

Set Pair Analysis of Lattice Order Decision-making Model and Application



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ABSTRACT: *When the theories and methods of set pair analysis are applied to the lattice order decision-making, a new decision-making model— set pair analysis of lattice order decision making model is created. Based on the portray of the lattice order of the set pair utility function, the best solution of the decision model is found.*

Keywords: Lattice Order Decision-Making, Set Pair Utility Function, Connection Degree, Set Pair Analysis

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1. Introduction

Professor Guo Yaohuang, using algebra lattice theory, promoted the total order depict of Von Neumann-Morgenstern's rational behavior axiom system to the lattice ordered depict, and then he proposed a new lattice order decision-making theory. In this way, Professor Guo build a lattice order decision-making^[1] system of rational behavior. Connectivity axiom and transitivity of the traditional VNM rational behavior axiom system require that all decision consequences should be comparable with the preference relation and should be transitive as well. This actually makes preference relationship into a total order relationship, which seems to be unreasonable. This condition is too strong to far apart with the actual decision situations. Therefore, under the condition of keeping the rational decision-making behavior following the independence axiom, lattice order decision making theory weakens the traditional connectedness axiom to connectivity axiom of lattice. But connectivity axiom of lattice does not require decision scheme be comparable in pairs. On the one hand, it ensures that there is a certain relationship between the various solutions; on the other hand it avoids the decision problem which has no comparative information. The creation of lattice order decision making theory filled the blank of lattice theory applied in the decision science and opened a research direction in the same field. In recent years, people, for the lattice order decision model, have carried out in-depth research and tried to give out a practical method of Lattice order decision-making. At present, the major problem is how to establish and perfect the lattice order decision making model of the giving set pair utility function.

This essay, applying the theory and method of set pair utility function to the lattice order decision making, analyzes lattice order structure characteristics of the sets of utility function, then establishes lattice order decision making model of the set pair analysis, and at last gives some application example.

2. Set pair Analysis and Set Pair Utility Function

2.1 The Basic Theory of Set Pair Analysis

Set pair is a pair which has a certain connection of two sets. The core idea of set pair analysis is to analyze the certainty and uncertainty of the objects as a certain-uncertain system^[2]. In some specific context, the characteristics of the two target set are analyzed from such perspectives as their similarities, their differences. Meanwhile a quantitative analysis is made. In this way, the connection degree expression of the two sets is got. Based on these analysis, such aspects as contact, decision-making, forecast, control, simulation, evaluation, evolution and mutation, etc^[4]. are the target of the further analysis.

Definition 2.1: two sets A and B are given, and $H = (A, B)$ is a set pair made up with the two Sets. In some specific context W , set pair H has N features, among which S features are mutual of A and B . They are opposite on P features. They are neither opposite nor similar in the rest F features ($F = N - S - P$). We define the ratio as follow:

S/N is the identity degree of A and B under background W , shortened as identity degree;

F/N is the discrepancy degree of A and B under background W , shortened as discrepancy degree;

P/N is the contrary degree of A and B under background W , shortened as contrary degree;

All these can be represented by the formula $\mu(W) = \frac{S}{N} + \frac{F}{N}i + \frac{P}{N}j$. $\mu(W)$ is the degree contact of set A and set B .

For simplicity, if we let $a = S/N, b = F/N, c = P/N$, then it can be recorded as the following: $u = a + bi + cj$. i is the mark of difference degree, and $i \in [-1, 1]$, j is the mark of contrary degree, and $j = -1$. Obviously, $0 \leq a, b, c \leq 1$ and $a + b + c = 1$.

2.2. The Related Knowledge of the Set Utility Function

2.2.1. Utility Function

In decision theory, utility is a concept which represents the results of the scheme, satisfies and achieves the decision maker's preference degree; at the same time it's value, which can be tested with some specific methods, and can be used as the basis of decision analysis. Assume that each feasible scheme of the decision problem may be results in different ways, each result can be of different value and effects to their decision-makers according to their subjective desire and value-orientation, each value of the result of decision makers have different value and effect and reaction result value for decision makers. Thus, utility should be some volume of value and effect that perform to the decision makers^[9].

The utility function $u(x)$ is a kind of relative measurement, generally ranging between 0 and 1 ($0 \leq u(x) \leq 1$), in which x is profit or monetary value, and the utility function is a increasing function of x .

2.2.2. Set Pair Utility Function

In the decision making process, decision makers tend to show the hesitant and perplexed psychological, which can be agreement, objection or neutrality whose counterparts are respectively identity degree, difference degree and contrary degree. Therefore, it is necessary to build a set pair utility function^[8], and to depict positive utility, negative utility and uncertainty utility in order to make the decision-making more objective and feasible.

Decision problem can be expressed in a format, which is called a decision table, also called decision matrix. For simplicity, assume that there are certain kinds of possible states which is incompatible, marks as $V = \{V_1, V_2, V_3, \dots, V_n\}$ (named as set of states), and the probability of occurrence of various natural states is represented by P . At the same time, assume that there are certain kinds of possible actions, which make up an action set represented as $S = \{S_1, S_2, S_3, \dots, S_n\}$. Decision-makers must select only one of these actions. If the consequences of S_i (actions) and V_j (real state) marked as x_{ij} , then we can get the decision table shown in table 1.

Based on the decision format context above, to structure a set pair utility function, the procedure is as follows:^[8]

Step 1: In the format context of decision problems, select the best value x_{ij} , make the minimum value x_{sq} , and then make $U(x_{ij}) = 1, U(x_{sq}) = 0$;

V_j (real state) S_i (actions)	$V_1 (P_1)$	$V_2 (P_2)$	$V_n (P_n)$
S_1	x_{11}	x_{12}	...	x_{1n}
S_2	x_{21}	x_{22}	...	x_{2n}
...
S_m	x_{m1}	x_{m2}	...	x_{mn}

Table 1. The general form of the decision table

Step 2: Seek for a suitable value x_{jg} between the best value and the minimum value to make $U(x_{jg} = 0.5)$;

Step 3: Choose the type of utility curve. Generally, we present parameters as $U(x) = A + B \ln(x + C)$. According to the three function values above, we can get the formula for the utility function: $U(x)$;

Step 4: Construct the set pair utility function: $f(x) = a(x) + b(x)i + c(x)j$

Under the condition of the probability P_i to take the biggest, $a(x)$; is the positive utility, $a(x) = P_i U(x)$; under the condition of the probability P_i to take the minimum, $c(x)$ is the negative utility, $c(x) = P_m U(x)$; $b(x)$ is the utility of uncertainty, $b(x) = 1 - a(x) - c(x) = 1 - (P_i + P_m)U(x)$. In this way the set pair utility function can be shown as follows:

$$f(x) = P_i U(x) + (1 - (P_i + P_m)U(x))i + P_m U(x)j$$

Step 5: The set of set pair utility function is got:

$$U(S) = \{U_1, U_2, U_3, \dots, U_m\},$$

$$U_i = U(S_i) = \sum_{j=1}^n P_j f(x_{ij}), i = 1, 2, \dots, m$$

3. Analysis Device of Lattice Order Decision of the Set of Set Pair Utility Function

3.1. The Partial Order Relation of the Set of Set Pair Utility Function

Generally, the set of set pair utility function $U(S) = \{U_1, U_2, U_3, \dots, U_n\}$ can be simply expressed as $U = \{(a_1, b_1, c_1), (a_2, b_2, c_2), \dots, (a_m, b_m, c_m)\}$, $a_i, b_i, c_i = 1, i = 1, 2, \dots, m$.

Definition 3.1^[1]: If A is a set, $R \subseteq A \times A$, R is for a binary relation in A . If $(a, b) \in R$, A and b have a relation R , marked as aRb ; If $(a, b) \notin R$, a and b have no relationship like R , marked as $aR'b$ or non- aRb .

Definition 3.2: Suppose U as a set of set pair utility function in some context, $\mu_1, \mu_2 \in U$, and $\mu_1 = a_1 + b_1 i + c_1 j, \mu_2 = a_2 + b_2 i + c_2 j$, then:

(1) If $a_1 = a_2, b_1 = b_2, c_1 = c_2$, μ_1 and μ_2 are equivalence, marked as $\mu_1 = \mu_2$;

(2) If $a_1 \leq a_2, a_1 + b_1 \leq a_2 + b_2$, μ_2 takes priority to μ_1 , marked as $\mu_1 \leq \mu_2$;

(3) If $a_1 < a_2, a_1 + b_1 < a_2 + b_2$, μ_2 takes absolute priority to μ_1 , marked as $\mu_1 < \mu_2$;

Definition 3.2 shows that precedence relation of the set of set pair utility function's has the following characters:

Character 3.1: reflexivity: for any μ_i , then $\mu_i = \mu_i$;

Character 3.2: antisymmetry: if $\mu_1 \leq \mu_2$, $\mu_2 \geq \mu_1$, then $\mu_1 = \mu_2$;

Character 3.3: transitivity: if $\mu_1 \leq \mu_2$, $\mu_2 \leq \mu_3$, then $\mu_1 \leq \mu_3$; Obviously, set U under the precedence relationship constitutes a poset.

In a poset (U, \leq) , suppose A and B are poset elements in U . If $\mu_1 \leq \mu_2$ or $\mu_2 \leq \mu_1$, μ_1 and μ_2 is comparable; otherwise μ_1 and μ_2 are not comparable, marked as $\mu_1 \parallel \mu_2$.

3.2 Lattice Ordered Structure of the Set of Set Pair Utility Function

U is a set of set pair utility function, let $\forall U_1, U_2 \subseteq U$, $U_1 = a_1 + b_1i + c_1j$, and $U_2 = a_2 + b_2i + c_2j$, suppose:

(1) Supremum of the set U is a connection degree expression of $U_i = a_i + b_i i + c_i j$, as $U_i = \sup(\{U_1, U_2\})$, then:

$$a_i = [\max(a_1 + b_1, a_2 + b_2) + \max(a_1 - b_1, a_2 - b_2)] / 2$$

$$b_i = [\max(a_1 + b_1, a_2 + b_2) - \max(a_1 - b_1, a_2 - b_2)] / 2$$

$$c_i = 1 - a_i - b_i$$

(2) Infimum of the set U is a connection degree expression of $U_j = a_j + b_j i + c_j j$, as $U_j = \inf(\{U_1, U_2\})$, then:

$$a_j = [\min(a_1 + b_1, a_2 + b_2) + \min(a_1 - b_1, a_2 - b_2)] / 2$$

$$b_j = [\min(a_1 + b_1, a_2 + b_2) - \min(a_1 - b_1, a_2 - b_2)] / 2$$

$$c_j = 1 - a_j - b_j$$

Supremum and Infimum of connection degree can be calculated with the following rules:

Idempotent rate: $U_i = \sup(\{U_1, U_1\})$ $U_i = \inf(\{U_1, U_1\})$

Commutation rate: $\sup(\{U_1, U_2\}) = \sup(\{U_2, U_1\})$ $\inf(\{U_1, U_2\}) = \inf(\{U_2, U_1\})$

Combine rate: $\sup(\{U_1, U_2\}, \{U_3\}) = \sup(\{U_1\}, \{U_2, U_3\})$

Definition 3.3: (U, \leq) is a poset of set pair utility functions, if any of the two elements have a supremum and an infimum, U on partial order " \leq " constitutes a lattice of set pair utility function, and " \leq " is a lattice order of set pair utility in U .

Theorem 3.1: In a poset $U = \{U(s_1), U(s_2), \dots, U(s_m)\}$, adding 2 $(m - 1)$ elements at most, then we can constitute an extended poset \bar{U} , making a lattice ordered structure.

Demonstration: Considering the worst case of poset, namely the elements in $U = \{U(s_1), U(s_2), \dots, U(s_m)\}$ form a anti-chain, i. e., any two elements are not comparable. These m elements can be regarded as a m leaf nodes. Using the structure theory of complete binary tree, we can structure superposition of two binary tree and then we get a lattice.

According to the properties of the binary tree, suppose the number of nodes of 2^0 is n_2 , the number of nodes of 0^0 is n_1 , then the relation between the number of leaf nodes and the number of the nodes of 2^0 can be represented as $n_2 = n_0 - 1$. So in the anti-chain, adding 2 $(m - 1)$ elements will result in a lattice structure.

Definition 3.4: In any lattice $(U, \leq), \forall u_i, u_j \in U, u_i = a_i + b_i i + c_j j, u_j = a_j + b_j i + c_j j$, the distance function can be defined as

$$d(u_i, u_j) = \sqrt{(a_i - a_j)^2 + (b_i - b_j)^2}$$

In any lattice $(U, \leq), \forall u_i, u_j, u_k \in U$, distance function has the following characters:

Character 3.4: $d(u_i, u_i) = d(u_i, u_i)$;

Character 3.5: $d(u_i, u_j) = d(u_j, u_i)$;

Character 3.6: $d(u_i, u_j) + d(u_j, u_k) \leq d(u_i, u_k)$

4. Sample Analysis

Suppose decision problems with four states and four projects, the probability of occurrence of each state and the revenue of each project are shown in the following table:

	$V_1 (P=0.2)$	$V_2 (P=0.4)$	$V_3 (P=0.3)$	$V_4 (P=0.1)$
	revenue	revenue	revenue	revenue
S_1	50	70	55	70
S_2	20	95	45	100
S_3	45	70	65	95
S_4	30	70	45	95

First, construct a set pair utility function:

Step 1: In the format context of decision problems, select the best value $x_{ij} = 100$ and the minimum value $x_{sq} = 20$ and make $U(100) = 1, U(20) = 0$;

Step 2: Seek for a suitable value x_{fg} between the best value and the minimum value to meet that $x_{fg} = 55$;

Step 3: According to three point method, we can get $A \approx -11.72, B = 2.34, C = 130$, and then the solution to the utility function is $U(x) = -11.72 + 2.34 \ln(x + 130)$;

Step 4: Construct the set pair utility function: $f(x) = 0.4U(x) + ((1 - 0.5U(x)))i + 0.1c(x)j$

Step 5: Output the set of set pair utility function:

$$U(S) = \{U(S_1), U(S_2), U(S_3), U(S_4)\}$$

$$U_1 = U(S_1) = 0.2f(50) + 0.4f(70) + 0.3f(55) + 0.1f(70) = 0.2304 + 0.712i + 0.0576j$$

$$U_2 = U(S_2) = 0.2f(20) + 0.4f(95) + 0.3f(45) + 0.1f(100) = 0.2364 + 0.7045i + 0.0591j$$

$$U_3 = U(S_3) = 0.2f(45) + 0.4f(70) + 0.3f(65) + 0.1f(95) = 0.2508 + 0.6865i + 0.0627j$$

$$U_4 = U(S_4) = 0.2f(30) + 0.4f(70) + 0.3f(45) + 0.1f(95) = 0.204 + 0.745i + 0.051j$$

	$u(x)$	$a(x)$	$b(x)$	$c(x)$
20	0	0	1	0
30	0.16	0.064	0.92	0.016
45	0.37	0.148	0.815	0.037
50	0.43	0.172	0.785	0.043
55	0.5	0.2	0.75	0.05
65	0.62	0.248	0.69	0.062
70	0.68	0.272	0.66	0.068
95	0.95	0.38	0.525	0.095
100	1	0.4	0.5	0.1

Secondly, construct lattice structure:

- (1) Any two elements in the set of set pair utility function cannot be compared;
- (2) The supremum of $U(S_1)$ and $U(S_2)$ is $U(R_1) = 0.23715 + 0.70525i + 0.0576j$;
- (3) The supremum of $U(S_3)$ and $U(S_4)$ is $U(R_2) = 0.25665 + 0.69235i + 0.05j$;
- (4) $U(R_2)$ priority than $U(R_1)$;
- (5) The infimum of $U(S_1)$ and $U(S_3)$ is $U(T_1) = 0.22785 + 0.70945i + 0.0627j$;
- (6) The infimum of $U(S_2)$ and $U(S_4)$ is $U(T_2) = 0.19995 + 0.74095i + 0.0591j$;
- (7) The infimum of $U(T_1)$ and $U(T_2)$ is $U(T_0) = 0.19815 + 0.73915i + 0.627j$.

Then partial order structure of the set of set pair utility function $U(S) = \{U(S_1), U(S_2), U(S_3), U(S_4)\}$ of the Hasse diagram is figure 4.1.

Finally, the analysis result is as followings:

- (1) $d(U(S_2), U(R_1)) \leq d(U(S_1), U(R_1))$, $U(S_2)$ takes priority to $U(S_1)$;
- (2) $d(U(S_3), U(R_2)) \leq d(U(S_4), U(R_2))$, $U(S_3)$ takes priority to $U(S_4)$;
- (3) $d(U(S_3), U(R_2)) \leq d(U(S_2), U(R_2))$, $U(S_3)$ takes priority to $U(S_2)$;
- (4) $d(U(S_1), U(R_2)) \leq d(U(S_4), U(R_2))$, $U(S_1)$ takes priority to $U(S_4)$.

(Note: by comparing the distance between each node with the infimum, the optimal solution of the model will be found)

Through the analysis above, $U(S_3)$ is the optimal solution of the model.

5. Conclusion

Set pair analysis is a better way to make a general and practical research in lattice order decision. It is a kind of development and improvement for rational behavior decision-making theory. This article applies set pair analysis method to the lattice order

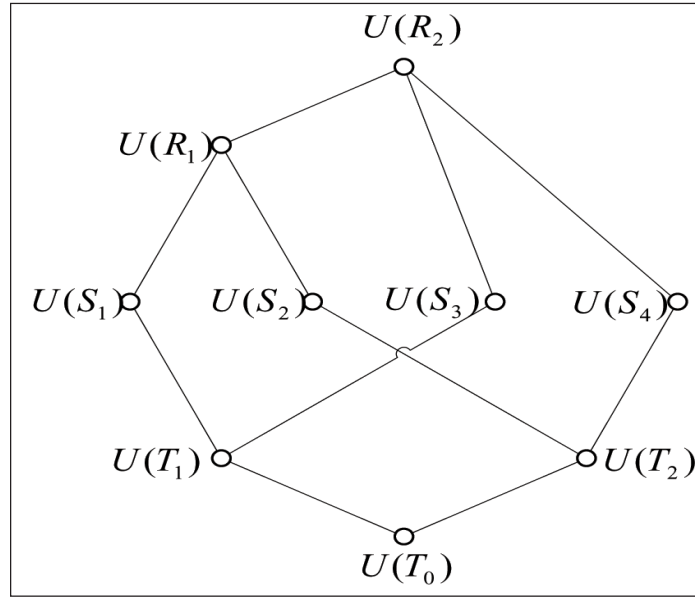


Figure 4.1 Hasse diagram

decision making theory, makes a in-depth study on the lattice ordered characteristics of the set of set pair utility function. Then a lattice structure is established. At last, more rational and objective l solution of the model is found.

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References

- [1] Guo, Yaohuang. (2003). Lattice order decision making. Shanghai Science and Technology Press.
- [2] Zhao, Keqin. (2000). Set Pair Analysis and Its Preliminary Application. Zhejiang Science and Technology Press.
- [3] Guo, Chunxiang., Guo Yaohuang. (2004). The Study of Lattice Ordering Under Lattice Order Decision Making Theory, *Systems Engineering-Theory Methodology Applications* 13 (5), 463–466.
- [4] Jiang, Yunliang., Xu Congfu. (2006). Advances in Set Pair Analysis Theory and its Applications, *Computer Science*, 33(1), 205–209.
- [5] Hong Wei., Wu Chengzhen., Li Zhenyou. (1995). Set Pair Analysis and Effectiveness Function, *Journal of Fujian College of Forestry* 15 (3) 203–207.
- [6] Guo, ChunXiang., Guo Yaohuang. (2006). A Method of Lattice Order Decision Making on Prospect Spaces, *Systems Engineering* 24 (2), 92–95.
- [7] Wang, Guohua., Liang Liang. (2006). Decision-making Theory and Method. University of Science and Technology of China press.
- [8] Liu Baoxiang., Yang Yafeng., Li Yankun. (2010). Construction of set pair utility function and application, *In: International Conference on Engineering and Business Management* 1440–1442.
- [9] Hao, Guang., Mou Qifeng., Zhang Dianye., Guo Yaohuang. (2006). Approach of Fuzzy Multi-objective Decision-Making Based on Lattice-Order Preference, *Journal of Southwest Jiaotong University* 41 (4), 517–522.
- [10] Dai, Yu., Zhou DeQun. (2007). A Method of Lattice-Order Selection Based on Rough Sets Theory, *Chinese Journal of Management Science* 15 (6), 105–111.



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