Pricing and Advertisement in a Manufacturer-Retailer Closed-loop Supply Chain

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ABSTRACT: *In order to study price decision and advertising coordinate strategy, we divided the closed-loop supply chain into two cases including the retailer recycling and manufacturer recycling. With the impact of advertising for market demand and recycling, by using the game theory, we established two mathematical models, the results show that the manufacture recycling mode is conducive to retailer, retailer recycling mode helps manufacturer. Compared to retailer recovery, the share proportion of advertising costs of retailer is smaller of manufacture recovery. The retailer advertising share proportion of manufacturer recovery is negatively correlated with the advertising recovery efficiency factor, positive correlation with the advertising and marketing efficiency factor. The retailer advertising share proportion of retailer recovery has nothing to do with the cost of advertising marketing efficiency factor, recovery efficiency factor. At last we proved the related conclusion through simulation.*

Keywords: Advertising, Closed-loop Supply Chain, Game Theory, Coordination

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1.Introduction

Closed loop supply chain refers to a supply chain which includes product recycling reverse logistics chain. It's a complete supply chain that start from purchase to sales, and the product recovery of circulation. By implement the closed-loop supply chain management not only meet the protect environment and reduce the waste of resources, but also can reduce cost of product through the recycling of waste products [1]. The enterprises often consider optimizing the closed-loop supply chain by lowering the price, channel optimization, improve the quality of products, but ignored the advertising factors role in the supply chain.

Many scholars have researched on maximum benefits for both manufacturer and retailer about supply chain coordination (Jeuland(1983),Moorthy(1987),Ingene(1995)) [2-6], but most of them focus on pricing while less pay attention to pricing and advertising for supply chain coordination. Doraiswamy(1979) have study on advertising strategy of the supply chain under the unit product fixed discount. Bergen(1997) [7-9]has analyzed the supply chain advertising cooperative strategies while the retailer decides the proportion of advertisement cost. Jorgensen consider that advertising cost share of supply chain can be divided into four types, namely short-term share, long-term share, both short-term and long-term share, both short-term and long-term don't share, their research have shown that advertising cost share is benefit for manufacture, on the basis of this research, Yue (2006)obtained optimize share proportion, retail price and marketing cost. Huang and Li (2001), Huang et al. (2002), Li et al. (2002), Xie and Ai (2006) [10-16] studied a game-theoretic model for cooperative advertising in a supply chain consisting of one manufacturer and one retailer. SeyedEsfahani and Biazaran (2011) considers vertical co-op advertising along with pricing decisions in a supply chain, four game-theoretic models are established in order to study the effect of supply chain power balance on the optimal decisions of supply chain members. Yue and Austin (2013) use a game theoretical approach to study pricing and advertisement decisions in a manufacturer–retailer supply chain when price discounts are offered by both the manufacturer and retailer. In Zhang and Gou(2013) model, both the consumer's goodwill and reference price for the product are assumed to be influenced by the advertising and are modeled in differential dynamic equations. Zhang and Li(2014) assume that both the manufacturer and the retailer can choose to participate in the advertising initiative by reducing their advertising levels, the research show that the effectiveness of the advertising initiative critically depends on the leader's participation in the initiative.

Through the above literatures, most of the existing researches only consider the price factor when analysis of the optimal decision problem of closed-loop supply chain, only a small number of scholars considering both price factor and advertising factor. And few papers study the two factors impact to closed-loop supply chain from variety of advertising model and recovery mode. In view of the current problems, based on existing related papers, we will build price and advertisement decision models on manufacturer recycling and retailer recycling, analysis of closed-loop supply chain optimal price decision and advertising coordination strategy.

2. Overview of Our Model

We consider a closed-loop supply chain with one manufacture and one retailer in which all activity occurs within a single period. The manufacturer or retailer is responsible for recycling of waste products. Because products retail price p and advertisement level m effect the amount of demand, suppose the demand function is $D(p,m)=\phi-\beta p+\gamma m$,which ϕ is the market base, β represents the sensitive factor of demand on price, γ is the sensitive factor of demand on advertising level. In virtue of retailer price p will not less than the production cost c_m of manufacture, while $p = c_m$, the amount of demand should reach the maximum, so $\phi-\beta c_m>0$ is permanent establishment.

There are two aspects information included by closed-loop supply chain advertisement, the one is the information for promoting the sale, and another is the information for improving waste products recovery rate. Suppose the advertising recovery satisfaction $\tau(m)=\lambda m$, which λ represents advertising recovery efficiency factor. When they recovery waste products from consumers, the unit cost A of recycling and storage of waste products must be paid to consumers. If retailer recycling of waste products, the retailer will resale them to manufacturer with unit price b, so $A < b$ guarantee retailer would like to recycling waste products .Suppose there are no difference between new products which produced by waste products and raw material. The unit cost of new product produced by waste product is c_r, in order to ensure manufacturer is more willing to reduce product costs by recycling waste products remanufacturing way, $c_r < c_m$ must be set up. Then $\Delta c_m - c_r$ means unit produce cost savings by waste products, so $c_m - \tau\Delta$ is an average cost per unit for a new product. $A < \Delta$ must be given, so as to ensure that manufacturer willing to recycling of waste products. Set $v = \Delta - A$, v means actual unit produce cost savings by recycling waste products. Then the total cost of closed-loop supply chain is $\kappa m^2/2+Am\lambda D(p,m)$ the cost of advertising level m is $m^2\kappa/2$, κ, is strictly positive parameter we refer to as "*advertisement cost factors*" (G.E.Frucher.,2009).

Suppose each party is information symmetry in the closed-loop supply chain, the manufacturer has absolute leadership to the channel. Parameters λ is small enough to ensure $\tau < 1$ and $(\gamma+\beta\lambda v)^2-2\beta\kappa<0$,which Guarantee if all the waste product recycling, the cost will be very large. Let M represents manufacturer, R represents the retailer, T represents the whole supply chain. Π_l^i represents i 's profits while retailers compete if l advertising, which .

The process of game for manufacturer and retailer are as follows: first, the manufacturers to make decisions on new products wholesale price w and advertising cost sharing proportion x; second, the retailer decide on the sales price of the product p, advertising input level m decision making. This game is a complete information dynamic game and the subgame perfect Nash equilibrium, can be solved by backward induction method.

3. Our Proposed Framework and Model

3.1. Advertisement Sharing when Manufacturer Recovery Model

In this model, manufacturer responsible for recycling of waste products and remanufacturing, and selling new products to retailer through the wholesale price. The retailer is responsible for product sales and advertising including information about products demand and the recycling of waste products. The advertising cost sharing proportion decided by the manufacturer.

Given wholesale price *w* and advertising cost sharing proportion *x* by manufacturer, the retailer's problem is:

$$\max_{p>0,m>0} \prod_R^M = (p-w)(\phi - \beta p + \gamma m) - x\kappa m^2/2 \tag{1}$$

Due to $\partial^2 \prod_R^M/\partial p^2 = -2\beta < 0$, $\partial^2 \prod_R^M/\partial m^2 = -x\kappa < 0$, $(\partial^2 \prod_R^M/\partial m^2)(\partial^2 \prod_R^M/\partial p^2) - (\partial^2 \prod_R^M/\partial p \partial m)^2 = -\gamma^2 + 2x\beta\kappa$, in order to ensure $\prod_R^M$ is a oncave function on *p* and *m*, we suppose $x > \gamma^2/2\beta\kappa$ establish first, verify this condition at last. We can obtain the optimal retailer's decision by first-order derivative to (1) on *p* and *m*, we get:

$$p^M = \frac{w\gamma^2 - wx\beta\kappa - x\kappa\phi}{\gamma^2 - 2x\beta\kappa}, m^M = \frac{\gamma(w\beta - \phi)}{\gamma^2 - 2x\beta\kappa} \tag{2}$$

Manufacturer has known the retailer would decide by equation (2), so manufacturer's problem in the first phase is:

$$\max_{w>0,x>0} \prod_M^M = (\phi - \beta p + \gamma m)(w - c_m + \Delta m\lambda) - Am\lambda(\phi - \beta p + \gamma m) - (1-x)\kappa m^2/2 \tag{3}$$

According to the first order conditions, we get:

$$w^{M*} = \frac{(-3\gamma^2 - 8\beta\lambda\gamma v - 4\beta(-2\kappa + \beta\lambda^2 v^2)\phi + 2\beta(-3\gamma^2 + 4\beta\kappa - 2\beta\gamma\lambda v)\lambda)c_m}{\beta(-(3\gamma + 2\beta\lambda v)^2 + 16\beta\kappa)}, x^{M*} = \frac{2\gamma}{3\gamma + 2\beta\lambda v}$$

From the second section, it can easy verify $\frac{2\gamma}{3\gamma + 2\beta\lambda v} > \frac{\gamma^2}{2\beta\kappa}$. Then we get:

$$m^{M*} = \frac{2(3\gamma + 2\lambda\beta v)(\phi - \beta c_m)}{-(3\gamma + 2\beta\lambda v)^2 + 16\beta\kappa}$$

$$p^{M*} = \frac{(-3\gamma^2 - 8\beta\lambda\gamma v - 4\beta(-3\kappa + \beta\lambda^2 v^2)\phi + 2\beta(-3\gamma^2 + 2\beta\kappa - 2\beta\gamma\lambda v\lambda)c_m}{\beta(-(3\gamma + 2\beta\lambda v)^2 + 16\beta\kappa)}$$

$$D^{M*} = \frac{4\beta\kappa(\phi - \beta c_m)}{-(3\gamma + 2\beta\lambda v)^2 + 16\beta\kappa}, \prod_M^{M*} = \frac{2\kappa(\phi - \beta c_m)^2}{-(3\gamma + 2\beta\lambda v)^2 + 16\beta\kappa},$$

$$\prod_R^{M*} = \frac{4\kappa(-3\gamma^2 + 4\beta\kappa - 2\beta\gamma\lambda v)(\phi - \beta c_m)^2}{(-(3\gamma + 2\beta\lambda v)^2 + 16\beta\kappa)^2}, \prod_T^{M*} = \frac{2\kappa(-15\gamma^2 + 16\beta\kappa - 16\beta\gamma\lambda v + 4\beta^2\lambda^2 v^2)(\phi - \beta c_m)^2}{(-(3\gamma + 2\beta\lambda v)^2 + 16\beta\kappa)^2}.$$

3.2 Advertisement Sharing when Retailer Recovery Model

In this model, manufacturer responsible for recycling of waste products from retailer and remanufacturing, and selling new products to retailer through the wholesale price. The retailer is responsible for product sales, recycling waste products from customers, and advertising including information about products demand and the recycling of waste products. The advertising cost sharing proportion still decided by the manufacturer.

Given wholesale price *w* and advertising cost sharing proportion *x* by manufacturer, the retailer's problem is:

$$\max_{p>0,m>0} \prod_R^K = (p-w)(\phi - \beta p + \gamma m) - x\kappa m^2/2 + b\lambda m(\phi - \beta p + \gamma m) - Am\lambda(\phi - \beta p + \gamma m) \tag{4}$$

Due to $\partial^2 \prod_R^R/\partial p^2 = -2\beta < 0$, $\partial^2 \prod_R^R/\partial m^2 = -x\kappa + 2(b-A)\gamma\lambda < 0$, $(\partial^2 \prod_R^R/\partial m^2)(\partial^2 \prod_R^R/\partial p^2) - (\partial^2 \prod_R^R/\partial p \partial m)^2 = -(\gamma + (b-A)\beta\lambda)^2 + 2\beta x\kappa > 0$, in order to ensure $\prod_R^R$ is a concave function on *p* and *m*, we suppose $x > \gamma^2/2\beta\kappa$ establish first, verify this condition at last. We can obtain the optimal retailer's decision by first-order derivative to (4) on *p* and *m*, we get:

$$p^{R*} = \frac{(\phi + w\beta)(\kappa x + (A-b)\lambda\gamma) - \phi\beta\lambda^2(b-A)^2 - w\gamma^2}{-(\gamma + (b-A)\beta\lambda)^2 + 2\beta\kappa x}, m^{R*} = \frac{(\phi - w\beta)(\gamma + (b-A)\beta\lambda)}{-(\gamma + (b-A)\beta\lambda)^2 + 2\beta\kappa x} \tag{5}$$

Manufacturer has known the retailer would decide by equation (5), so manufacturer's problem in the first phase is:

$$\max_{w>0,x>0} \Pi_M^R = (\phi-\beta p+\gamma m)(w-c_m+\Delta m\lambda)-b\lambda m(\phi-\beta p+\gamma m)-(1-x)\kappa m^2/2$$

According to the first order conditions, we get:

$$x^{R*}=\frac{2(\gamma+(b-A)\beta\lambda)}{3\gamma+\beta\lambda(2\Delta-3A+b)},\ w^{R*}=\frac{\{(3\gamma^2-2\beta\gamma(3A+b-4\Delta)\lambda+\beta(-8\kappa+\beta(3A^2-b^2+2A(b-4\Delta)+4\Delta^2)\lambda^2))\phi+2\beta(3\gamma^2+2\beta\gamma(-3A+2b+\Delta)\lambda+\beta(-4\kappa+(A-b)\beta(3A-b-2\Delta)\lambda^2))c_m\}}{\beta((3\gamma+\beta\lambda(2\Delta-3A+b))^2-16\beta\kappa)},$$

$$p^{R*}=\frac{\{(3\gamma^2+4\beta\gamma(-3A+b+2\Delta)\lambda+\beta(-12\kappa+\beta(-3A+b+2\Delta)^2\lambda^2))\phi+2\beta(3\gamma^2-2\beta\kappa+\beta\gamma(-3A+2b+\Delta)\lambda)c_m\}}{\beta((3\gamma+\beta\lambda(2\Delta-3A+b))^2-16\beta\kappa)},\ m^{R*}=\frac{2(3\gamma+\beta(-3A+b+2\Delta)\lambda)(\phi-\beta c_m)}{\beta(-(3\gamma+\beta\lambda(2\Delta-3A+b))^2+16\beta\kappa)}$$

$$\Pi_M^{R*}=\frac{2\kappa(\phi-\beta c_m)^2}{-(3\gamma+\beta\lambda(2\Delta-3A+b))^2+16\beta\kappa},\ \Pi_R^{R*}=\frac{\{4\kappa(3\gamma^2+2\beta\gamma(-15A+7b+8\Delta)\lambda+\beta(-24\kappa+\beta(15A^2-14Ab+3b^2-16A\Delta^2)\lambda^2))(-\phi+\beta c_m)\}}{((3\gamma+\beta\lambda(2\Delta-3A+b))^2-16\beta\kappa)^2}.$$

Especially, when $b=\Delta$ both of them achieve the maximization of profit, then we get:

$$x^{R*}=\frac{2}{3},\ w^{R*}=\frac{3((\gamma+\beta v\lambda)^2-8\beta\kappa)\phi+6((\gamma+\beta v\lambda)^2-4\beta\kappa)\beta c_m}{\beta(9(\gamma+\beta\lambda v)^2-16\beta\kappa)},\ m^{R*}=\frac{6(\gamma+v\beta\lambda)(\phi-\beta c_m)}{-9(\gamma+\beta\lambda v)^2+16\beta\kappa},\ D^{R*}=\frac{4\kappa\beta(\phi-\beta c_m)}{-9(\gamma+\beta\lambda v)^2+16\beta\kappa},$$

$$p^{R*}=\frac{3((\gamma+\beta v\lambda)^2+2\beta^2v^2\lambda^2)-4\beta\kappa)\phi+2(3\gamma^2+3\beta v\lambda\gamma-2\beta\kappa)\beta c_m}{\beta(9(\gamma+\beta\lambda v)^2-16\beta\kappa)},\ \Pi_M^{R*}=\frac{2\kappa(\phi-\beta c_m)^2}{-9(\gamma+\beta\lambda v)^2+16\beta\kappa},$$

$$\Pi_R^{R*}=\frac{4\kappa(-3(\gamma+v\beta\lambda)^2+4\beta\kappa)(-\phi+\beta c_m)^2}{(-9(\gamma+\beta\lambda v)^2+16\beta\kappa)^2},\ \Pi_T^{R*}=\frac{6\kappa(-5(\gamma+v\beta\lambda)^2+8\beta\kappa)(-\phi+\beta c_m)^2}{(-9(\gamma+\beta\lambda v)^2+16\beta\kappa)^2}.$$

From the second section, it can easy verify $2/3>\gamma^2/2\beta\kappa$.

4. Model Comparative Analysis

Proposition 1: ① $x^{M*}<x^{R*}$; ② $\partial x^{M*}/\partial\lambda<0$, $\partial x^{M*}/\partial\gamma>0$ 。

Proof: From the models results, $x^{M*}-x^{R*}=\frac{2\gamma}{3\gamma+2\beta\lambda v}-\frac{2}{3}<0$, then $x^{M*}<x^{R*}$.

Due to $\frac{\partial x^{M*}}{\partial\lambda}=-\frac{4\beta\gamma v}{(3\gamma+2\beta(-A+\Delta)\lambda)^2}<0$, $\frac{\partial x^{M*}}{\partial\gamma}=\frac{4\beta v\lambda}{(3\gamma+2\beta(-A+\Delta)\lambda)^2}>0$, so $\frac{\partial x^{M*}}{\partial\lambda}<0$, $\frac{\partial x^{M*}}{\partial\gamma}>0$ 。

Proposition 1 shows that the advertising costs share proportion of retailer is smaller when manufacture recycling. The proportion is negatively correlated with the advertising recovery efficiency factor, positive correlation with the marketing efficiency factor. means that the proportion when retailer recycling independent of advertising recovery efficiency factor and marketing efficiency factor.

Proposition 2: ① $w^{M*}<w^{R*}$;② $\partial w^{M*}/\partial\lambda<0$, $\partial w^{R*}/\partial\lambda>0$;③ $\partial w^{M*}/\partial\gamma>0$, $\partial w^{M*}/\partial\gamma>0$ 。

Proof: from the models results and assumes, we get:

$$w^{M*}-w^{R*}=-\frac{(4\beta v\lambda)(\phi-\beta c_m)\left(9\gamma\left(-(\gamma+\beta v\lambda)^2+2\beta\kappa\right)+2\beta\kappa\gamma+\beta v\lambda(-12\gamma\beta v\lambda-6\gamma^2+14\beta\kappa)\right)}{\beta\left(9\gamma^2+12\beta\gamma v\lambda+4\beta\left(-4\kappa+\beta v^2\lambda^2\right)\right)\left(9\gamma^2+18\beta\gamma v\lambda+\beta\left(-16\kappa+9\beta v^2\lambda^2\right)\right)}<-\frac{(4\beta v\lambda)(\phi-\beta c_m)\left(9\gamma\left(-(\gamma+\beta v\lambda)^2+2\beta\kappa\right)+2\beta\kappa\gamma+\beta v\lambda(-7(\gamma+\beta v\lambda)^2+14\beta\kappa)\right)}{\beta\left(9\gamma^2+12\beta\gamma v\lambda+4\beta\left(-4\kappa+\beta v^2\lambda^2\right)\right)\left(9\gamma^2+18\beta\gamma v\lambda+\beta\left(-16\kappa+9\beta v^2\lambda^2\right)\right)}<0$$

, then $w^{M*} < w^{R*}$. Due to $\frac{\partial w^{M*}}{\partial \lambda} = -\frac{4v(\phi-\beta c_m)\left(-9\gamma(\gamma+\beta v\lambda)^2+18\gamma\beta\kappa+5\gamma(\gamma^2-(\gamma-\beta v\lambda)^2)+16\beta^2 v\kappa\lambda\right)}{\left(9\gamma^2+12\beta\gamma v\lambda+4\beta\left(-4\kappa+\beta v^2\lambda^2\right)\right)^2}<0$, then we could conclude that $\frac{\partial w^{M*}}{\partial\gamma}>0$, $\frac{\partial w^{M*}}{\partial\gamma}>0$.

Proposition 2 shows that wholesale price when the manufacturer recycling products is lower than the retail recycling products wholesale price. The wholesale price when the manufacturer recycling is negative correlation with advertising recovery efficiency factor while it is positive correlation with advertising marketing efficiency factor; The wholesale price when the retailer recycling is positive correlation with advertising marketing efficiency factor as well as advertising recovery efficiency factor.

Proposition 3:

① $\Pi_M^{R*} > \Pi_R^{R*}$, $\Pi_M^{M*} > \Pi_R^{M*}$, $\Pi_M^{R*} > \Pi_M^{M*}$, $\Pi_R^{R*} < \Pi_R^{M*}$;

② $\partial\Pi_M^{R*}/\partial\gamma > 0$, $\partial\Pi_R^{R*}/\partial\gamma > 0$, $\partial\Pi_M^{M*}/\partial\gamma > 0$, $\partial\Pi_R^{M*}/\partial\gamma > 0$;

③ $\partial\Pi_M^{R*}/\partial\lambda > 0$, $\partial\Pi_R^{R*}/\partial\lambda > 0$, $\partial\Pi_M^{M*}/\partial\lambda > 0$, $\partial\Pi_R^{M*}/\partial\lambda > 0$。

Proof: Because $\Pi_M^{M*}-\Pi_R^{M*} = \frac{2\kappa\left(-4(\gamma+\beta v\lambda)^2+8\beta\kappa+\gamma^2\right)(\phi-\beta c_m)^2}{\left(9\gamma^2+12\beta\gamma v\lambda+4\beta\left(-4\kappa+\beta v^2\lambda^2\right)\right)^2}>0$,

$\Pi_M^{R*}-\Pi_R^{R*} = \frac{2\kappa\left(-3(\gamma+\beta v\lambda)^2+8\beta\kappa\right)(\phi-\beta c_m)^2}{\left(9\gamma^2+18\beta\gamma v\lambda+\beta\left(-16\kappa+9\beta v^2\lambda^2\right)\right)^2}>0$ $\Pi_M^{R*}-\Pi_M^{M*} = \frac{2\kappa(\phi-\beta c_m)^2\beta v\lambda(6\gamma+5\beta v\lambda)}{\left(-9\gamma^2+18\beta\gamma v\lambda+\beta\left(16\kappa-9\beta v^2\lambda^2\right)\right)\left(-9\gamma^2+12\beta\gamma v\lambda-4\beta\left(-4\kappa+\beta v^2\lambda^2\right)\right)}>0$,

$\Pi_R^{R*}-\Pi_R^{M*} < 4\kappa(\phi-\beta c_m)^2\frac{-4\beta\gamma v\lambda-3\beta^2v^2\lambda^2}{\left(9\gamma^2+12\beta\gamma v\lambda+4\beta\left(-4\kappa+\beta v^2\lambda^2\right)\right)^2}<0$, then ① is established. Due to $\frac{\partial\Pi_M^{R*}}{\partial\gamma} = \frac{36\kappa(\gamma+\beta v\lambda)(\phi-\beta c_m)^2}{\left(9\gamma^2+18\beta\gamma v\lambda+\beta\left(-16\kappa+9\beta v^2\lambda^2\right)\right)^2}>0$, $\frac{\partial\Pi_R^{R*}}{\partial\gamma} = \frac{24\kappa(\gamma+\beta v\lambda)\left(-9(\gamma+\beta v\lambda)^2+18\kappa\beta\right)(\phi-\beta c_m)^2}{\left(-9\gamma^2-18\beta\gamma v\lambda+\beta\left(16\kappa-9\beta v^2\lambda^2\right)\right)^3}>0$,

$\frac{\partial\Pi_R^{M*}}{\partial\gamma} = \frac{8\kappa\left(-27\gamma(\gamma+\beta v\lambda)^2+54\beta\kappa\gamma+32\beta^2v\kappa\lambda+27\gamma\beta^2v^2\lambda^2+27\gamma^2\beta\lambda\kappa+4\beta^3v^3\lambda^3\right)(\phi-\beta c_m)^2}{\left(-9\gamma^2-12\beta\gamma v\lambda+4\beta\left(4\kappa-\beta v^2\lambda^2\right)\right)^3}>0$, then② is established.

Because $\frac{\partial\Pi_R^{R*}}{\partial\lambda} = \frac{24\beta v\kappa(\gamma+\beta v\lambda)\left(-9(\gamma+\beta\kappa\lambda)^2+18\kappa\beta\right)(\phi-\beta c_m)^2}{\left(-9\gamma^2-18\beta\gamma v\lambda+\beta\left(16\kappa-9\beta v^2\lambda^2\right)\right)^3}>0$, $\frac{\partial\Pi_M^{R*}}{\partial\lambda} = \frac{36\beta v\kappa(\gamma+\beta v\lambda)(\phi-\beta c_m)^2}{\left(9\gamma^2+18\beta\gamma v\lambda+\beta\left(-16\kappa+9\beta v^2\lambda^2\right)\right)^2}>0$,

$\frac{\partial\Pi_R^{M*}}{\partial\lambda} = \frac{8\beta v\kappa\left(-27\gamma(\gamma+\beta v\lambda)^2+54\beta\kappa\gamma+32\beta^2v\kappa\lambda+15\gamma\beta^2v^2\lambda^2+18\gamma^2\beta\lambda\kappa\right)(\phi-\beta c_m)^2}{\left(-9\gamma^2+12\beta\gamma v\lambda+4\beta\left(4\kappa-\beta v^2\lambda^2\right)\right)^3}>0$ $\frac{\partial\Pi_M^{M*}}{\partial\lambda} = \frac{8\beta v\kappa(3\gamma+2\beta v\lambda)(\phi-\beta c_m)^2}{\left(9\gamma^2+12\beta\gamma v\lambda+4\beta\left(-4\kappa+\beta v^2\lambda^2\right)\right)^2}>0$

, So ③ is proofed。

Proposition 3 has showed that either the recycle mode of manufacturers or retailers, the manufacturer's profit is always lager than the retailer's profit. The profit of the manufacturer is larger than the manufacturer recycle mode in the retailer recycle mode, and the profit of the retailer is larger than the retailer recycle mode in the manufacturer recycle mode. The profit of the manufacturer and retailer is positive relation to the factor of advertisement efficiency and marketing efficiency.

Summarizing the propositions above, it can be seen that the manufacturer recycle mode can improve the profit of the retail, and the retailer recycle mode can enhance the profit of the manufacturer. Under the manufacturer recycle mode, the improvement of the advertisement recycle efficiency has reduced the making cost of the product, then lessen the wholesale price and augment the retail price, Although the improvement of the retail price will reduce the quantity of demand, the whole the quantity of demand will be presented as the tendency of rise with the improvement of the pouring quantity of advertisement. The ratio of the advertisement share that the manufacture is going to undertake will increase under the condition, and the enhancement of its profit results from the old product cost saving of recycle which increases the profit of wholesale, while the enhancement of retail profit derives from the increase of sale profit and the reduce of the ratio of the advertisement share, in the other words at the same time, the close-loop supply chain is cooperating the profit of every member mainly by reducing the wholesale price and the ratio of the advertisement share. The increase of advertisement sale efficiency results in the improvement of the wholesale price, the retail price and the quantity of demand of the advertisement pouring quantity., while the ratio of the advertisement share that the retailer will undertake will rise, at the time, the profit of the manufacturer and retailer all mainly results from the increase of the sale quantity, in the other words at the same time, the close-loop supply chain is cooperating the profit of every member mainly by enhancing the wholesale price, the retail price and the ratio of the advertisement share.

Under the retailer recycle mode, the ratio of the advertisement share does not change, the improvement of advertisement recycle efficiency makes the retail recycle more old products, then makes more recycle profit by reselling the manufacturer, which increase the product wholesale price, the retail price, the market demand quantity, the level of the pouring of advertisement and the profit of the retailer. The profit of the manufacturer mainly derives from the enhancement of the wholesale price and the demand quantity which bring the profit of wholesale simultaneously, and the improvement of the retailer's profit mainly results from the enhancement of reselling recycle old commodities quantity, meanwhile, the close-loop supply chain is cooperating the profit of every member mainly by improving the wholesale price. The increase of advertisement sale efficiency results in the demand quantity, the retail price and the retail price , the advertisement pouring quantity and the profit, at the time, the profit of the manufacturer results from the increase of wholesale price and quantity of demand, while the profit of the retailer mainly derives from the increase of retail price and demand, the close-loop supply chain is cooperating the profit of every member mainly by enhancing the wholesale price, the retail price and the ratio of the advertisement share, in the other words, the close-loop supply chain is cooperating the profit of every member mainly by enhancing the wholesale price, the retail price.

6. Experimental Analysis and Simulation

In order to verify the models, we set $\phi=10000$, $c=10$, $\gamma=5$, $\Delta=7$, $\lambda=0.003$, $A=4$, $\kappa=1$. Because this paper mainly studies the advertising impact factors to profits of different subjects and cost allocation problem in closed-loop supply chain, therefore, we mainly simulation analysis the influence of advertising marketing efficiency factor and the advertising recovery efficiency factor for profit. The simulation results as shown from figure 1-figure 10, which "—"means retailer recycling mode,"…"means manufacture recycling mode.

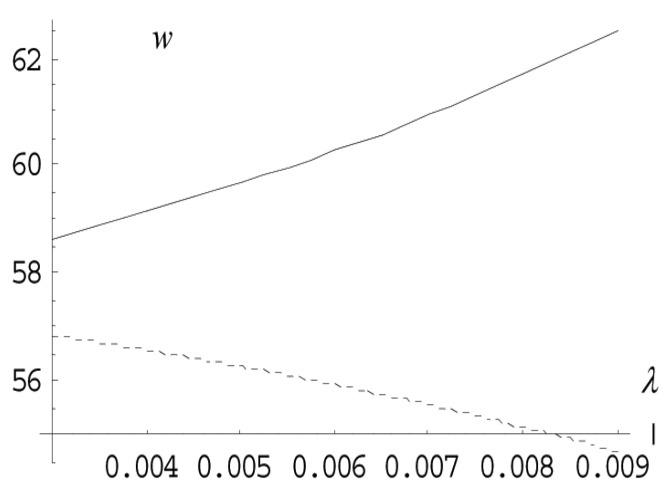

Figure 1. Wholesale price change with λ

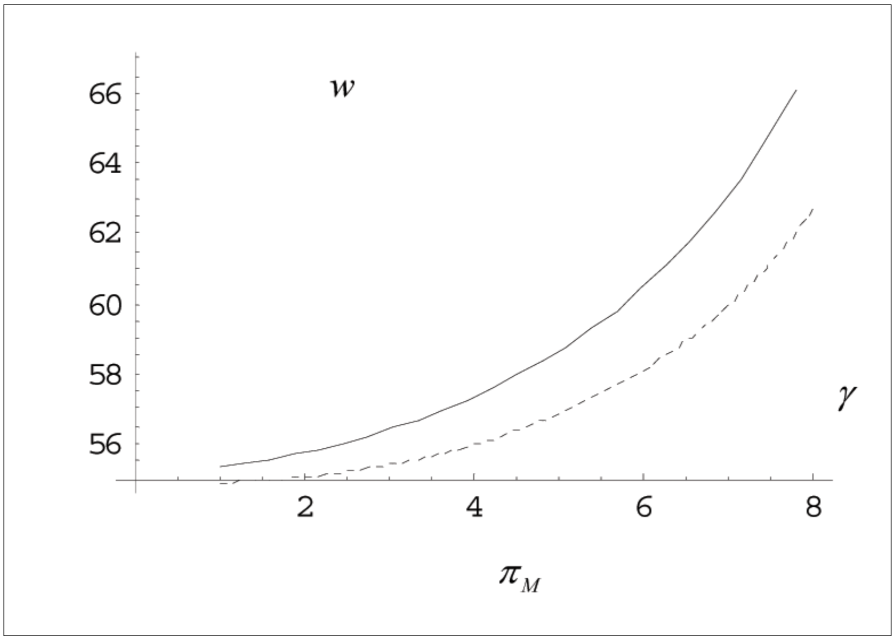

Figure 2. Wholesale price change with γ

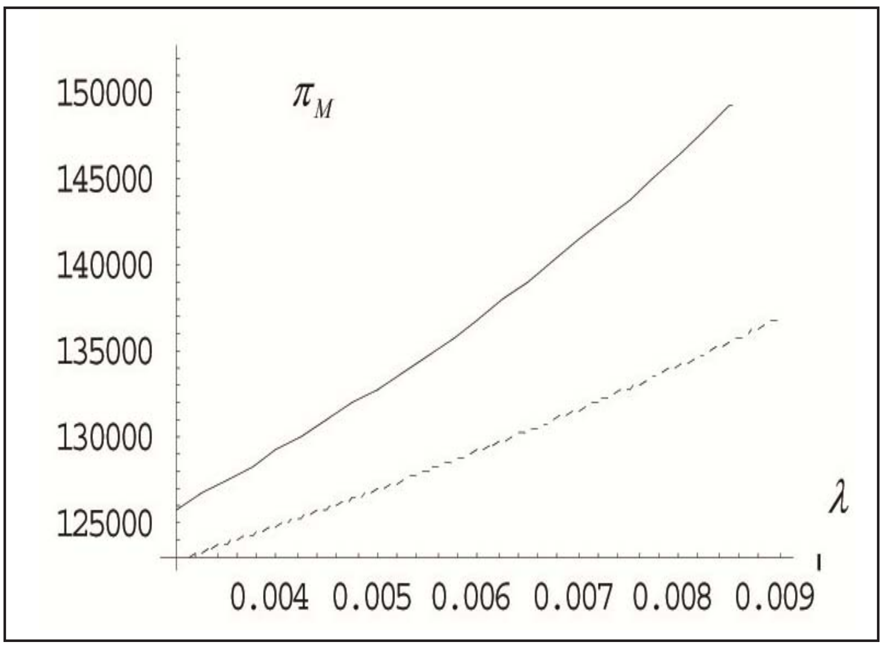

Figure 3. Manufacture profit change with λ

From Figure 1 and Figure 2 it can be seen that in the manufacturer recycling mode, the wholesale price decreasing with the advertising recycling efficiency factor increases, but increasing along with advertising and marketing efficiency factor increases. In the retailer recovery mode, wholesale price is positively related advertising recycling efficiency factor as well as advertising marketing efficiency, then the conclusion of Proposition 2 was established.

From Figure 3 - Figure 8 it can be seen, the profit of the manufacturer, retailer and the whole all increasing with the advertising recycling efficiency factor and advertising marketing efficiency factor. Manufacturer's profit is larger when retailer recycling, both the profit of retailer and the whole are larger when manufacture recycling while the total profit is larger .This conclusion is consistent with the proposition 3.

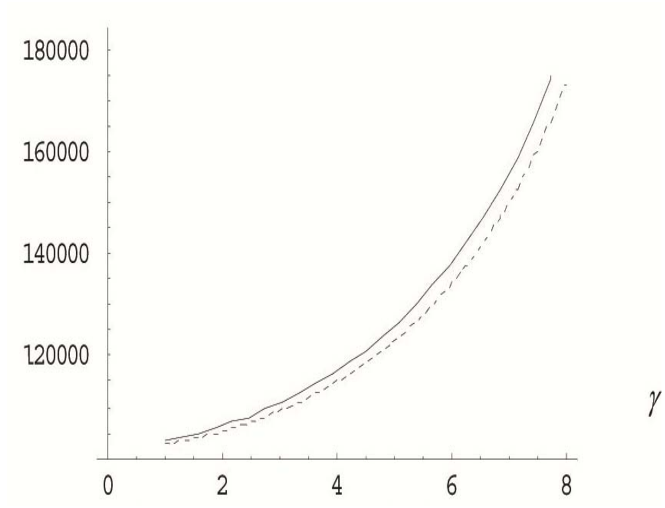

Figure 4. Manufacture profit change with γ

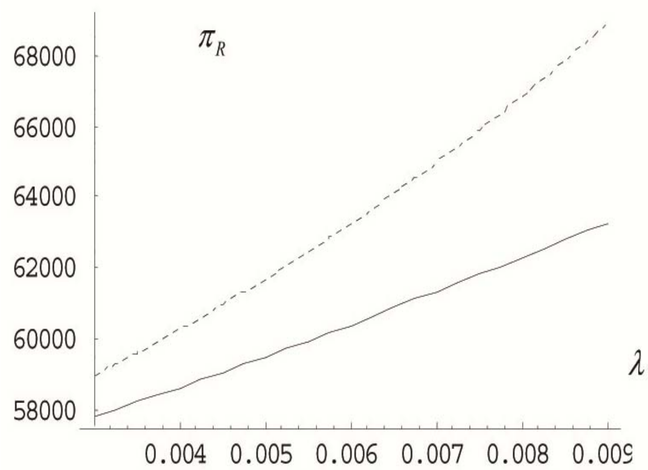

Figure 5. Retailer profit change with λ

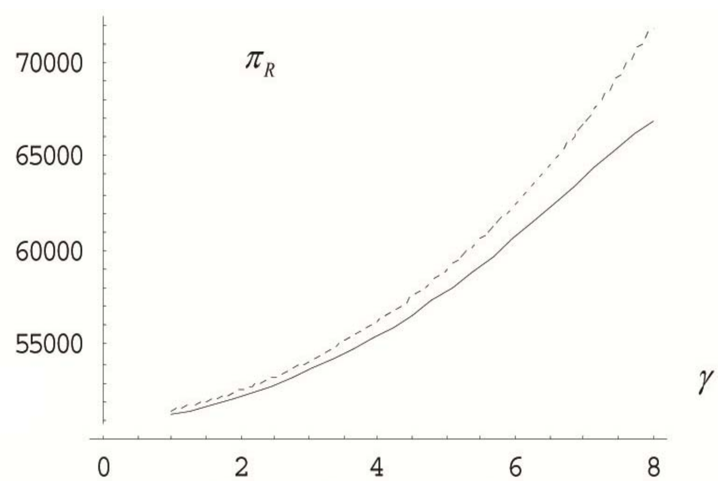

Figure 6. Retailer profit change with γ

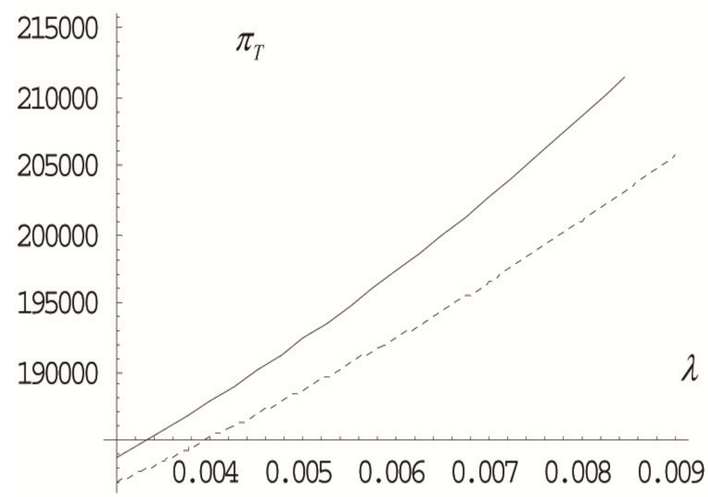

Figure 7. Total profit change with λ

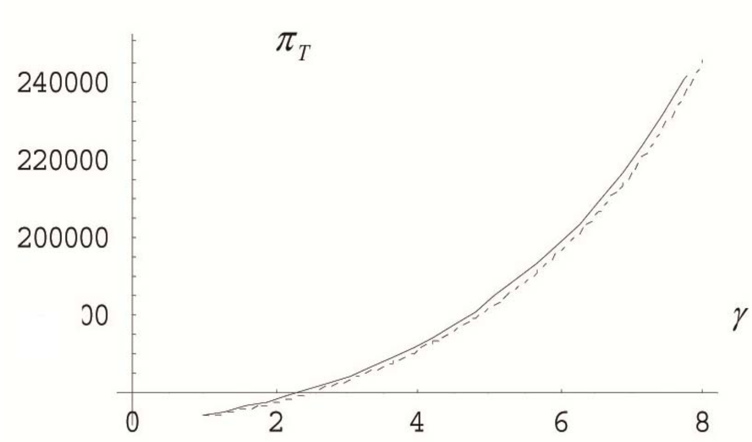

Figure 8. Total profit change with γ

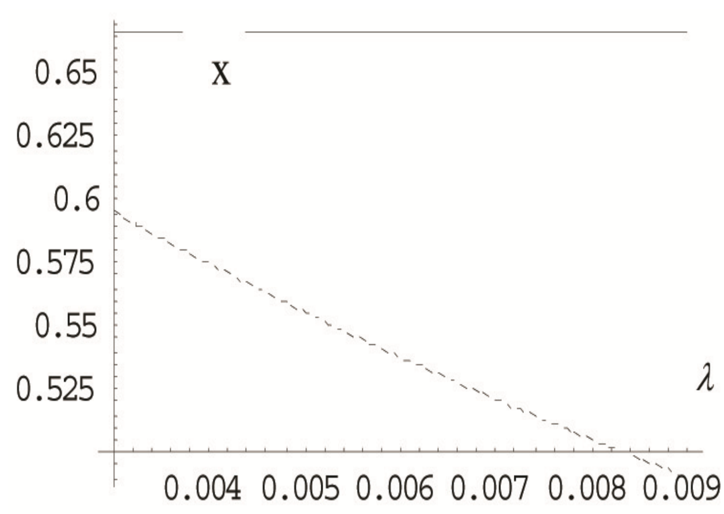

Figure 9. Adverting cost share proportion change with λ

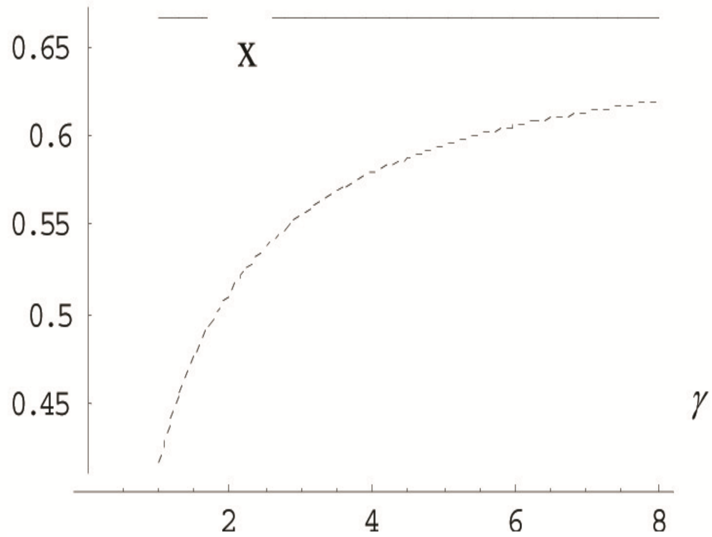

Figure 10. Adverting cost share proportion change with γ

From Figure 9 - in Figure 10 it can be seen, advertising cost share proportion is fixed and higher when retailer recycling. Advertising cost share proportion when manufacture recycling increasing with advertising marketing efficiency factor, but decreasing with advertising recycling efficiency factor, this also verified proposition 1 conclusion.

5. Conclusion and Summary

In this paper, In order to study price decision and advertising coordinate strategy, we divided the closed-loop supply chain into two cases including the retailer recycling and manufacturer recycling. With the impact of advertising for market demand and recycling, by using the game theory, we established two mathematical models, the results show that the manufacture recycling mode is conducive to retailer, retailer recycling mode helps manufacturer. Compared to retailer recovery, the share proportion of advertising costs of retailer is smaller of manufacture recovery. The retailer advertising share proportion of manufacturer recovery is negatively correlated with the advertising recovery efficiency factor, positive correlation with the advertising and marketing efficiency factor. The retailer advertising share proportion of retailer recovery has nothing to do with the cost of advertising marketing efficiency factor, recovery efficiency factor. At last we proved the related conclusion through simulation.

Research conclusions provide the guiding significance of closed-loop supply chain coordination between manufacturers and retailers as below: Firstly, for coordination between manufacturers and retailers to provide the theoretical guidance in the closed-loop supply chain. Based on the model and simulation in this paper, provide retail, wholesale, advertising investment levels and cost-sharing ratio formulation with the theoretical guidance, is advantageous to the cooperation and coordination between manufacturers and retailers, to maintain the stability of the closed-loop supply chain has a realistic significance.

Second, provide coordination strategy for the closed-loop supply chain under different recycling mode ads recovery efficiency and marketing efficiency changes.

6. Acknowledgements

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References

[1] Ahmadi-Javid, A., Hoseinpour., P. (2012). On a cooperative advertising model for a supply chain with one manufacturer

and one retailer, *European Journal of Operational Research* 219 (2) 458-466.

[2] Bergen M., John, G. (1997). Understanding Cooperative Advertising Participation Rates in Conventional Channels. *Journal of Marketing Research*, 1 (46), 357-369.

[3] Doraiswamy, K., McGuire, T. W., Staelin, R. (1979). An Analysis of Alternative Advertising Strategies Competitive Franchise Framework. Educators' Conference Proceedings, American Marketing Association.

[4] Frucher, G. E. (2009). Signaling Quality: Dynamic price-Advertising Model, *Journal of Optimization Theory and Applications*, 143. 479-496.

[5] Huang, Z., Li, S. X. (2001). Co-op advertising models in manufacturer–retailer supply chains: A game theory approach. *European Journal of Operational Research*, 135, 527–544.

[6] Huang, Z., Li, S. X., Mahajan, V. (2002). An analysis of manufacturer–retailer supply chain coordination in cooperative advertising, *Decision Sciences*, 33, 469–494.

[7] Ingene, C. A., Parry, M. E. (1995). Channel Coordination when Retailers Compete. *Marketing Science*, 14 (4), 360-377.

[8] Ingene, C. A., Parry, M. E. (1995). Coordination and Manufacturer Proût Maximization: The Multiple Retailer , *Journal of Retailing*, 71 (2), 129-151.

[9] Jeuland, A. P., Shugan, S. M. (1983). Managing Channel Proûts.*Marketing Science*, 2 (3), 239-272.

[10] Jorgensen S., Zaccour, G. (2000). Dynamic cooperative advertising in a channel, *Journal of Retailing*, 76 (1), 71-92.

[11] Krishnasamy, Ganesh., Kulkarni, Anand J., Paramesran, Raveendran. (2014). A hybrid approach for data clustering based on modified cohort intelligence and K-means, *Expert Systems with Applications*, 41 (13), 6009-6016.

[12] Lin, Phen-Lan., Huang, Po-Whei., Kuo, Chun-Hung., Lai, Y. H. (2014) .A size-insensitive integrity-based fuzzy c-means method for data clustering, *Pattern Recognition* 47 (5), 2042-2056

[13] Qin, Hongwu., Ma, Xiuqin., Herawan, Tutut., Zain, Jasni Mohamad, MGR. (2014). An information theory based hierarchical divisive clustering algorithm for categorical data, *Knowledge-Based Systems* 67, 401-411.

[14] Yue, J., Austin, J., Huang, Z. (2013). Pricing and advertisement in a manufacturer–retailer supply chain. *European Journal of Operational Research*. 231 (2), 492-502.

[15] Yue, J. (2006). Coordination of cooperative advertising in a two-level supply chain when manufacturer offers discount. *European Journal of Operational Research*, 168 (1), 65-68.

[16] Zhang, W., Li, S., Zhang, D., et al. (2014). On the impact of advertising initiatives in supply chains. *European Journal of Operational Research*, 234 (1), 99-107.