A New Discounting Evidence Combination Method Based on Classification of Decision-making

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ABSTRACT: *The D-S evidence theory is an important information fusion method at the decision level. A novel evidence combination method is proposed to solve the counterintuitive problem of the Dempster rule. It firstly classifies the evidences according to decision making; then, constructs evidence importance degree and evidence reliability models as the discounting factors for evidences with different conflict degrees within the same class or between classes; finally, combines the evidences by the Dempster's rule. The experimental analysis shows that the method is reasonable.*

Keywords: D-S Theory, Evidence Combination, Classification Of Decision-Making, Evidence Importance Degree, Discount Revision

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1. Introduction

The evidence theory was put forward by Dempster in 1967 and was further developed and improved by his student, Shafer. As a result, it is also called as D-S evidence theory. It has become an important information fusion method at the decision making level[1]. The Dempster's rule of combination is one of the core contents of the D-S theory. It combines the information pieces represented by basic probability assignments (BPA) into more completed information. However, when treating some special evidence combinations, especially high conflict evidences, the Dempster's rule will lead to some counterintuitive results[2].

Many researchers thought that the counterintuitive results are produced because the Dempster's rule does not reasonably allocate the conflict evidences. A more reasonable rule should be established to solve this problem[3]. However, new combination rules not only increase the amount of calculation, but also are hard to construct to have good properties such as commutativity and associativity. Professor Haenni thought that it was not correct to attribute the counterintuitive problem to the combination rule. Rather, the data models should be corrected. In other words, revisions need to be made for the available evidences before using the Dempster's rule[4]. The discounting factor method proposed by Shafer is the most classic data model correction method[5]. The method corrects the original BPA to suppress the effects of conflict evidences by using reliability of evidences[6]. The degrees of reliability are usually obtained by the distance metrics between evidences[7]. The class of evidence convex combination methods first weighted averages the obtained evidences and then combines the averaged evidence and gets the final evidence by using the Dempster's rule multiple times. For example, Murphy simply averaged the evidences[8]; Deng used weighted average based on the results of Murphy[9]; and Han et. al proposed a weight calculation method based on the variance of a series of evidences[10]. All of them have relatively good aggregation results.

The D-S theory is a high level information fusion method. Its treatment for multisource evidences should be similar to the human cognitive process of things. First of all, the multisource evidences are collected and classified. Then, the orders of preference of evidences in a single class are formed according to their features and the evidences are processed by class. Finally, the comprehensive information is provided to the decision maker for cognition or decision making[11]. Based on the above consideration, a discounting evidence combination method based on classification of decision-making is proposed in this paper. This method adopts the largest assumption supported by evidences as the classification criterion and uses different discounting factors for evidences with different degrees of conflict within a class and between classes. It also constructs an evidence importance model and an evidence reliability model when calculating the discounting factors. Experimental results show that the proposed method can solve the counterintuitive problem in the fusion of evidences. The evidence combination results are reliable and effective.

2. Correction Methods for Data Model

Basic concepts of evidence theory can be referred to related literature. In this section, we mainly focus on the correction methods for data model. Among all the data model correction methods, the discounting factor method proposed by Shafer corrects the original BPA to suppress the effects of conflict evidences by distributing parts of the belief to the complete set according to the discounting factors. Assuming that the discounting factor is α ($0 < \alpha < 1$), the discounted BPA is:

$$m'(A) = \begin{cases} \alpha m(A) & A \neq \Theta \\ 1-\alpha+\alpha m(\Theta) & A = \Theta \end{cases} \tag{1}$$

Then, the discounted evidences are combined using the Dempster's rule to obtain the final evidence[5].
The simple averaging method proposed by Murphy simply averages the N collected evidences:

$$m_a = \frac{m_1 + m_2 + \cdots + m_N}{N} \tag{2}$$

The Dempster's rule is then used for N-1 time for combination[8].

Deng introduces the Jousselme distance to calculate the belief of evidence, which is used in Equation (2) as the weighted average of multisource evidences:

$$Cred(m_i) = \frac{\sum_{j=1, j\neq i}^{N} (1 - d_J(m_i, m_j)}{\sum_{j=1}^{N} (1 - d_J(m_i, m_j))} \tag{3}$$

where $d_J(m_i, m_j)$ is the Jousselme distance between evidences m_i and m_j:

$$\begin{cases} d_J(m_i, m_j) = \sqrt{\frac{1}{2}(m_i - m_j)^T D(m_i - m_j)} \\ D(A,B) = \begin{cases} 1 & if\ A = B = \phi \\ \frac{|A \cap B|}{|A \cup B|} & \forall A, B \in 2^{\Theta} \end{cases} \end{cases} \tag{4}$$

The Dempster's rule is then used to combine evidence information. The method has a good performance[9].

Han defines the variance of evidence series based on the Jousselme evidence distance:

$$\begin{cases} Var([m_1, m_2, \cdots, m_N]) = \sqrt{\frac{1}{N}\sum_{i=1}^{N} d_J^2(m_i, \overline{m})} \\ \overline{m} = \frac{1}{N}\sum_{i=1}^{N} m_i \end{cases} \tag{5}$$

Weights are determined based on the difference between variances of the combined evidence series before and after adding a new piece of evidence. The method also solves the counterintuitive problem in evidence combination[10].

3. The discounting Evidence Combination Method Based on the Classification of Decision-making

3.1 Evidence Classification Method Based on Decision-making

When making decisions directly based on evidences, if the belief of the largest assumption supported by evidences is greater than some set threshold *r*, the decision then can be made. Otherwise, evidence need to be further accumulated[12]. Therefore, the largest assumption supported by evidences can be used as the criterion of evidence classification. If both two evidences support the same assumption by the greatest belief, then they can be regarded as identical from a decision-making point of view and the two can be classified in the same class. The most special BPA is defined as the Bayesian BPA under the identification framework Θ, which satisfies:

$$m_{spec}^{i}(X)=\begin{cases}1 & X=\theta_i\\ 0 & else\end{cases} \tag{6}$$

Assuming that the most special BPA is one-to-one correspondent, for example, the corresponding most special BPA for 'θ_1 is true' is [1, 0, 0, …, 0], the largest assumption supported by a piece of evidence can be obtained by constructing its corresponding most special BPA. Professor Smarandache proposed two methods for the construction of the most special BPA[13]. However, both methods involve the probability transformation problem of BPA, but how to transform BPA into reasonable probabilities is still a current research focus. In this paper, based on the Jousselme distance, the most special BPA with the least distance to evidence m is defined as the corresponding most special BPA of the evidence:

$$m_{spec}=\underset{i=1,2,\cdots,|\Theta|}{\arg\min}\left(d_J(m,m_{spec}^{i})\right) \tag{7}$$

where $|\Theta|$ is the number of elements in the set $|\Theta|$.

Under the identification framework Θ, all evidences compose a set $\Pi=\{m_1,m_2,\ldots,m_N\}$. The core of each evidence is $C_i, 1\le i\le N$, respectively. The union of all cores composes the assumption set supported by evidence set Π.

$$\Omega=\bigcup_{i=1}^{N}C_i \tag{8}$$

We define the set composed by evidences supporting assumption $\theta_j, j=1,2,\cdots,|\Theta|$ as

$$T_{\theta_j}=\{m_i \mid \theta_j\in\Theta\} \tag{9}$$

All non-empty set *T* compose the set class

$$\Gamma_T=\{T_k \mid k=1,2,\cdots Z,1\le Z\le|\Theta|\} \tag{10}$$

Each element in set class Γ_T represents a non-empty evidence class. The evidences in each class are identical from a decision-making point of view and support the same assumption by the largest belief. The conflicts between different evidences are relatively small.

3.2 Discounted Combination Method Of Classified Evidences

The discounting combination method by Shafer retains the good characteristics and the independence of evidence of the Dempster's rule, which can increase the reliability of decisions and is easy to be realized.The key of the discounting combination method is the selection of discounting factors. In recent years, several researchers have successively proposed a number of calculation methods for discounting factors, but they roughly thought that the importance degree of evidence is equivalent to the reliability of evidence, which ignored the effect of the importance degree of evidence on the fusion process[14]. However, reliability and importance degree are two different concepts, which should be treated differently in the discounting evidence fusion[15]. Therefore, a new method is proposed by using different discounting factors for evidences with different degrees of conflict within a class and between classes.

(1) Discounting Factor for Evidence Fusion Within a Class

According to the evidence classification method in 3.1, evidences within a class support the same assumption with the largest belief. The conflicts between evidences are insignificant. The importance degree of evidence has a larger effect on the evidence combination process. During the daily information treatment process, people usually have different importance evaluations for evidences with different amount of information. The more support one piece of evidence has for its corresponding most special BPA, the more beneficial it is for the decision-making, and the more important the evidence is.

Martin analyzed the axiom that conflict degree should satisfy in literature [16] and gave a new measurement for conflict:

$$Conf(m_1, m_2) = (1 - d_{inc}(m_1, m_2))d_J(m_1, m_2) \tag{11}$$

where $d_{inc}(m_1, m_2)$ is the inclusion degree of m_2 to m_1:

$$d_{inc}(m_1, m_2) = \frac{1}{|C_1||C_2|} \sum_{X_1 \in C_1} \sum_{Y_2 \in C_2} Inc(X_1, Y_2) \tag{12}$$

where C_1 and C_2 are the cores of m_1 and m_2 respectively. If $X_1 \subseteq Y_2$, then X_1 supports Y_2 in belief distribution and $Inc(X_1, Y_2) = 1$, otherwise, $Inc(X_1, Y_2) = 0$.

The support degree and conflict degree of evidence are a pair of opposite concepts. The support degree of m_2 to m_1 can be defined from Equation (11):

$$Sup(m_1, m_2) = 1 - Conf(m_1, m_2) \tag{13}$$

Using Equation (13), we can define that the support degree of evidence m to its corresponding most special BPA is the quality of the evidence.

$$Quat(m) = Sup(m_{spec}, m) \tag{14}$$

Thus, we choose the evident quality as the discount factor for discounted evidence fusion within a class.

$$\alpha_{C_{in}}(m_i) = Quat(m_i) \tag{15}$$

(2) Discount Factor For Evidence Fusion between Classes

There are relatively large conflicts in the evidence combination between classes. The fusion of evidences is largely influenced by reliability. The reliability of evidence is not only related to the percentage portion of the evidences of the class in all the evidences, but also related to the geometry relation between the evidences within the class and the most special BPA of the class. In other words, if all the evidences within the class support the assumption with relatively large belief, i.e. the corresponding most special BPAs of the evidences in the category have good focusing, then the reliability of the evidence is good. Otherwise, the corresponding most special BPA will be scatter and the reliability of the evidence will be low[17].

According to Equation (13), we define that within class *l*, the average support degree of the evidences to their corresponding most special BPA m^l_{spec} is the degree of aggregation of category *l*:

$$S_{mean}(l) = \frac{\sum_{i=1}^{N(l)} Sup(m^l_{spec}, m_i)}{N(l)} \tag{16}$$

where $N(l)$ is the number of evidences in class *l*. the degree of aggregation of class *l* reflects the reliability of the combined evidence of the class.

According to the number of evidences in each class, the evidence percentage can be obtained as:

$$\delta_{per}(l) = \frac{N(l)}{\sum_{i=1}^{L} N(i)} \tag{17}$$

Combining the degree of aggregation of evidence and the evidence percentage, the reliability of combined evidence m_l of class l can be defined as:

$$Cred(m_l) = S_{mean}(l) \cdot \delta_{per}(l) \tag{18}$$

For combined evidence between categories, we choose the normalized evidence reliability as the discount fusion factor.

$$\alpha_{C_{out}}(m_l) = \frac{Cred(m_l)}{\max\left(Cred(m_l)\right)}, 1 \le l \le |\Theta| \tag{19}$$

4. Experimental Comparisons

4.1 Experiment 1

This experiment analyzes the situation of a battlefield. Assuming that node interception is the overall situation of the confrontation for both sides, it has three sub-situations, which are Attack, Evade, and Stalemate. The corresponding identification framework is then {A, E, S} for the situation analysis. Two evidences from different sources are obtained. Their BPA are:

$$m_1(A) = 0.7 \qquad m_1(E) = 0.0 \qquad m_1(S) = 0.3;$$
$$m_2(A) = 0.0 \qquad m_2(E) = 0.7 \qquad m_2(\mathrm{S}) = 0.3.$$

It can be seen that, there is a large conflict between evidences m_1 and m_2. The evidences are combined using Dempster's method, Murphy's method, Deng's method, Han's method, and the proposed method. Results are shown in Table 1.

	Dempster's method	Murphy's method [8]	Deng's method [9]	Han's method [10]	the proposed method
m(A)	0	0.3657	0.3657	0.3657	0.2682
m(E)	0	0.3657	0.3657	0.3657	0.2682
m(S)	1	0.2687	0.2687	0.2687	0.3678
m(Θ)	0	0	0	0	0.0958

Table 1. Results for Experiment 1

It can be seen that the result of Dempster's method says the current situation is stalemate, which is counterintuitive. The proposed method has similar results as Murphy's method, Deng's method, and Han's method, which provides good basis for future evidence fusion at the same time resolving the conflicts between evidences. Additionally, the proposed method distributes 0.0958 belief to the complete set Θ. The main reason is that there is a relatively large conflict between the two evidence, which increases the uncertainty degree of the problem.

4.2 Experiment 2

This experiment adopts the example used in literature [10]. Under the identification framework {A, B, C}, we have the following according to the six obtained evidences:

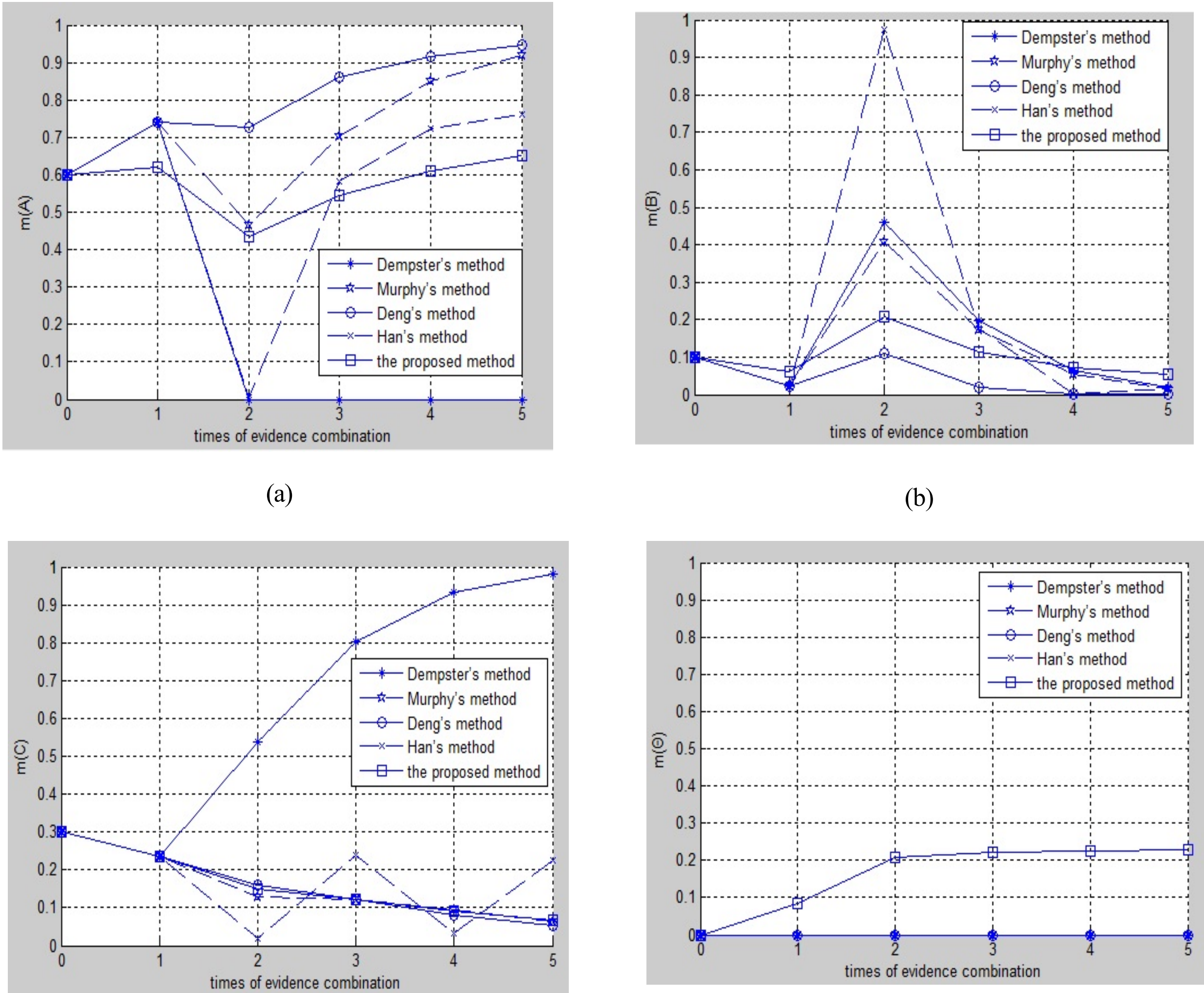
$$m_1(A) = 0.60 \qquad m_1(B) = 0.10 \qquad m_1(C) = 0.30;$$
$$m_2(A) = 0.55 \qquad m_2(B) = 0.10 \qquad m_2(C) = 0.35;$$
$$m_3(A) = 0.00 \qquad m_3(B) = 0.90 \qquad m_3(C) = 0.10;$$
$$m_4(A) = 0.55 \qquad m_4(B) = 0.10 \qquad m_4(C) = 0.35;$$
$$m_5(A) = 0.55 \qquad m_5(B) = 0.10 \qquad m_5(C) = 0.35;$$
$$m_6(A) = 0.55 \qquad m_6(B) = 0.10 \qquad m_6(C) = 0.35.$$

In order to illustrate the changing in evidence combination with the accumulation of evidences better, the number of evidences used in the combination gradually increases. The combination results are shown in Table 2. The belief of each assumption as a function of number of times of evidence combinations is shown in Figure 1.

		Dempster's method	Murphy's method [8]	Deng's method [9]	Han's method [10]	the proposed method
$m_{1\text{-}2}$	$m(A)$	0.7416	0.7409	0.7409	0.7409	0.6201
	$m(B)$	0.0225	0.0224	0.0224	0.0224	0.0614
	$m(C)$	0.2360	0. 2367	0.2367	0.2367	0.2346
	$m(\grave{E})$	0.0000	0.0000	0.0000	0.0000	0.0839
$m_{1\text{-}3}$	$m(A)$	0.0000	0.4646	0.7270	0.0066	0.4364
	$m(B)$	0.4615	0.4066	0.1115	0.9751	0.2068
	$m(C)$	0.5385	0.1289	0.1615	0.0183	0.1485
	$m(\grave{E})$	0.0000	0.0000	0.0000	0.0000	0.2083
$m_{1\text{-}4}$	$m(A)$	0.0000	0.7025	0.8609	0.5839	0.5460
	$m(B)$	0.1967	0.1744	0.0183	0.1757	0.1122
	$m(C)$	0.8033	0.1231	0.1207	0.2404	0.1203
	$m(\grave{E})$	0.0000	0.0000	0.0000	0.0000	0.2215
$m_{1\text{-}5}$	$m(A)$	0.0000	0.8507	0.9151	0.7221	0.6104
	$m(B)$	0.0654	0.0548	0.0031	0.0032	0.0731
	$m(C)$	0.9346	0.0946	0.0817	0.0317	0.0908
	$m(\grave{E})$	0.0000	0.0000	0.0000	0.0000	0.2257
$m_{1\text{-}6}$	$m(A)$	0.0000	0.9206	0.9456	0.7603	0.6528
	$m(B)$	0.0196	0.0144	0.0005	0.0146	0.0528
	$m(C)$	0.9804	0.0650	0.0539	0.2251	0.0661
	$m(\grave{E})$	0.0000	0.0000	0.0000	0.0000	0.2283

Table 2. Results for Experiment 2

In this experiment, evidence m_3 indicates that the probability of assumption {b} happening is 0.9. It is a piece of evidence with strong interference and largely conflict with other evidences. It can be seen from the belief values at the second evidence combination of Figure 1-(a) and 1-(b) that the addition of m_3 leads to large changes in the results by Dempster's method, Murphy's method, and Han's method. It indicates that those methods have relatively resistance to this conflicting evidence. With the addition of following evidences, the belief distributed to assumption {C} by Dempster's method gradually increases (Figure 1-(c)), which is counterintuitive. The belief assigned to assumption {A} in Han's method recovers rapidly (Figure 1-(a)), but the belief assigned to assumption {C} becomes oscillatory. Both Deng's method and the proposed method are able to handle the interfering evidence. It can be seen from Figure 1-(a) that Deng's method has a good aggregation speed. However, except the interfering evidence, the evidence mass of other five evidences are low, but the belief for assumption {A} in the final combination results is greater than 0.94. Although the result is good for decision making, the reliability of the final decision will be affected. It can be seen from Figure 1-(d) that due to the existence of the interfering evidence, the uncertainty degree of the problem increases. The proposed method allocates part of the belief to the complete set Θ. In fact, in the combination of evidences, it is not good to pursue convergence rate blindly. The reliability of the combined evidences for decision making should also be considered. The proposed approach considers several factors in the combination of evidences such as the importance degree of the evidence and the aggregation degree of the evidence. It can effectively suppress interfering evidences and, at the same time, reach a moderate convergence rate.



(a) (b)

(c) (d)

Figure 1. Belief profile for the combined evidences (a) $m(A)$ (b) $m(B)$ (c) $m(C)$ (d) $m(\Theta)$

5. Conclusions

In this paper, a discounted evidence combination method based on classification of decision-making was proposed to solve the paradox problem in evidence combination. The proposed method uses whether the largest assumption supported by evidences is the same as the criterion of classification and adopts different discount factors for evidences with different conflict degree. The importance of evidences is also considered independently of the reliability of evidences. The proposed method does not target on fast convergence rate, but ensures the reliability of the combined evidence. Experimental results showed that the method was able to solve the paradox problem in evidence fusion. However, the proposed method allocated part of the belief to the complete set. Due to the diversity of evidence structure and numerical distribution, the method might not converge to the right assumption when the amount of evidences is large, which need to be improved in the future. Additionally, there still lacks an accepted evaluation system for evidence combination rule. The reasonability of methods can only be checked by the consistency of typical evidence combination results and intuition. Establishing an objective and reasonable evaluation system for evidence combination rules is a tough task that must be done.

References

[1] G S. A Mathematical Theory of Evidence [M]. Princeton:Princeton University Press,1976.

[2] Yang, Feng-Bao., Wang, Xiao-Xia. (2010). Combination Method of Conflictive Evidences in D-S Evidence Theory, Beijing: *National Defence Industry Press*. 65-72

[3] Wang, X., Ji ,Y. D., Xu, X. B., Sun, X. Y. (2012). Combining Belief Functions Based on Three Mechanisms: Average, Multiplication and Intersection. 2012 Fifth International Conference on Intelligent Computation Technology and Automation (ICICTA), 494-500.

[4] R H. (2002). Are Alternatives to Dempster's rule of combination real alternative? Comments on about the belief function combination and the conflict management problem . *Information Fusion*, 3 (4) 237-239.

[5] G S. (1992).Rejoinder to Comments on Perspectives in the theory and practice of belief functions. *Int J Approximate Reasoning*, 6: 445-480.

[6] Liu, Z. G., Pan, Q., Cheng, Y. M., Dezert, J. (2010). Sequential adaptive combination of unreliable sources of evidence. **In**: Proceeding of workshop on the theory of belief functions. Brest, France.

[7] Liu, Z. g., Dezert, J., Pan, Q., et al. (2011). Combination of sources of evidence with different discounting factors based on a new dissimilarity measure. Decision Support Systems, 52, 133-141.

[8] C.K. M.(2000). Combining Belief Functions When Evidence Conflicts. Decision Support Systems, 29.9.

[9] Y D. (2004)Combining belief functions based on distance of evidence. Decision Support Systems, 38. 489-493.

[10] Han De-Qiang., Han Chong-Zhao., Deng Yong., Yang Yi. (2011). Weighted combination of conflicting evidence based on evidence variance. *Acta Electronica Sinica*, 39 (3A) 153-157.

[11] Chen Sheng-Qun, Wang Ying-Ming. A grouping method for combining evidence. Control and Decision, under publishment.

[12] Zhou Zhe., Xu Xiao-Bin., Wen Cheng-Lin., Lv Feng. (2012). An optimal method for combining conflicting evidences. **Acta Automatica Sinica,** 38 (6) 976-985

[13] Smarandache, F., Martin, A.,Osswald, C. (2011). Contradiction measures and speciûcity degrees of basic belief assignments. International Conference on Information Fusion, Boston, USA.

[14] Elouedi, Z., Lefèvre, E., Mercier, D. (2010). Discountings of a belief function using a confusion matrix. *IEEE International Conference on Tools with Artificial Intelligence (ICTAI)*. Tunis, Tunisia. 1. 287- 294.

[15] Smarandache, F., Dezert , J,. Tacnet , J. (2010). Fusion of sources of evidence with different importances and reliabilities. 2010 13th Conference on Information Fusion (FUSION) 1-8.

[16] Martin, A. (2012). About conûict in the theory of belief functions . BELIEF FUNCTIONS: THEORY AND APPLICATIONS *Advances in Intelligent and Soft Computing*, 164, 161-168.

[17] Han De-Qing., Deng Yong., Han Chong-Zhao., et al. (2011). Conflicting evidence combination by using uncertainty degree. *Control Theory and Applications*, 28 (6) 788-792.