Stability Along the Pass of Differential Linear Repetitive Processes Using an LMI

Jamel Dridi, Houssem ouerfelli, Selma Ben Attia, Salah Salhi
Laboratory of Analysis and Control of Systems (LACS)
BP-37, le Belvedere, 1002, Tunis, Tunisia
National Engineering School of Tunis
Tunis, Tunisia
dridi_jamel@live.fr, houssemouerfelli@yahoo.com, benattiaselma@gmail.com, salhis@lycos.com

ABSTRACT: *This paper develops new sufficient conditions for the stability analysis along the pass and the synthesis problem of differential linear repetitive processes, based on application of the Kalman-Yakubovich-Popov lemma. The given results are expressed in terms of linear matrix inequality (LMI). The inclusion of extra design specifications is developed for the case of regional constraints on the eigenvalues of state matrix and a finite frequency range design. Simulation results demonstrate the good performance of application in iterative learning control.*

Keywords: Iterative Learning Control (ILC), Repetitive Systems, Stability Along the Pass, LMIs

Received: 18 July 2013, Revised 15 August 2013, Accepted 19 August 2103

1. Introduction

The original work of Iterative learning control has been the subject of intense research effort [1]. And an overview of the work until 1998 is given by Moore in [2]. Iterative Learning Control (ILC) is an advanced methodology introduced in 1984 by Arimoto et al. [1] The ILC has been especially developed to improve the performance of systems that operate in a repetitive manner where the task is to follow some specified trajectory (tracking problem) in a specified finite time interval, known also as a pass or a trial in the literature, with high precision [1]. ILC employs the knowledge of the control input and the system error in the past executions to modify the control input in the next trial. In particular, the aim is to improve performance from trial-to-trial in the sense that the tracking error (the difference between the output on a trial and the specified reference trajectory) is stable along the pass [3], [4], [5], [6].

The theory of stability along the pass to the processes produces three conditions [3], [4], [5], [6], discussed above, that can be tested by direct application of standard, or 1D, linear systems stability tests. Two of these tests require that the eigenvalues of the matrices which describe the previous pass profile contribution to the current pass profile and the current pass state vector contribution to the along the pass dynamics lie in the open unit circle and open left-half of the complex plane, respectively. The third test requires the computation of the eigenvalues of the transfer function matrix representation of the contribution of the previous pass profile dynamics to current one for $s = j\omega$, $\omega \geq 0$ where denotes the Laplace transform variable. Assuming that the first two conditions hold, stability along the pass requires that the loci generated by the eigenvalues of this transfer-function matrix lie in the open unit circle in the complex plane [3], [4], [5], [6].

Figure 1 shows a schematic of an ILC scheme. Here the subscript k represents the trial or repetition number, and the reference signal $y_d(t)$ is defined on the interval $[0, T]$. At any given repetition, k, a control input of $u_k(t)$ is applied to the system to produce output, $y_k(t)$, $t \in [0, T]$ where T is the length of the periodic reference. The input and output of the k^{th} trial are stored in memory and used along with the fixed reference to calculate the input for the $(k+1)^{th}$ trial. Where the control objective for ILC can be expressed as $\lim_{k \to \infty} e_k(t) = \lim_{k \to \infty} (y_d(t) - y_k(t)) = 0$, $\forall t$ and $\lim_{k \to \infty} e_k(t) = 0$, $\forall k$.

Thus the goal of the algorithm written by [1] is to design an update law to produce the lowest possible error as $k \to \infty$. In most ILC systems, it is assumed that the plant initial conditions are reset at the start of every period $(x_k(0) = x_0)$.

Also, the system is assumed to be stable along the pass, or stabilized along the pass, using feedback control.

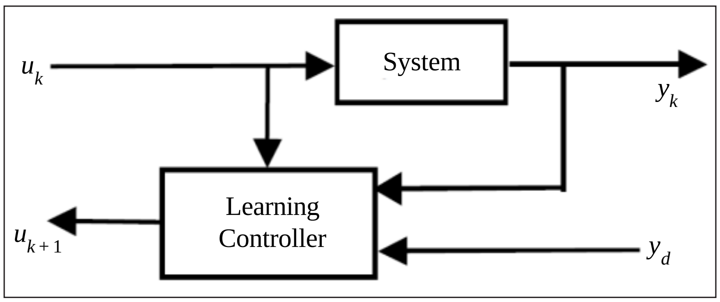

Figure1. Iterative Learning control Configuration

In a repetitive process the pass profile $y_{k \geq 0}(t)$, $0 \leq t \leq \alpha$ that generated on pass acts as a forcing function contributing to the dynamics of the next pass profile $y_{k+1}(t)$. Typical iterative learning control algorithms construct the input to the plant on a given trial from the input used on the last trial plus an additive incremental which is typically a function of the past values of the output error, that is, the difference between achieved output and desired output.

This note is organized as follows. In Section 2, the theoretical study of stability along the pass of a differential linear repetitive process is introduced, and the Kalman-Yakubovich- Popov (KYP) lemma is presented. Section 3, applying the Iterative Learning Control for differential SISO system, and a performance analysis of ILC systems by mean of a quadratic Lyapunov function is investigated. In Section 4, a new sufficient LMI condition is demonstrated, in order to obtain stabilizing classes of linear repetitive systems. Then, a numerical evaluation is presented to illustrate the effectiveness of the proposed approach in Section 5. Finally, the paper is concluded.

1.1 Notations

$\sigma(A)$ and $\rho(A)$ denote the spectrum and the spectral radius of a given matrix A. X (respectively,) denotes a real symmetric positive (respectively, negative) definite matrix. A^T denotes the transpose of A. Furthermore, the symbol $\mathbb{C}$ indicates the set of a complex numbers and $\mathbb{C}$_the open left-half of the complex plane. To simplify the scriptures, we will use the symbol $sym\{A\} = A^T + A$. * is used for the blocks induced by symmetry. Also, the identity and null matrix of the required dimensions are denoted by I and 0, respectively.

2. Stability Theory of a Differential Linear Repetitive Process

In this section, we discuss the concept of a repetitive control system, and recall the main stability theorem for such systems.

The state space model, of a differential linear repetitive process [3], [4], described by the following form over $0 \leq t \leq \alpha$, $k \geq 0$.

$$\begin{cases} \dot{X}_{k+1}(t) = Ax_{k+1}(t) + Bu_{k+1}(t) + B_0 y_k(t), \\ y_{k+1}(t) = Cx_{k+1}(t) + Du_{k+1}(t) + D_0 y_k(t). \end{cases} \qquad (1)$$

Here on pass k, $\alpha < +\infty$: denotes the pass length (α : is the finite pass length), $x_k(t) \in \mathbb{R}^n$: is the state vector, $u_k(t) \in \mathbb{R}^r$: is the input vector, $y_k(t) \in \mathbb{R}^r$: is the output or pass profile. To complete the process description, it is necessary to specify the initial, or boundary, conditions, i.e. the state initial vector on each pass and the initial profile.

For a differential linear repetitive process of the form considered here, stability along the pass holds if, and only if, the so-called 2D characteristic polynomial [3], [4]:

$$C_{diff\,LRP}(s, z)_2 = \left(\begin{bmatrix} SI - A & -B_0 \\ -z_2 C & I - z_2 D_0 \end{bmatrix} \right) \neq 0,$$

$$\forall \{s, z_2\} \in \{(s, z_2) : Re(s) \geq 0, |z_2| \leq 1\}. \tag{2}$$

Where $s \in \mathbb{C}$ is the Laplace transform indeterminate and $z_2 \in \mathbb{C}$ arises, as before, form the use of the z-transform in the pass to-pass direction.

$$\begin{cases} x_k(t) = s\, x_k(t+1), \\ x_k(t) = z_2 x_{k+1}(t)\,. \end{cases} \tag{3}$$

Theorem 1: [4] *A differential linear repetitive process of the form* (1) *is stable along the pass if and only if,*

(i) $\rho(D_0) < 1$

(ii) $\sigma\ (A) \in \mathbb{C}\_,$ *all eigenvalues of the matrix A have strictly negative,*

(iii) $G_{diff}(s) = C(sI - A)^{-1} B_0 + D_0 < 1, \forall \omega \geq 0$ *all eigenvalues of* $G_{diff}(s)$ *shave modulus strictly less than one.*

All three conditions of the Theorem 1 have well-defined physical interpretations and, unlike equivalents [5], [6], [7], can be tested by direct application of 1D linear time invariant systems.

It is easy to show that stability along the pass guarantees that the corresponding limit profile of (i) is stable as a 1D linear system, i.e. all eigenvalues of the state matrix $A + B_0(I - D_0)^{-1}C$ have strictly negative real parts.

In terms of checking the conditions of these two results, the first two conditions in each case are easily solves.

Consider condition (i), this is the necessary and sufficient condition for asymptotic stability, i.e. BIBO stability over the finite pass length.

Applying the second conditions of Theorem 1, stability of the matrix A (i.e. a uniformly bounded first pass profile) is, in general, only a necessary condition for stability along the pass.

The last condition for stability along the pass of the three discussed above can be computationally intensive and is, not suitable for the synthesis of control laws for stability along the pass. To overcome these problems, Kalman-Yakubovich- Popov (KYP) lemma [6], [7], [8] establish the equivalence between Frequency Domain Inequalities (FDIs) for a transfer-function matrix and Linear Matrix Inequalities (LMIs) defined in terms of its state-space realization, as in [4] for 1D linear systems. The new results in this paper start with the development of LMI based tests for stability along the pass.

The Kalman-Yakubovich-Popov (KYP) lemma [6], [7], [8], is essential and a basis idea to develop necessary and sufficient conditions for stability along the pass of the SISO of the differential linear repetitive processes (1). The KYP lemma [6], [7], [8] is expressed as follows, for a differential linear time-invariant system with transfer-function matrix$G(j\omega)$ and frequency response matrix $G(j\omega) = \tilde{C}(j\omega I - \tilde{A})^{-1} \tilde{B}_0 + \tilde{D}_0$.

The following inequalities are equivalent [7], [8], [9] :

$$[G(j\omega) \quad I] \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} G^*(j\omega) \\ I \end{bmatrix} < 0.\ \forall \omega_l \leq \omega \leq \omega_h$$

where Π is a given real symmetric matrix and ω denotes the following frequency ranges

	LF (low freq.)	MF (middle freq.)	HF (high freq.)
Ξ	$\begin{bmatrix} -Q & P \\ P & \omega_l^2 Q \end{bmatrix}$	$\begin{bmatrix} -Q & P + j\omega_c Q \\ P - j\omega_c Q & -\omega_l \omega_h Q \end{bmatrix}$	$\begin{bmatrix} Q & P \\ P & -\omega_h^2 Q \end{bmatrix}$

Table 1. Frequency Ranges in the Continuous-Time Setting

2. The LMI $\begin{bmatrix} \tilde{A} & \tilde{B}_0 \\ I & 0 \end{bmatrix}^T \Xi \begin{bmatrix} \tilde{A} & \tilde{B}_0 \\ I & 0 \end{bmatrix} + \begin{bmatrix} \tilde{C} & \tilde{D}_0 \\ 0 & I \end{bmatrix}^T \begin{bmatrix} 1 & 0 \\ 0 & -\gamma^2 \end{bmatrix} \begin{bmatrix} \tilde{C} & \tilde{D}_0 \\ 0 & I \end{bmatrix} < 0.$ where $Q > 0$, P is a symmetric matrix, $\omega_c = \dfrac{\omega_l + \omega_h}{2}$ for a finite frequency range, and for an entire frequency range, that is, $\omega_l = 0$, $\omega_h = \infty$.

3. LMI Based Iterative Learning Control Design

In this paper, we considered a differential linear time invariant system described by the state space $\{A, B, C\}$ is considered:

$$\begin{cases} \dot{x}_k(t) = A x_k(t) + B u_k(t), \\ y_k(t) = C x_k(t). \end{cases} \quad 0 \le t \le \alpha,\ k \ge 0. \tag{4}$$

Where on trial k, $x_k(t) \in \mathbb{R}^n$ is the state vector, $y_k(t) \in \mathbb{R}^r$ is the output vector, $u_k(t) \in \mathbb{R}^r$ is the vector of control inputs, and $\alpha < \infty$ is the trial length. The signal desired is denoted by $dy(t)$ then $e_k(t) = y_d(t) - y_k(t)$ is the error on trial k, and the most basic requirement is to force the error to converge as $k \to \infty$. In particular, the objective of constructing a sequence of input functions such that the performance is gradually improving with each successive trial can be refined to a convergence condition on the input and error $\lim_{k \to \infty} \| e_k \| = 0$.

Let a control law given by:

$$\Delta u_{k+1}(t) = u_{k+1}(t) - u_k(t) = K_1 \dot{\eta}_{k+1}(t) + K_2 \dot{e}_k(t) \tag{5}$$

Where $\Delta u_{k+1}(t)$ denotes a variation of the control input, K_1 and K_2 are matrices with compatible dimensions.

Then clearly (4) and (5) can be written as

$$\begin{cases} \dot{\eta}_{k+1}(t) = \int_0^t [\dot{x}_{k+1}(\tau) - \dot{x}_k(\tau)]\, d\tau = (A + BK_1)\, \eta_{k+1}(t) + BK_2 e_k(t) & (6) \\ e_{k+1}(t) - x_k(t) = -y_{k+1}(t) + y_k(t) = -C\,(x_{k+1}(t) - x_k(t)) & \\ e_{k+1}(t) = -C\,(A + BK_1)\, \eta_{k+1}(t) + (I - CBK_2)\, e_k(t) & (7) \end{cases}$$

We introduced:

$$\tilde{A} = A + BK_1,\ \tilde{B}_0 = BK_2,\ \tilde{C} = -C\,(A + BK_1) \text{ and } \tilde{D}_0 = I - CBK_2 \tag{8}$$

Then, clearly (6)-(7) and (8) can be written as

$$\begin{bmatrix} \dot{\eta}_{k+1}(t) \\ e_{k+1}(t) \end{bmatrix} = \begin{bmatrix} \tilde{A} & \tilde{B}_0 \\ \tilde{C} & \tilde{D}_0 \end{bmatrix} \begin{bmatrix} \eta_{k+1}(t) \\ e_k(t) \end{bmatrix} \tag{9}$$

Which is of the form (1) and hence the repetitive process stability theory can be applied to this ILC control scheme (5). In particular, stability along the pass is equivalent to uniform BIBO stability (defined in terms of the norm on the underlying function space), i.e. independent of the trial length, and hence it may be possible to achieve pass -to- pass error convergence with acceptable along the pass dynamics.

It has been proved recently that any robust control problem can be turned into an LMI dilated one, in terms of converting the Lyapunov conditions to be generalized in equations by mean of lemmas [2], [3], [4].

However, it is very difficult to provide computationally effective tests for stability in this way.

One of the ways to derive tractable tests is by applying Lyapunov theory associated with LMI techniques that became a standard tool for the stability analysis of 1D system when manipulating state space models.

These Lyapunov functions must contain contributions from the current pass state and previous pass profile vectors, for example, composed of which is the sum of quadratic terms in the current pass state and previous pass profile respectively [8].

An alternative approach that does lead to control law design algorithms is [4], [8] this approach is developed by using candidate Lyapunov function for differential models, of the form:

$$\dot{V}(k,t) = \dot{x}_{k+1}^T(t)\, P_1 x_{k+1}(t) + y_k^T(t)\, P_2 y_k(t) \tag{10}$$

Where $P_1 > 0$ and $P_2 > 0$

With associated increment:

$$\Delta V(k,t) = \dot{x}_{k+1}^T(t)\, P_1 x_{k+1}(t) + x_{k+1}^T(t)\, P_1 \dot{x}_{k+1}(t) + y_{k+1}^T(t)\, P_2 y_{k+1}(t) - y_k^T(t)\, P_2 y_k(t)$$

Then the stability along the pass holds if $\Delta V(k, t) < 0$ for all k and t which is equivalent to the requirement that:

$$\Phi_i^T P_{i+1} \Phi_i - P_i < 0. \textit{ Where: } P_i = diag\,(P_1, P_2) \tag{11}$$

The following Theorem, allows the necessary and sufficient connection for the stability along the pass of 2D/ repetitive systems.

Theorem 2: [6] *The SISO version of* (9) *is stable along the pass if and only if there exist matrices* $r > 0$, $X > 0$, $Q > 0$ *and a symmetric matrix P such that the following LMIs are feasible*:

$$\begin{cases} (1)\ \tilde{D}_0^T r \tilde{D}_0 - r < 0, \\ (2)\ \tilde{A}^T X + X\tilde{A} < 0, \\ (3) \begin{bmatrix} \tilde{A}Q\tilde{A}^T + P\tilde{A}^T + \tilde{A}P & \tilde{A}QC^T + P\tilde{C}^T & \tilde{B}_0 \\ \tilde{C}Q\tilde{A}^T + \tilde{C}P & \tilde{C}Q\tilde{C}^T - I & \tilde{D}_0 \\ \tilde{B}_0^T & \tilde{D}_0^T & -I \end{bmatrix} < 0. \end{cases} \tag{12}$$

The difficulty with the condition of Theorem 2 is that it is non-linear in its parameters. It can, however, be controlled in to the following results, where the inequality is a strict LMI a linear constraint which also gives a formula for computing the gain K_1 and K_2.

4. A New Condition for the Stablity along the Pass

In this section, enforcing the frequency attenuation as required by condition (3) of Theorem 1 over the complete frequency range is either unobtainable or very restrictive [9].

Hence the subject of this section is presented, the attenuation is only required over a finite frequency range $\omega_l \le \omega \le \omega_h$, where the lower and upper frequency values are selected based on knowledge of the particular example considered, where the idea is avoid oscillatory time responses for $|G_{diff}(j\omega)| \le 1 + \varepsilon$ for all $\omega_l \le \omega \le \omega_h$ and ε is very small.

The next Theorem in this paper start with the development of LMI based tests for stability along the pass. This allows us to use the strong concept of stability along the pass for these processes, in an ILC setting, as a possible means of dealing with errors transients in the dynamics produced along the trials [10], [11].

Theorem 3: *The SISO version of* (9) *is stable along the pass if there exist matrices* $X > 0$, $Q > 0$, G, Ng, K_2 *and a symmetric matrix* P *such that the following LMIs are feasible:*

$$(1)\begin{bmatrix} -CBK_2 & 0 \\ 0 & CBK_2 - 2 \end{bmatrix} < 0, \tag{13}$$

$$(2)\begin{bmatrix} sym\,(AG + BN_g) & X + AG + BN_g - G^T \\ * & -G - G^T \end{bmatrix} < 0, \tag{14}$$

$$(3)\begin{bmatrix} sym\,\{\alpha(AG + BN_g)\} & * & P + AG + BN_g - \alpha G^T & BK_2 \\ -\alpha CAG - \alpha CBN_g & -I & -CAG - CBN_g & I - CBK_2 \\ * & * & Q - G - G^T & 0 \\ * & * & 0 & -I \end{bmatrix} < 0. \tag{15}$$

Where $\alpha > 0$. If these LMIs are feasible, the gain is computed by: $K_1 = N_g G^{-1}$

Proofs

1- First LMI: First note that both r and, $\tilde{D}_0$ are real numbers and hence $r\,(\tilde{D}_0^2 - 1) < 0$ with Hence, using (8), it is obvious that $(1 - CBK_2)^2 - 1 < 0$, or $CBK_2\,(CBK_2 - 2) < 0$.

Hence, we require $0 < CBK_2 < 2$. The value of CBK_2 greatly influences the pass to pass error convergence, which is equivalent to (13) since here CBK_2 is a scalar.

2- Second LMI : $A^T X + X\tilde{A} < 0$ by applying the projection lemma [12], we obtain:

$$(*):\quad [-1 \quad 1]\begin{bmatrix} 0 & X \\ X & 0 \end{bmatrix}\begin{bmatrix} -1 \\ 1 \end{bmatrix} < 0$$

$$(**):\begin{bmatrix} A \\ I \end{bmatrix}\begin{bmatrix} 0 & X \\ X & 0 \end{bmatrix}[A^T \quad I] < 0$$

Substituting (*) – (**), applying the projection lemma and the Schur complement, we obtain:

$$\begin{bmatrix} 0 & X \\ X & 0 \end{bmatrix} + \begin{bmatrix} A \\ -I \end{bmatrix} G\,[I \quad I] + \begin{bmatrix} I \\ I \end{bmatrix} G^T [A^T \quad I] < 0 \tag{16}$$

(16) is equivalent to:

$$\begin{bmatrix} AG & X + AG - G^T \\ * & -G - G^T \end{bmatrix} < 0$$

Substituting (8) in this last LMI, we obtain (14).

3- Third LMI: Multiplied by $\begin{bmatrix} I & 0 & A \\ 0 & 1 & C \end{bmatrix}$, the right side of (15) and the left by its transpose.

Introducing (8), we obtain by applying the projection lemma [12] the inequality (12). Moreover, (15) follows by setting $N_g = K_1 G$.

5. Simulation Results

In this section, an example is given to demonstrate the effectiveness of the proposed method. We consider a differential linear time invariant systems described by the state space $\{A, B, C\}$:

$$\begin{cases} \dot{x}_k(t) = Ax_k(t) + Bu_k(t), 0 \le t \le \alpha \\ y_k(t) = Cx_k(t) \end{cases}$$

5.1 Case A

$$\text{Where: } A = \begin{bmatrix} -2.929 & -0.3186 \\ -0.3186 & -0.8829 \end{bmatrix}, B = \begin{bmatrix} -0.2 \\ -1.52 \end{bmatrix}, C = [0.9 \quad 1.2]$$

By applying the control law (5), the system is stable along the pass in the closed-loop and the conditions in Theorem 3 provide the following gains:

$$K_1 = [-1.5062 \quad -0.6715] \text{ and } K_2 = -0.3193$$

Here:

$$\rho(\tilde{A}) = \begin{Bmatrix} -2.4896 \\ -0.0001 \end{Bmatrix}, \rho(\tilde{D}_0) = 0.3601$$

The three condition proposed in Theorem 3 are verified for $\omega_l \le \omega \le \omega_h$, this process is stable along the pass, and as confirmed by the Nyquist plot of Figure 2:

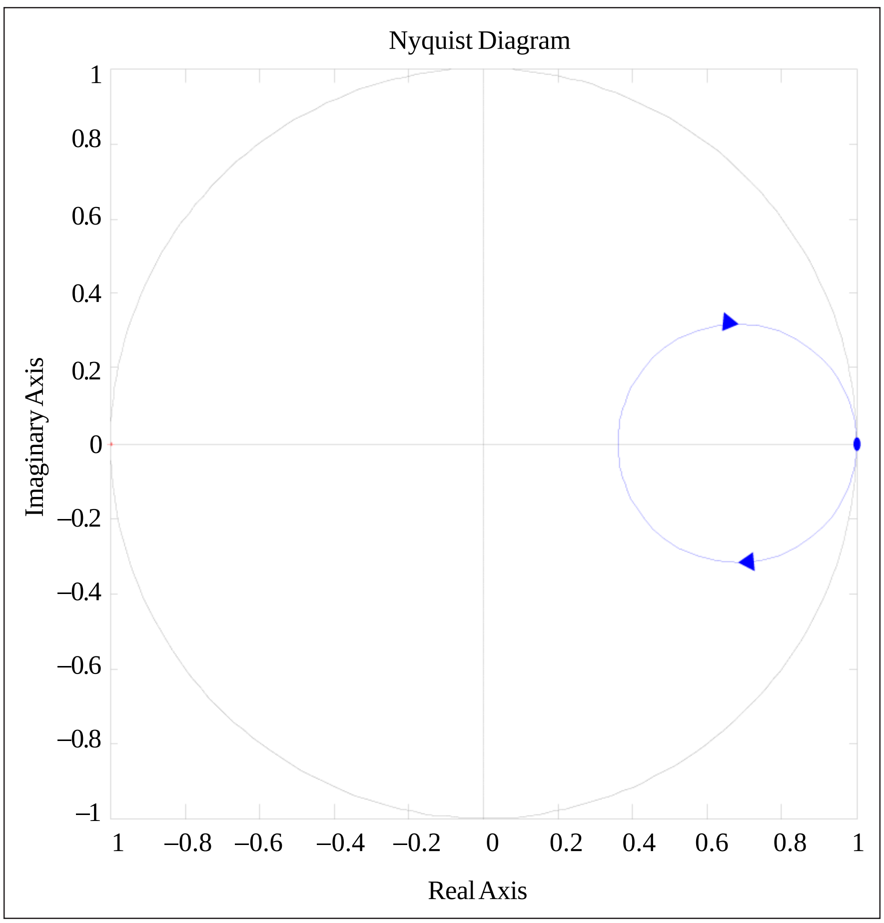

Figure 2. Nyquist plots for the stable along the pass process

5.2 CaseB

Applying the control law (5), the system

$$\text{Where: } A = \begin{bmatrix} -1.5 & 0.5 \\ -0.2 & 0.1 \end{bmatrix}, B = \begin{bmatrix} -2 \\ 0.1 \end{bmatrix} \text{ and } C = [-0.2 \quad 0.6]$$

is stable along the pass in the closed-loop and the conditions in Theorem 3 provide the following gains (for $\varepsilon = 0.001$):

$$K_1 = [-4.366\, e^{-001} \quad -1.763\, e^{-002}] \text{ and } K_2 = -9.770\, e^{-001}$$

Here:

$$\rho(\tilde{A}) = \begin{Bmatrix} -2.951\, e^{-001} \\ -2.332\, e^{-001} \end{Bmatrix}, \rho(\tilde{D}_0) = 5.505\, e^{-001}$$

Thus, the resulting ILC process can be guaranteed with its tracking error converging to zero along the iteration axis. Figure 3 shows the time evolution of the reference trajectory $y_d(t)$ and the output $y_k(t)$ for $k = 1$, $k = 2$, $k = 3$, $k = 5$ and $k = 6$. As shown in this figure, the output tracks the reference trajectory more and more accurate as the iteration number increases.

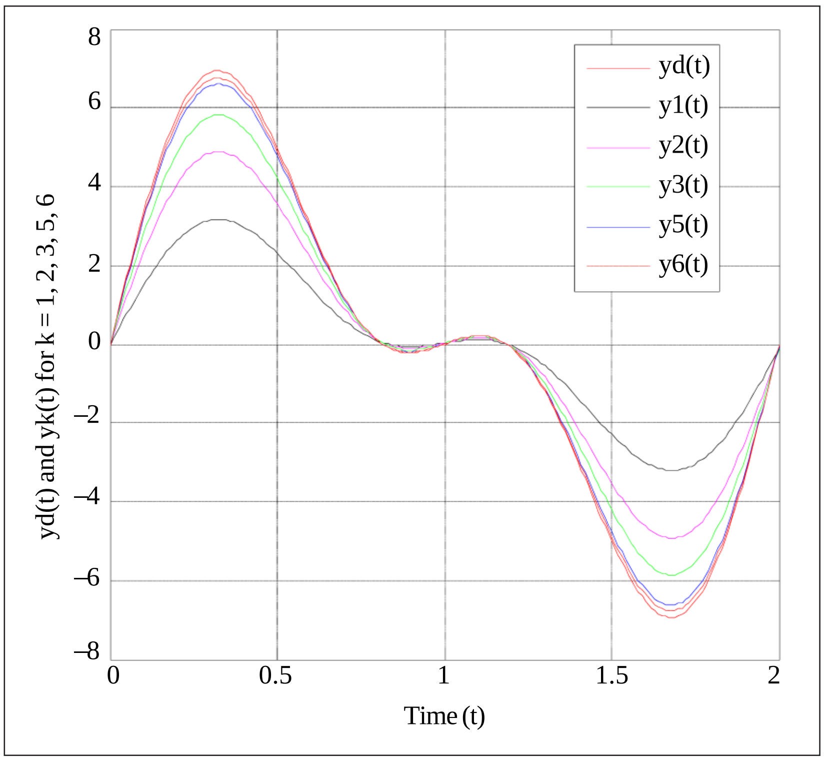

Figure 3. Reference and output signal for $k = 1$, $k = 2$, $k = 3$, $k = 5$ and $k = 6$

Figure 4 shows the time evolution of the output error $e_k(t) = y_d(t) - y_k(t)$. As shown in this figure, the tracking error converging to zero along the time and more accurate as the iteration number increases.

This simulation is performed with a reference trajector $y_d(t) = 3 * \sin(2\Pi * t) + 5 * \sin(\Pi * t)$, where $t \in [0, 2]$ and $t_s = 0.01$.

6. Acknowledgment

This paper has proposed a new LMI based condition for stability Along the Pass a class on 2D model differential linear repetitive process. Here we have given a new sufficient condition for synthesis of MIMO system and a new necessary and sufficient condition for the MISO system. A numerical example has been given to validate the effectiveness and advantages of the proposed method. Future study will be focused on how the learning behavior and the control behavior affect the error convergence speed and steady-state error, respectively.

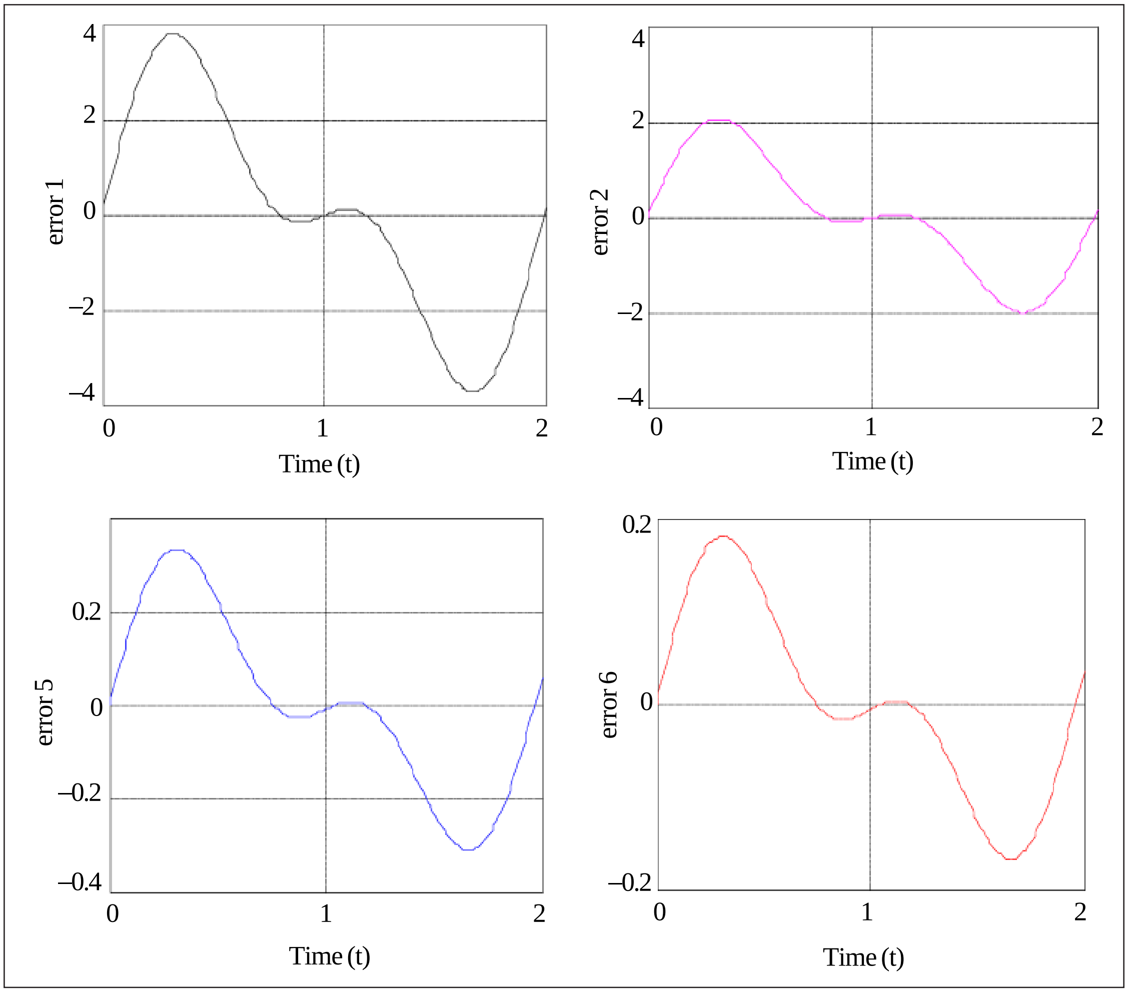

Figure 4. Tracking error with respect to the iteration number for $k = 1$, $k = 2$, $k = 3$, $k = 5$ and $k = 6$

References

[1] Arimoto, S., Kawamura, S., Miyazaki, F. (1984). Bettering operations of robots by learning, *Journal of Robotic Systems*, 1 (2) 123- 140.

[2] Moore, K. L. (1993). Iterative Learning Control for Deterministic Systems. Advances in Industrial Control Series. Springer-Verlag, London.

[3] Rogers, E., Owens, D. H. (1992). Stability Analysis for Linear Repetitive Processes, V. 175 of Lecture Notes in Control and Information Sciences. Springer-Verlag, Berlin, Germany.

[4] Rogers, E., Galkowski, K., Owens, D. H. (2007). Control Systems Theory and Applications for Linear Repetitive Processes, ser. Lecture Notes in Control and Information Sciences. Springer Verlag,V. 349.

[5] Paszke, W., Rapisarda, P., Rogers, E., Steinbuch, M. (2008). Using dissipativity theory to formulate necessary and sufficient conditions for stability of linear repetitive processes, Research Report, School of Electronics and Computer Science, University of Southampton, UK.

[6] Hladowski, L., Zhonglun Cai., Galkowski, K., Rogers, E., Freeman, C. T., Lewin, P. L., Paszke, W. (2009). Repetitive Process Based Iterative Learning Control Designed by LMIs and Experimentally Verified on a Gantry Robot. Inst. of Control & Comput. Eng., Univ. of Zielona, Gora, Poland. American control conference, p. 949 – 954, hyatt regency riverfront.

[7] Iwasaki, T., Meinsma, G., ed al. (2000). Generalized S”procedure and finite frequency KYP lemma. *Mathematical Problems in Engineering*, 6, 305–320.

[8] Iwasaki, T., Hara, S. (2005). Generalized KYP lemma: unified frequency domain inequalities with design applications. *IEEE Transactions on Automatic Control*, 50 (1) 41–59.

[9] Tetsuya Iwasaki. (2005). Generalized KLY lemma: unified frequency domain inequalities with design applications. *IEEE Transactions On Automatic Control*, 50 (1), January.

[10] Wojciech Paszke, al. (2011). Finite Frequency Range Control Law Synthesis for Differential Linear Repetitive Processes. 2011 50th IEEE Conference on Decision and Control andEuropean Control Conference (CDC-ECC) Orlando, FL, USA, December p. 12-15.

[11] Dridi, J, Ben attia, S, Salhi, S., Ksouri, M. (2012). Repetitive Processes Based Iterative Learning Control designed by LMIs ISRN Applied Mathematics, Article ID 365927, p .18.

[12] Gahinet, Apkarian, P. (1994). A linear matrix inequality approach to h_∞ control, *Int. J.Robust Nonlin. Control*, 4 (4) 421–448.