Optimal Source - Power Splitting in Cooperative Relaying Communications

Jihyun Shin, Yunsung Choi, Dongwoo Kim
Hanyang University
ERICA
Republic Of Korea
{jhshin, yschoi}@wnl.hanyang.ac.kr, dkim@hanyang.ac.kr

ABSTRACT: *In this paper, a usage of splitting source - node power is proposed for a two-phase cooperative relaying system where the transmit powers of the source and the relay node are individually constrained. In the proposed usage, the limited source power is divided into two parts that are used in the first and the second phase, respectively. Unlike conventional relaying methods, the source again in the second phase transmits its signal with the split power and, at the same time, the relay forwards the signal received at the first phase, which causes intervention between the signals. In order to avoid the intervention, so-called a co-phasing weight for aligning the phases of the two signals is used at at the source before the second transmission. The forwarding operation at the relay however is exactly the same to the conventional ones. Optimal power-splitting as well as the co-phasing weight is provided in this paper. With numerical investigation, the proposed power-splitting is shown to significantly reduce the outage probability compared with the conventional individual power allocation.*

Keywords: Relay Systems, Co-phase, Power Allocation, Splitting Source, Signal Transmission, Cooperative Relay

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1. Introduction

AF and DF are relaying techniques widely accepted in cooperative communication systems that consist of a source S, a relay R and a destination D. The transmission protocol in both AF and DF systems is typically built on two consecutive phases: source-transmitting and relay-forwarding phases. Let us denote the transmit power at the source and at the relay by P_s and P_r, respectively. The end-to-end SNR received at the destination after the two phases is then given by a function of P_s and P_r [1] - [2]. If an aggregate power constraint for P_s and P_r is applicable, power allocation between P_s and P_r that achieves certain balance between the SNRs attainable at the respective phase is very plausible in terms of maximizing the end-to-end SNR [3] - [5].

However, the power allocation between P_s and P_r is not possible if an individual power constraint is applied. The relay is usually located in a remote site from the source and then the aggregate constraint that allows power-sharing between the source and the relay is not permitted. We propose, when an individual power constraint is imposed, splitting the source power between two phases to enhance the end-to-end SNR. More specifically, the source power is divided into two parts: $(1-\alpha)\, P_S$ and αP_S $(0 \leq \alpha \leq 1)$, and they are used in the first and the second phase, respectively. α is called a powersplitting factor in this letter and αP_S is used to reinforce the signal power received at the destination in the second phase. Intuitively, the greater α is expected when the greater the firstphase SNR is than the second-phase one.

Unlike the conventional relaying methods in [1]-[5], the source transmits its signal again with power αP_S in the second phase and, at the same time, the relay forwards the signal received at the first phase. In order to avoid intervention between the signals simultaneously received at the destination, a co-phasing weight w for aligning *S-D* channel to *R-D* channel is used at the source before the second transmission. The forwarding operation at the relay however is exactly the same to the conventional ones.

We provide an optimal pair of α and ω that maximizes the end-to-end SNR of the proposed power-splitting for AF and DF methods, respectively. With numerical investigation, the proposed power-splitting is shown to significantly reduce the outage probability of the cooperative relaying systems. Moreover, the outage-reduction is also interpreted as power saving in the relay node, which ranges from 3 to 5 dB.

Notations: Upper-case letters in bold-face indicate matrices and lower-case letters in bold-face indicate column vectors. The superscripts used in A^T, A^H and A^{-1} denote transpose, hermitian and inverse operations of matrix A, respectively. $\tilde{\alpha}$ denotes complex conjugate of a. $diag\,[\,X\,]_N$ stands for an N by N diagonal matrix with x on its main diagonal.

2. System Model and Problem Definition

We assume that all the nodes are equipped with a single antenna and a node cannot transmit and receive signals simultaneously. And we also assume that the nodes work in two equal-length phases of cooperative relaying and, as a relaying technique, we consider AF and DF, respectively. Complex channel coefficients between nodes are assumed mutually independent and denoted by h_{sd}, h_{sr} and h_{rd}. The channels are modeled as zero-mean circularly symmetric Gaussian random variables whose variances are σ^2_{sd}, σ^2_{sr} and σ^2_{rd}, respectively, and they are also assumed constant during the two cooperation phases. For simplicity, we also denote $\gamma_{sd} = |h_{sd}|^2$, $\gamma_{sr} = |h_{sr}|^2$ and $\gamma_{rd} = |h_{rd}|^2$.

2.1 Phase I

Let us recall that P_S and α are the source power and the power-splitting factor, respectively. The source transmits symbol x with power $(1-\alpha)P_S$ in the first phase and αP_S is used in the second phases. In the first phase, the received signals at the relay and the destination are then given by

$$y_{r,1} = \sqrt{(1-\alpha)P_S}\, h_{sr} x + n_{r,1} \tag{1}$$

$$y_{d,1} = \sqrt{(1-\alpha)P_S}\, h_{sd} x + n_{d,1} \tag{2}$$

where $n_{r,1}$ and $n_{d,1}$ denote additive white Gaussian noise (AWGN) at the relay and the destination in the first phase, respectively. We assume that all the AWGN terms including those appear in the following are mutually independent and have the same power N_o.

2.2 Phase II and Optimization Problem

In the second phase, the relay forwards the signal received in the first phase and the source transmits x again with the remaining power αP_S. The signal from the source is multiplied by a co-phasing weight coefficient w before transmission. The relay's operation depends on the relaying methods: either AF or DF, and accordingly the power allocation and weight calculation at the source is performed, which is described in the following. Our focus in this paper is to find an optimal pair of α and w that maximizes the end-to-end SNR achieved from the cooperation. We assume that the optimization problem is solved at the source that has the instantaneous channel information on h_{sd}, h_{sr} and h_{rd}. For clarity, let (α_a, w_a) and (α_d, w_d) denote the pair of decision variables for AF and DF systems, respectively.

2.2.1 Optimization Problem in AF Relaying

The AF relay in the second phase multiplies $y_{r,1}$ in (1) with an amplifying gain

$$G = \sqrt{\frac{P_r}{(1-\alpha_a)P_S \gamma_{sr} + N_o}}$$

to limit the power of the transmitted signal to P_r and forwards it to the destination. And the source transmits x multiplied by co-phasing weight w_a with power $\alpha_a P_S$. Then the received signal at the destination is

$$y_{d,2} = \sqrt{((1-\alpha_a)P_S} Gh_{sr} h_{rd} + \sqrt{(\alpha_a P_S} w_a h_{sd})x + Gh_{rd} n_{r,1} + n_{d,2}, \tag{3}$$

where $n_{d,2}$ denotes AWGN at the receiver. Finally, $y_{d,1}$ (in (2)) received in the first phase through direct *S-D* link and the above $y_{d,2}$ are combined with weights g_{a1} and g_{a2}, which gives combined signal $y_a = g_{a1}y_{d,1} + g_{a2}y_{d,2}$. Let $g_a = [g_{a1}, g_{a2}]^T$ be a combining weight vector. Optimal combining vector also will be provided shortly. After combined, the output signal-to-noise ratio (SNR) is

$$SNR_{AF} = P_S \frac{g_a^H V_a V_a^H g_a}{g_a^H U_a U_a^H g_a}, \tag{4}$$

Where

$$V_a = [\sqrt{(1-\alpha_a)}\, \tilde{h}_{sd}, h_{rd} + \sqrt{(1-\alpha_a)}\, G\tilde{h}_{sr}\, \tilde{h}_{rd} + \sqrt{\alpha}_a\, \tilde{w}_a\, \tilde{h}_{sd}]^T,$$

$$U_a = diag\,[\sqrt{N_o}, \sqrt{G^2\, \gamma_{rd} N_o + N_o}]_2 \tag{5}$$

The problem of interest in AF systems is now represented by

$$R_{AF} = \max_{\alpha_a w_a g_a} \frac{1}{2} log_2(1 + SNR_{AF}) \tag{6}$$

2.2.2 Optimization Problem in DF Relaying

If the DF relay successfully decodes symbol x from $y_{r,1}$ (in (1)) received in the first phase, the relay re-encodes and forwards it in the second phase. At the same time, the source also transmits x multiplied by weight coefficient w_d with power $\alpha_d P_s$. Then, the destination receives

$$y_{d,2} = \sqrt{P_r h_{rd}} + \sqrt{(\alpha_d P_S} w_a h_{sd})x + n_{d,2}, \tag{7}$$

$y_{d,1}$ and $y_{d,2}$ are combined with weights $g_{d1}$and g_{d2}, which results in combined signal $y_d = g_{d1}y_{d,1} + g_{d2}y_{d,2}$.

Let $g_d = [gd_1, gd_2]^T$ be a combining vector in DF system. The output SNR is then written by

$$SNR_{DF} = \frac{g_d^H V_d V_d^H g_d}{g_d^H U_d U_d^H g_d}, \tag{8}$$

Where

$$V_d = [\sqrt{(1-\tilde{\alpha}_d)}\, P_S h_{sd}, \sqrt{P_r}\, \tilde{h}_{rd} + \sqrt{\tilde{\alpha}_d P_S}\, \tilde{w}_d\, \tilde{h}_{sd}]^T, \tag{9}$$

$$U_d = diag\,[\sqrt{N_o}, \sqrt{N_o}]_2$$

If the DF relay cannot decode x from $y_{r,1}$, the relay keeps silent and only the source transmits x with power $\alpha_d P_S$ without the co-phasing weight. In this case, the received SNR becomes $\alpha_d\, \gamma_{sd} / N_o$ after MRC combining of the signals from the two phases [6]. Thus, the optimization problem is given by

$$R_{DF} = \max_{\alpha_d w_d g_d} \frac{1}{2} log_2 (1 + max \left[\left(\frac{P_s\, \gamma_{sd}}{N_o}, min\left(\frac{(1-\alpha_d)P_s\, \gamma_{sr}}{N_o}, SNR_{DF}\right)\right)\right]. \tag{10}$$

3. Optimal Power Splitting and Co - phasing Weights

3.1 AF Relaying

Let us first determine an optimal combining vector for the signals from two phases. The numerator and the denominator in (4) are decomposed by using the Cauchy-Schwarz inequality as follows [7]:

$$\underbrace{(V_a^H g_a^H)}_{\text{Numeration in (4)}} \leq \{(U_a^{-1} V_a)^H (U_a^{-1} V_a)\}\{\underbrace{(U_a^H g_a)^H (U_a^H g_a)^H}_{\text{Denominator in (4)}}\}. \tag{11}$$

From (11), an upper bound of SNR_{AF} is given by

$$\begin{aligned} SNR_{AF} &\leq P_s (U_a^{-1} V_a)^H (U_a^{-1} V_a) \\ &= \frac{(1-\alpha_a) P_s \gamma_{sd}}{N_o} + \frac{|\sqrt{(1-\alpha_a) P_s} G \tilde{h}_{sr} \tilde{h}_{rd} + \sqrt{\alpha_a P_s} \tilde{w}_a \tilde{h}_{sd}|^2}{G^2 \gamma_{rd} N_o + N_o} \\ &\triangleq SNR_{AF}^U \end{aligned} \tag{12}$$

The above upper bound is achieved by an optimal combining vector g_a^* that can be derived from the equality condition in (11) as follows.

$$\begin{aligned} g_a^* &= \beta (U_a U_a^H)^{-1} V_a \\ &= \beta \left[\frac{\sqrt{(1-\alpha_a)} \tilde{h}_{sd}}{N_o}, \frac{\sqrt{(1-\alpha_a)} G \tilde{h}_{sr} \tilde{h}_{rd} + \sqrt{\alpha_a P_s} \tilde{w}_a h_{sd}}{G^2 \gamma_{rd} N_o + N_o} \right]^T, \end{aligned} \tag{13}$$

where β is an arbitrary complex number but not equal to zero. Using (12) and (13), with combining vector g_a^* the optimization problem can be reduced into

$$\max_{\alpha_a \, w_a} SNR_{AF}^U \text{ subject to } 0 \leq \alpha_a \leq 1, |w_a|^2 = 1 \tag{14}$$

Since the weight coefficient has unity norm, scalar α_a does not affect on choosing the optimal weight. Thus, we first consider the problem in (14) assuming that α_a is given, which leads to

$$\max_{w_a} | \sqrt{(1-\alpha_a) P_S} G \tilde{h}_{sr} \tilde{h}_{rd} + \sqrt{\alpha_a P_s} w_a \tilde{h}_{sd} | \text{ Subject to } |w_a|^2 = 1 \tag{15}$$

In order to maximize the objective function in (15), the phases of the first and the second term should be the same. An optimal co-phasing weight is then given by $w_a = \dfrac{\tilde{h}_{sd} h_{sr} h_{rd}}{|\tilde{h}_{sd} h_{sr} h_{rd}|}$. Now, we have the optimization problem for power splitting factor:

$$\max_{\alpha_a} \frac{(1-\alpha_a) P_s \gamma_{sd}}{N_o} + \frac{(G\sqrt{(1-\alpha_a) P_s \gamma_{sr} \gamma_{rd}} + \sqrt{\alpha_a P_s \gamma_{sd}})^2}{G^2 \gamma_{rd} N_o + N_o} \text{ Subject to } 0 \leq \alpha_a \leq 1. \tag{16}$$

Since a closed-form solution of the above problem is hard to find, we use an ideal amplifying gain $G = \sqrt{\dfrac{P_r}{(1-\alpha_a) P_s \gamma_{sr}}}$, which ignores the noise amplification at the relay and hence is known to give an upper bound of the SNR [2]. The relaxed optimization problem is then given by

$$\max_{\alpha_a} \frac{(1-\alpha_a) P_s \gamma_{sd}}{N_o} + \frac{(\sqrt{P_r \gamma_{rd}} + \sqrt{\alpha_a P_s \gamma_{sd}})^2 (1-\alpha_a) P_s \gamma_{sr}}{P_r \gamma_{rd} N_o} \text{ Subject to } 0 \leq \alpha_a \leq 1. \tag{17}$$

To obtain optimal (now suboptimal due to the approximation with an ideal amplifying gain) α_a, let us differentiate the objective function in (15) with respect to α_a and find its roots. Let $x = \sqrt{a}$ and let $f(x)$ be the first derivative of the objective function. Then

$$f(x) = c_1 x^3 + c_2 x^2 + c_3 x + c_4, \tag{18}$$

Where

$$\begin{aligned} c_1 &= -2P_s \gamma_{sd} \gamma_{sr}, \\ c_2 &= -3\gamma_{sr} \sqrt{P_s P_r \gamma_{sd} \gamma_{rd}}, \\ c_3 &= -P_r \gamma_{rd} (\gamma_{sd} + \gamma_{sr}) - P_s \gamma_{sr} \gamma_{sd}, \\ c_4 &= \gamma_{sr} \sqrt{P_s P_r \gamma_{sd} \gamma_{rd}}. \end{aligned} \tag{19}$$

Since all the coefficients of $f(x)$ are real-valued, cubic equation $f(x) = 0$ has at least one real root that can be obtained by a closed-form formula [8], the presentation of which however is long and tedious and is omitted in this paper.

Proposition 1: There exists a real root x^* of $f(x) = 0$ between 0 and 1, and it is an optimal solution of the problem in (15).

Proof: Since $f(0) > 0$ and $f(1) < 0$, there exists a real root x^* such that $0 < x^* < 1$. And since $df(x)/dx \leq 0$ for $0 < x^* < 1$, the objective function in (15) is concave for $0 < x < 1$ and hence $\alpha_a^* = (x^*)^2$ is optimal for the problem.

3.2 DF Relaying

In the same manner as in AF case, we can find an optimal combining vector as follows:

$$\begin{aligned} g_d^* &= \beta (U_d U_d^H)^{-1} V_d \\ &= \beta \left[\frac{\sqrt{(1-\alpha_d) P_s}\, \tilde{h}_{sd}}{N_o}, \frac{\sqrt{P_r}\, \tilde{h}_{rd} + \sqrt{\alpha_d P_s}\, \tilde{w}_d \tilde{h}_{sd}}{N_o} \right]^T. \end{aligned} \tag{20}$$

And an upper bound of SNR_{DF} is given by

$$\begin{aligned} SNR_{DF} &\leq (U_d^{-1} V_d^U)^H (U_d^{-1} V_d) \\ &= \frac{(1-\alpha_d) P_s \gamma_{sd} + |\sqrt{P_r}\, \tilde{h}_{rd} + \sqrt{\alpha_d P_s}\, \tilde{w}_d \tilde{h}_{sd}|^2}{N_o} \\ &\triangleq SNR_{DF}^U \end{aligned} \tag{21}$$

Using (21), the optimization problem for power splitting factor and co-phasing weight is written by

$$\max_{\alpha_d, w_d} \min \left(\frac{(1-\alpha_d) P_s \gamma_{sr}}{N_o}, SNR_{DF}^U \right) \text{ Subject to } 0 \leq \alpha_d \leq 1,\ |w_d|^2 = 1 \tag{22}$$

When α_d is given, an optimal co-phasing weight is $w_d = \dfrac{\tilde{h}_{sd} h_{rd}}{|\tilde{h}_{sd} h_{rd}|}$. Now, we have the optimization problem for power spiliting factor as

$$\max_{\alpha_d} \min \left\{ \underbrace{\frac{(1-\alpha_d) P_s \gamma_{sr}}{N_o}}_{g_1(\alpha_d)}, \underbrace{\frac{(1-\alpha_d) P_s \gamma_{sd} + (\sqrt{\alpha_d P_s \gamma_{sd}} + \sqrt{P_r \gamma_{rd}})^2}{N_o}}_{g_2(\alpha_d)} \right\} \text{Subject to } 0 \leq \alpha_d \leq 1. \tag{23}$$

With respect to α_d, $g_1(\alpha_d)$ is monotonically decreasing but $g_2(\alpha_d)$ is monotonically increasing. If $g_1(0) \leq g_2(0)$, then $\alpha_d = 0$ is an optimal solution. If $g_1(0) > g_2(0)$, an optimal α_d can be found by solving $g_1(\alpha_d) = g_2(\alpha_d)$, which results in a quadratic equation with respect to $\sqrt{\alpha_d}$. Using a formula for finding roots of the quadratic equation [9], it is easy to see that the equation has two real roots: a positive and a negative root. If $g_1(0) > g_2(0)$, discarding the negative one, we then have

$$\sqrt{\alpha_d^{\dagger}} = \frac{-\sqrt{P_r \gamma_{sd} \gamma_{rd}} + \sqrt{P_r \gamma_{sd} \gamma_{rd} + \gamma_{sr} (P_s \gamma_{sr} - P_s \gamma_{sd} - \gamma_{rd})}}{\sqrt{P_s}\, \gamma_{sr}} \tag{24}$$

Proposition 2: If $g_1(0) > g_2(0)$, then $\alpha_d^{\dagger}$ in (21) is always $0 < \alpha_d < 1$.

Proof: Let $\delta_1 = P_r \gamma_{sd} \gamma_{rd}$, $\delta_2 = \gamma_{sr} (P_s \gamma_{sr} - P_s \gamma_{sd} - P_r \gamma_{rd})$ and $\delta_3 = P_s \gamma_{sr}^2$. Then, δ_1 and δ_3 are obviously positive and δ_2 is also positive if $g_1(0) > g_2(0)$. Moreover, $\delta_2 < \delta_3$. Now we have

$$0 < \alpha_d^{\dagger} = \left(\sqrt{\alpha_d^{\dagger}}\right)^2 = \left(\frac{\sqrt{\delta_1 + \delta_2} - \sqrt{\delta_1}}{\delta_3}\right)^2$$

$$= \frac{\delta_1 - 2\sqrt{\delta_1 - (\delta_1 + \delta_2)}}{\delta_3} < 1 \tag{25}$$

Proposition 2 means that $\alpha_d^{\dagger}$ is an optimal solution for the problem in (20) and power-splitting is always effective (i.e. between 0 and 1) if $g_1(0) > g_2(0)$. When $P_s \gamma_{sd} > P_s \gamma_{sr}$, the data rate between *S* and *D* is greater than the data rate between *S* and *R*, and hence the relay is useless [10] and only the direct communication is used with $\alpha_d = 0$. In summary, we have an optimal power-splitting factor $\alpha_d^{\dagger}$ and its corresponding data rate R_{DF}^{*} in DF relaying as the following.

Figure 1. An illustration of optimal / suboptimal power-splitting

$$\alpha_d^{*} = \begin{cases} 0 & if\, P_s \gamma_{sr} \leq P_s \gamma_{sd} + P_r \gamma_{rd} \\ \alpha_d^{\dagger} & if\, P_s \gamma_{sr} \leq P_s \gamma_{sd} + P_r \gamma_{rd} \end{cases} \tag{26}$$

$$R_{DF} = \begin{cases} \frac{1}{2} log_2 (1 + \frac{P_s \gamma_{sd}}{N_o}) & if\ P_s \gamma_{sd} > P_s \gamma_{sr} \\ \frac{1}{2} log_2 (1 + \frac{P_s \gamma_{sr}}{N_o}) & if\ P_s \gamma_{sd} \leq P_s \gamma_{sr} \leq P_s \gamma_{sd} \leq P_r \gamma_{rd} \\ \frac{1}{2} log_2 \left(1 + \frac{(1 - \alpha_d^{*}) P_s \gamma_{sr}}{N_o}\right) & if\, P_s \gamma_{sr} > P_s \gamma_{sd} P_r \gamma_{sd} \end{cases} \tag{27}$$

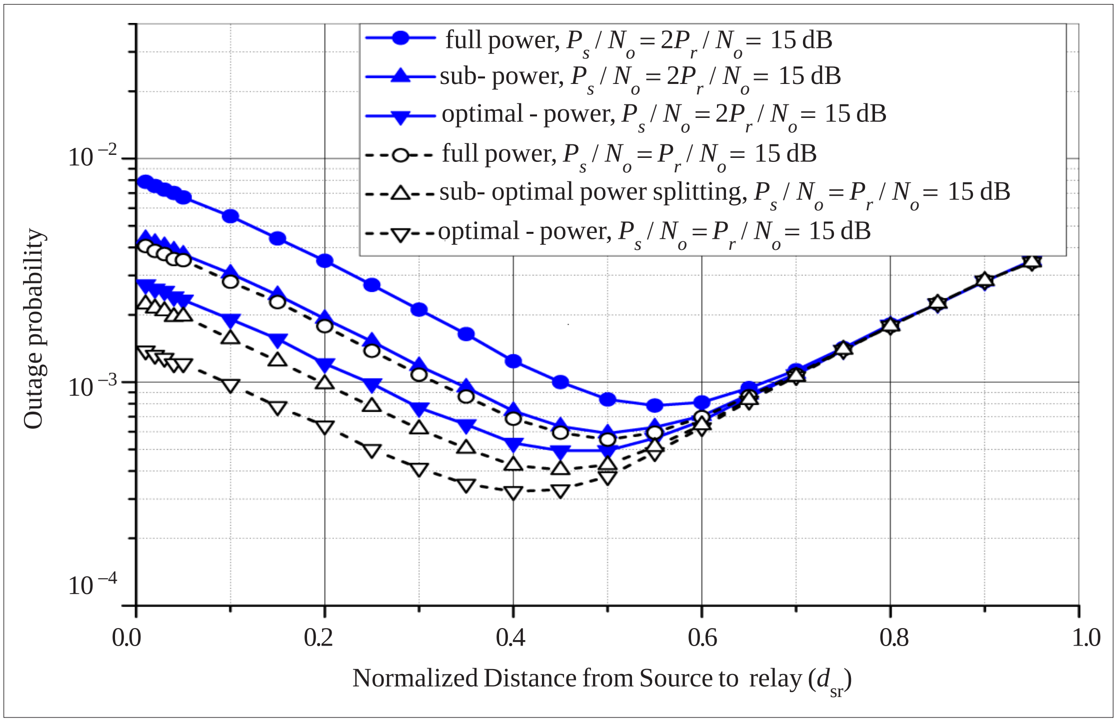

Figure 2. Outage probabilities of AF relaying for the distances between the source and the relay (d_{sr}): $R_{th} = 1$, $d_{sr} + d_{rd} = 1$ and $d_{sd} = 1$

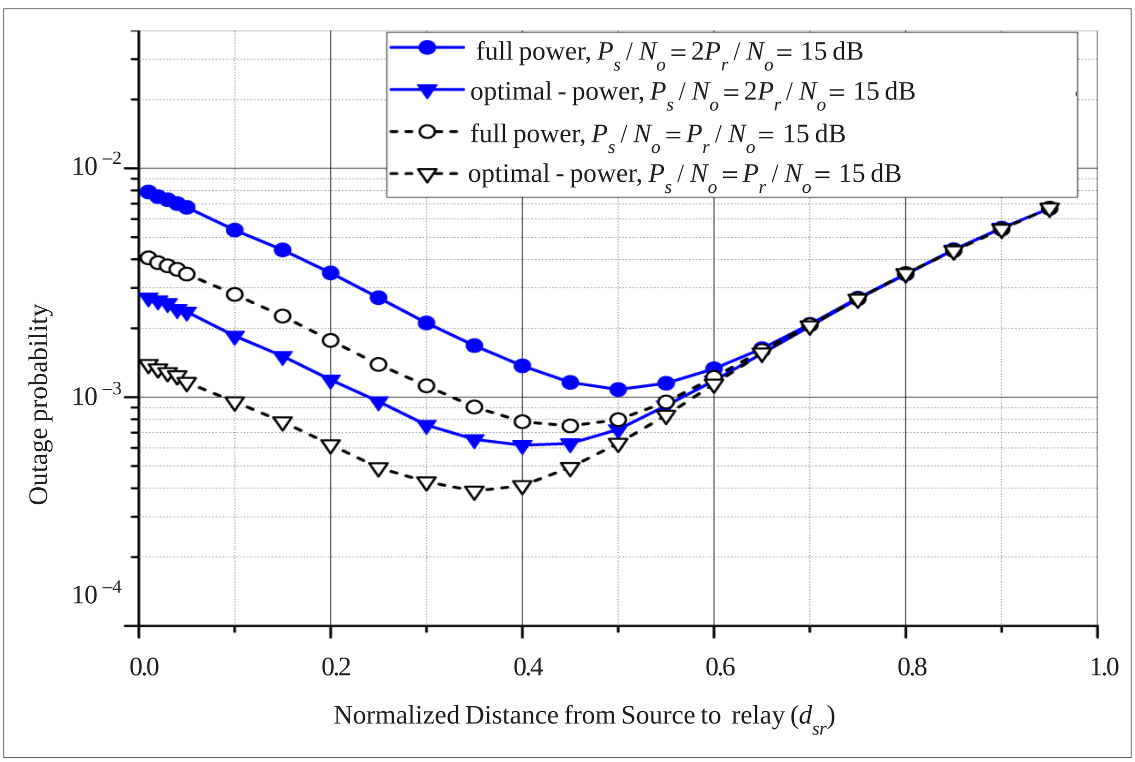

Figure 3. Outage probabilities of DF relaying for the distances between the source and the relay (d_{sr}): $R_{th} = 1$, $d_{sr} + d_{rd} = 1$ and $d_{sd} = 1$

4. Numerical Results

Figure 1. illustrates the optimality of α_a^* and α_d^* obtained in this paper. In the figure, assuming that instantaneous channel samples are given, the throughput is plotted as a function of power-splitting factor α for AF and DF systems, respectively.We assume that $\gamma_{sd} = 0.3$, $\gamma_{sr} = 2$, $\gamma_{rd} = 1$ and $\frac{P_s}{N_o} = \frac{P_r}{N_o} = 15$ db.

For AF system, the suboptimal α_a^* is 0.1298 but optimal $\alpha_a^*$is 0.2561 which is obtained by an exhaustive search. However, the resulting throughputs arevery close: 2.6203 for the suboptimal α_a^* and 2.6364 for the optimal α_a, which shows the suboptimal α_a^* obtained in this paper is a near-optimal power-splitting factor. For DF system, $\alpha_d^* = 0.1429$ provides exactly the maximum throughput 2.8933.

In Figure 2 and Figure 3, using Monte-Carlo methods, the outage performance of the proposed power-splitting is compared with conventional individual power allocation where P_s and P_r are fixed and fully used. In the figures, we assume that the outage occurs when achievable data rate $R_\delta < R_{th}$ ($\delta = \{\text{AF, DF}\}$) and $R_{th} = 1$ is assumed. We also assume that S, R, D nodes are on a straight line and the distance between S and D is normalized to one. The channel coefficients between the nodes are generated randomly from complex Gaussian distribution $CN \sim (0, 1/d^4)$, where d is a normalized distance between the two nodes and 4 is a path-loss exponent. In Figure 2, the outage probability of AF systems is shown as a function of the distance between S and R, denoted by d_{sr}. When $d_{sr} \leq 0.6$, the proposed sub-optimal power-splitting outperforms the individual full power allocation. The reduction in outage probability is about 43% when $d_{sr} = 0.3$ and about 23% when $d_{sr} = 0.5$. It is seen that the reduction is significant if $d_{sr} \leq 0{:}5$ but not negligible (more than 8%) even in $0.5 \leq d_{sr} \leq 0.6$. Comparing the two different power settings used in the figure: $\frac{P_s}{N_o} = \frac{P_r}{N_o} = 15$ db and $\frac{P_s}{N_o} = 2\frac{P_r}{N_o} = 15$ db, in which the relay of the latter case uses a half of the power compared to the former. The performance of sub-optimal power-splitting with the latter setting is very close to that of the full-power allocation, which means that the power gain of the power-splitting in this simulation is nearly 3 dB in terms of the relay power. If we can have the optimal power-setting, the power gain is greater than 3 dB as shown in the figure. In Figure 3. the outage probability of DF systems is shown as a function of d_{sr}. When $d_{sr} < 0.6$, the proposed optimal power-splitting outperforms the full power allocation. The reduction in outage probability is about 62% when $d_{sr} = 0.3$ and about 32% when $d_{sr} = 0.5$. It is also seen that the power gain of the power-splitting in this simulation is greater than 3 dB in terms of the relay power.

To further investigate the power-saving in the relaying power, which can be achieved by using the power-splitting, we set $\frac{P_s}{N_o} = 15$ dB, dB as fixed and reduce $\frac{P_r}{N_o}$ from15 dB to 5 dB. The results are given in Figure 4. In the figure, both AF and DF outages are plotted by assuming $d_{sr} = 0.5$. In AF systems, suboptimal power-splitting with $\frac{P_r}{N_o} = 12$ dB provides the same outage probability 6×10^{-4} of using full power with Hence the power-splitting saves 3- dB relaying power. If optimal power-splitting is assumed, the saving goes to 5 dB. For the tested range of relaying powers, the savings look almost constant. In DF systems, the powersaving is almost 5 dB. For example, if outage probability 10^{-3} is required, power-splitting needs $\frac{P_r}{N_o} = 8$ dB while the full power allocation needs 13 dB.

5. Conclusions

We have proposed splitting the source power constrained individually and using them in both of the phases in cooperative relaying transmission. We have shown that it significantly improves outage performance compared with the fixed full power transmission. The improvement is achieved by an optimal power splitting factor and a co-phasing weight that are provided in this paper. With numerical investigation, the outage improvement is also translated as relay-power saving that ranges from 3 dB to 5 dB. The proposed power-splitting would be regarded as a promising usage of transmit power in cooperative communications.

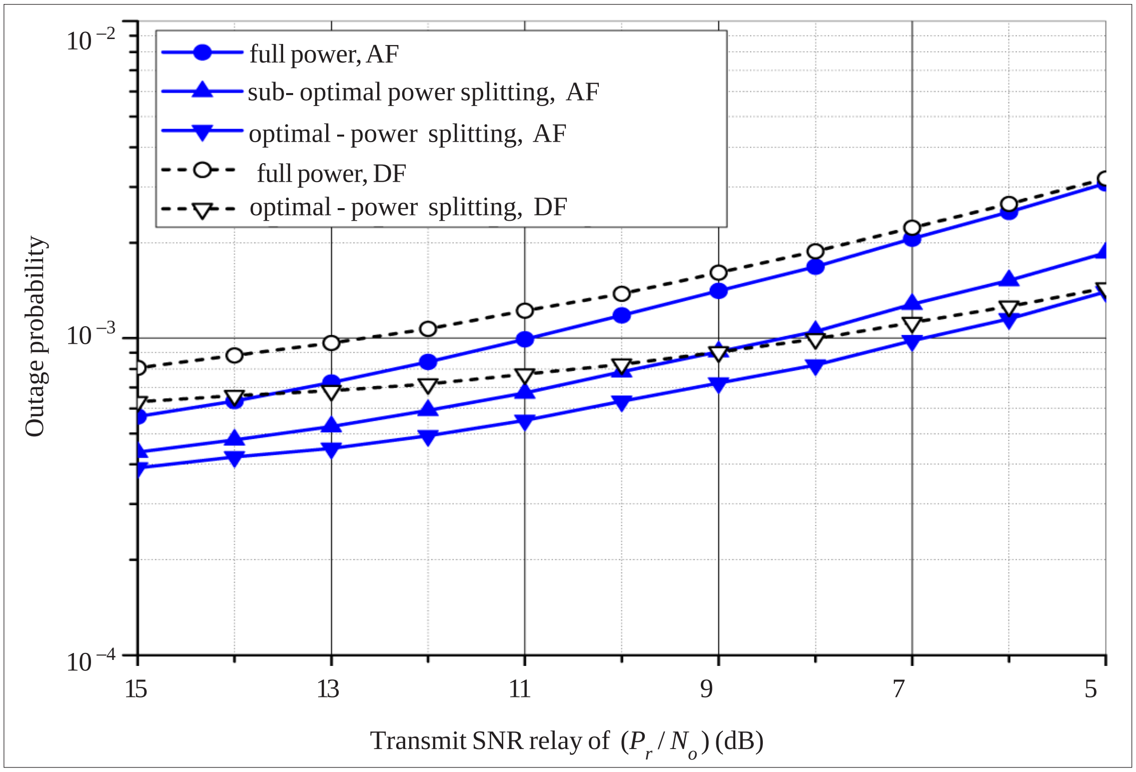

Figure 4. Comparison of outage probabilities by reducing relaying power : $R_{th} = 1$, P_s / N_o = 15 dB, $d_{sr} = d_{rd} = 0{:}5$ and $d_{sr} = 1$

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