a_i)$:

$$\phi^-(a_i) = \frac{1}{n-1}\sum_{\substack{j=1 \\ j\neq i}}^{n} \prod(a_i, a_j) \tag{10}$$

$\phi^+(a_i)$ expresses the outranking character of a_i over the other alternatives whereas $\phi^-(a_i)$ represents its outranked character: the higher $\phi^+(a_i)$ and the lower $\phi^-(a_i)$ the better a_i. Both flows do not usually induce the same ranking. PROMETHEE I is their intersection.

It is also possible de deduce a total preorder of the actions. This ranking is called PROMETHEE II. For that purpose, we should compute for each alternative its net flow $\phi(a_i)$ given by:

$$\phi(a_i) = \phi^+(a_i) - \phi^-(a_i) \tag{11}$$

$\phi(a_i)$ is the balance between $\phi^+(a_i)$ and $\phi^-(a_i)$ the higher its value, the better a_i. Finally, let us note that since the preference degrees can be determined based on the ascending (descending, resp.) belief function, we can deduce therefore ascending (descending, resp.) PROMETHEE I ranking and ascending (descending, resp.) PROMETHEE II ranking.

3.2.4 Aggregated PROMETHEE I and II rankings

In this step, the ascending and descending PROMETHEE I (PROMETHEE II, resp.) rankings are aggregated in order to obtain a global PROMETHEE I (PROMETHEE II, resp.) ranking. Four preference situations can be distinguished: the preference P of a_i to a_j or of a_j to a_i, the indifference I and the incomparability J. These situations are defined formally as follows (R_1 and R_2 refer respectively to the ascending and descending rankings):

$$\begin{cases} a_i P a_j \Leftrightarrow \begin{cases} a_i P_{R_1} a_j \text{ and } a_i P_{R_2} a_j \\ a_i P_{R_1} a_j \text{ and } a_i I_{R_2} a_j \\ a_i I_{R_1} a_j \text{ and } a_i P_{R_2} a_j \end{cases} \\ a_j P a_i \Leftrightarrow \begin{cases} a_j P_{R_1} a_i \text{ and } a_j P_{R_2} a_i \\ a_j P_{R_1} a_i \text{ and } a_j I_{R_2} a_i \\ a_j I_{R_1} a_i \text{ and } a_j P_{R_2} a_i \end{cases} \\ a_i I a_j \Leftrightarrow a_i I_{R_1} a_j \text{ and } a_i P_{R_2} a_j \\ a_i J a_j \Leftrightarrow \text{otherwise} \end{cases} \tag{12}$$

Finally, let us note that the aggregated PROMETHEE II ranking is a partial preorder since the ascending and descending PROMETHEE II rankings are not usually the same. Moreover, let us mention that the aggregated PROMETHEE I and II rankings can induce the same preorder.

4. Illustrative Example

In order to illustrate our approach, let us consider the following ranking problem. A company wants to recruit a new collaborator for the marketing department. Five candidates are considered. A decision for selecting a candidate c_i (with) $i = 1, 2, ..., 5$ has to be made based on three qualitative criteria to be maximized: the learning capacities, the past experience and the communication skills. The criteria weights are respectively 0.5, 0.3 and 0.2. The candidates are evaluated by the director of human resources department. For each criterion, five assessment grades are considered: x_2 "*very bad* ", x_2 "*bad* ", x_3 "*average*", x_4 "*good* " and x_5 "*excellent* ". The set of these grades is denoted by X.

	g_1	g_2	g_3
c_1	$m_1(\{x_3\}) = 0.94$ $m_1(\{x_3, x_4\}) = 0.06$	$m_1(\{x_4\}) = 1$	$m_1(\{x_3\}) = 0.84$ $m_1(\{x_3, x_4\}) = 0.16$
c_2	$m_2(\{x_3\}) = 0.6$ $m_2(\{x_2, x_3\}) = 0.4$	$m_2(\{x_3\}) = 1$	$m_2(\{x_4\}) = 0.6$ $m_2(\{x_4, x_5\}) = 0.4$
c_3	$m_3(\{x_2\}) = 1$	$m_3(\{x_3\}) = 0.28$ $m_3(\{x_4\}) = 0.44$ $m_3(\{x_3, x_4\}) = 0.28$	$m_3(\{x_2\}) = 0.33$ $m_3(\{x_3\}) = 0.67$
c_4	$m_4(\{x_1\}) = 0.67$ $m_4(\{X\}) = 0.33$	$m_4(\{x_4\}) = 0.67$ $m_4(\{x_3, x_4\}) = 0.33$	$m_4(\{x_2\}) = 0.9$ $m_4(\{x_3\}) = 0.05$ $m_4(\{x_2, x_3\}) = 0.05$
c_5	$m_5(\{x_2\}) = 1$	$m_5(\{x_4\}) = 1$	$m_5(\{x_3\}) = 0.6$ $m_5(\{x_3, x_4\}) = 0.4$

Table 1. Candidate's Performances

	c_1	c_2	c_3	c_4	c_5	ϕ^+	ϕ
c_1	-	0.8	1	0.7	0.5	0.75	0.650
c_2	0.2	-	0.7	0.7	0.7	0.575	0.150
c_3	0	0.3	-	0.2	0	0.125	-0.625
c_4	0	0.3	0.8	-	0.5	0.4	-0.05
c_5	0.2	0.3	0.5	0.2	-	0.3	-0.125
ϕ^-	0.1	0.425	0.75	0.450	0.425		

Table 2. Preference Degrees And Flows Related To the Ascending Belief Function

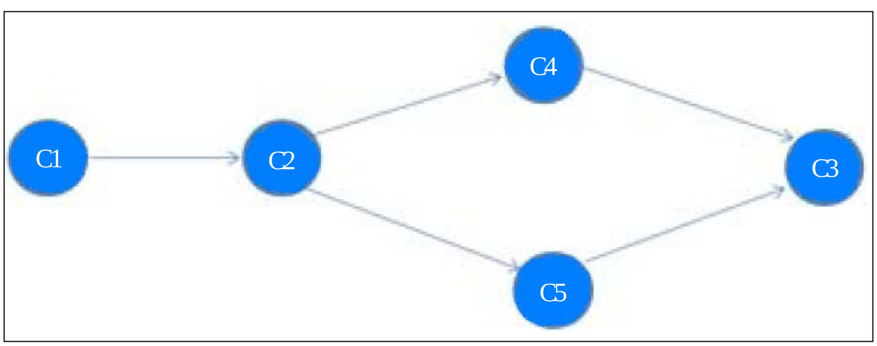

Figure 1. Ascending PROMETHEE I ranking

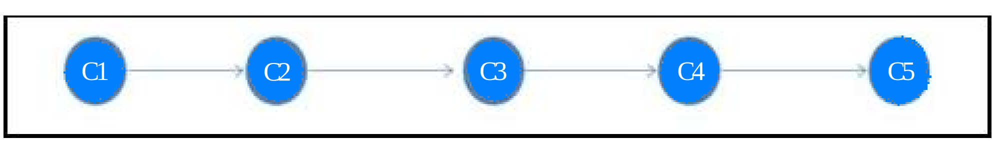

Figure 2. Ascending PROMETHEE II ranking

The candidates' performances on each criterion are given by BBAs and are presented in Table 1. For instance, the evaluation of candidate c_1 on criterion g_3 is established as follows: the director of human resources department hesitates between the third and the fourth assessment grades. He is sure that the candidate has either average or good communication skills without being able to refine his judgment.

As described above, the proposed approach allows us to deduce two types of rankings: the ascending ranking related to the acceding belief function and the descending ranking related to the descending belief function.

4.1 Ascending PROMETHEE I and II rankings

The first step of our model consists in comparing each pair of actions on each criterion. For that purpose, we compute the ascending belief function of each alternative on each criterion. Then, we determine the sum of this function and we compare the values of each pair of sums. Tables 4 give the values of these sums and Tables 6, 7 and 8 present the induced preference situations between each pair of alternatives on each criterion.

In the second step, the values of preference functions are at first deduced using formula (6). The preference degrees between each pair of actions are then computed. Finally, the entering, leaving and net flows are determined. The results are presented in Table 2. Figures 1 and 2 give the ascending PROMETHEE I and II rankings.

As one can remark, c_1 is the best candidate. c_4 and c_5 are incomparable according to the second ranking.

4.2 Descending PROMETHEE I and II rankings

This ranking is based on the descending belief function. We should compute at first for each alternative on each criterion this function, determine its sum, compare each pair of sums and deduce the preference situations between the actions on each criterion (see Tables 9, 10 and 11). Then, the values of preference functions are determined using formula (8). The preference degrees, the entering, leaving and net flows are computed. The results are described in Tables 3. The descending PROMETHEE I and II rankings are illustrated in Figures 3 and 4.

	c_1	c_2	c_3	c_4	c_5	ϕ^+	ϕ
c_1	-	0.8	1	1	0	0.7	0.650
c_2	0.2	-	0.2	0.2	0.2	0.2	–0.225
c_3	0	0.3	-	0.7	0	0.25	–0.25
c_4	0	0.3	0.3	-	0	0.15	–0.575
c_5	0	0.3	0.5	1	-	0.45	0.4
ϕ^-	0.05	0.425	0.5	0.725	0.05		

Table 3. Preference Degrees And Flows Related To the Descending Belief Function

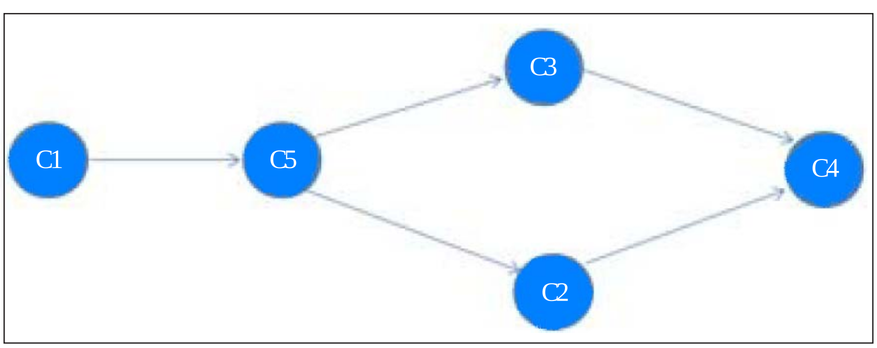

Figure 3. Descending PROMETHEE I ranking

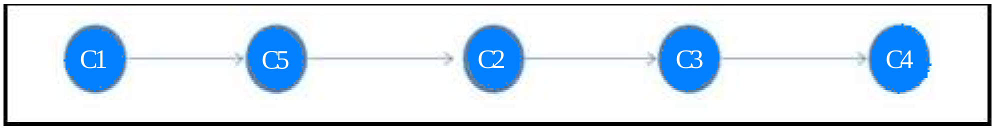

Figure 4. Descending PROMETHEE II ranking

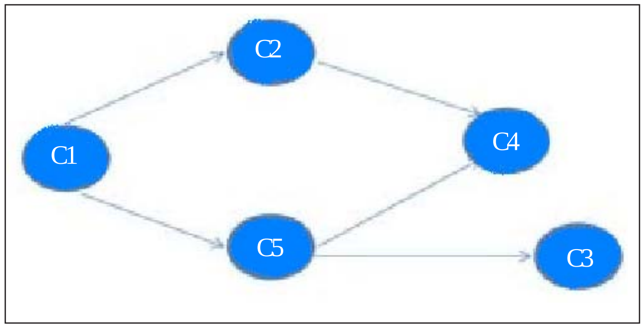

Figure 5. Agregated PROMETHEE I ranking

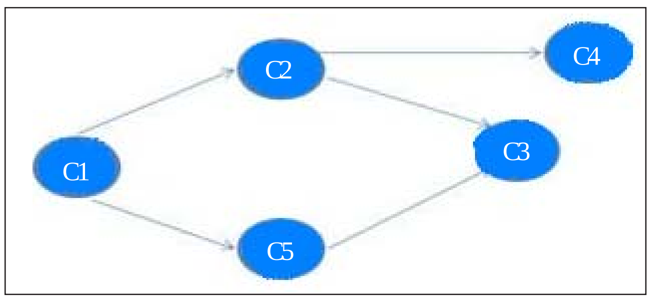

Figure 6. Agregation PROMETHEE II ranking

$\sum_{A_k \in \vec{S}(X)} \overrightarrow{Bel}_j^h(A_k)$	g_1	g_2	g_3
c_1	2.94	2	2.84
c_2	3.6	3	1.6
c_3	4	0.3	3.33
c_4	3.68	2.28	3.9
c_5	4	2	2.6

Table 4. The Sum of the Ascending Belief Function

$\sum_{B_l \in \overleftarrow{S}(X)} \overleftarrow{Bel}_j^h(B_l)$	g_1	g_2	g_3
c_1	3	4	3
c_2	2	3	4
c_3	2	3.44	2.05
c_4	1	3.67	2.67
c_5	2	4	3

Table 5. The Sum Of The Descending Belief Function

As can be seen, c_1 is the best candidate. c_2 and c_3 are incomparable according to the second ranking.

4.3 Aggregated PROMETHEE I and II rankings

It is also possible to deduce two aggregated rankings related respectively to the ascending and descending PROMETHEE I

	c_1	c_2	c_3	c_4	c_5
c_1	-	P	P	P	P
c_2	P^{-1}	-	P	P	P
c_3	P^{-1}	P^{-1}	-	P^{-1}	I
c_4	P^{-1}	P^{-1}	P	-	P
c_5	P^{-1}	P^{-1}	P	P^{-1}	-

Table 6. Preference Relations On Criterion g_1

	c_1	c_2	c_3	c_4	c_5
c_1	-	P	P	I	I
c_2	P^{-1}	-	P^{-1}	P^{-1}	P^{-1}
c_3	P^{-1}	P^{-1}	-	P^{-1}	I
c_4	P^{-1}	P^{-1}	P	-	P
c_5	P^{-1}	P^{-1}	P	P^{-1}	-

Table 7. Preference Relations On Criterion g_2

	c_1	c_2	c_3	c_4	c_5
c_1	-	P^{-1}	P	P	P^{-1}
c_2	P	-	P	P	P
c_3	P^{-1}	P^{-1}	-	P	P^{-1}
c_4	P^{-1}	P^{-1}	P^{-1}	-	P
c_5	P	P^{-1}	P	P	-

Table 8. Preference Relations On Criterion g_3

rankings and to the ascending and descending PROMETHEE II rankings. The results are given in Figures 5 and 6.

As can be noticed, c_1 is the best candidate in both rankings. In the aggregated PROMETHEE I ranking, c_2 is incomparable to c_3 since c_2 is preferred to c_3 in the ascending ranking and c_2 and c_3 are incomparable in the descending ranking. Moreover, c_2 is incomparable to c_5 since c_2 is preferred to c_5 in the ascending ranking and c_5 in preferred to c_2 in the descending ranking. c_4 and c_3 are also incomparable since c_4 is preferred to c_3 in the ascending ranking and c_3 is preferred to c_4 in the descending ranking.

In the aggregated PROMETHEE II ranking, c_5 is incomparable to c_2 and c_4 since c_2 and c_4 are preferred to c_5 in the ascending ranking and c_5 is preferred to c_2 and c_4 in the descending ranking. c_4 and c_3 are also incomparable since c_4 is preferred to c_3 in the ascending ranking and c_3 is preferred to c_4 in the descending ranking.

5. Conclusion

In this paper, we have addressed the multicriteria ranking problems where the actions are evaluated on ordinal criteria and where the evaluations are given imperfectly. Evidence theory has been used to tackle this problem. At first, the concept of BBA allows to represent imperfect evaluations. Then, the ascending and descending belief functions have been applied in order to compare the evaluations

Further research is needed to determine an aggregated total ranking. Another possible line of research is in extending this method in the context of group decision making.

Appendix A. The sum of the ascending and descending belief functions. Tables 4-5.

Appendix B. Tables 6-7: The established preference relations between the candidates on each criterion based on the ascending belief function.

Appendix C. Tables 9-11: The established preference relations between the candidates on each criterion based on the descending belief function

	c_1	c_2	c_3	c_4	c_5
c_1	-	P	P	P	P
c_2	P^{-1}	-	I	I	I
c_3	P^{-1}	I	-	P	I
c_4	P^{-1}	P^{-1}	P^{-1}	-	P^{-1}
c_5	P^{-1}	I	I	P	-

Table 9. Preference Relations On Criterion g_1

	c_1	c_2	c_3	c_4	c_5
c_1	-	P	P	P	P
c_2	P^{-1}	-	I	I	I
c_3	P^{-1}	I	-	P	I
c_4	P^{-1}	P^{-1}	P^{-1}	-	P^{-1}
c_5	P^{-1}	I	I	P	-

Table 10. Preference Relations On Criterion g_2

	c_1	c_2	c_3	c_4	c_5
c_1	-	P^{-1}	P	P	P
c_2	P	-	P	I	I
c_3	P^{-1}	I	-	P	P^{-1}
c_4	P^{-1}	P^{-1}	P^{-1}	-	P^{-1}
c_5	P^{-1}	I	I	P	-

Table 11. Preference Relations On Criterion g_3

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