Pulse Wave Velocity in Arteries using Centre Line Velocity and Radius Effect of Terminal Impedance and Measurement Errors

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ABSTRACT: *Pulse wave velocity (PWV) is a measure of arterial stiffness. A strong correlation between PWV and cardiovascular events that cause mortality, was recognized. The foot-to-foot (f-t-f) method is a standard techniques used for the determination of arterial PWV. This technique offers an estimation of the regional wave speed. F-t-f methods usually require longer vessels to allow the measurements at two sites. In this work, we present a mathematical description of a new method for the determination of the local PWV; Diameter Velocity loop* ***(Dv_{cl} - loop)****. The method only requires the measurements of centre line velocity and diameter which can be obtained non-invasively. We conclude that the* ***Dv_{cl} - loop*** *method provides a reliable local pulse wave velocity and has the merits of using non-invasive parameters.*

Keywords: PWV, Biomechanics, Arterial Properties, Non-invasive Measurements

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1. Introduction

The pressure and flow waveforms depend on the mechanical properties of the vessel walls; usually expressed in terms of stiffness and distensibility. Hypertension, for example, is associated with an increase in vascular stiffness leading to structural alterations of the vessel wall [1]. Therefore, a measurement of the PWV may help in the diagnosis of cardiovascular pathology of reduced distensibility and stiffness [2]. In order to estimate the local PWV, the *f-t-f* method was applied using a small distance between the measurement sites [3-7]. PWV, by definition, is the distance traveled (Δx) by the wave divided by the time (Δt) for the wave to travel that distance. This holds true for a system with zero wave reflections. The transmission of the arterial pressure pulse does not give the true PWV as it is a sum of vectors of the incident and reflected waves. Therefore, appropriate pressure and flow measurements must be made to estimate the characteristic impedance and to calculate the incident, or the reflected pressure wave at two sites. The advantage of *f-t-f* PWV measurement is the simplicity of measurement, requiring only two pressure wave forms recorded with invasive catheters, or mechanical tonometers or pulse detection devices applied non-invasively. However, this technique is limited by the minimum distance between probes for higher temporal resolution.

The Waterhammer equation gives another alternate expression of PWV [22]. The equation directly relates characteristic impedance to PWV through the ratio of pressure and linear flow velocity in the absence of wave reflection ($dP = \rho c dU$). Subsequently, an estimate of characteristic impedance through pressure and flow measurement provides a measure of PWV, which is proportional to arterial stiffness. In order to avoid these difficulties, several approaches were developed to estimate local PWV. Young [8] is the first author who derived an equation of the local PWV using an argument based on intuition and analogy to Newton's theory of the speed of sound in air. In 1878, Moens [9] and Kortweg [10] independently published an analysis of flow in thin-walled elastic vessels, deriving what is now known as the Moens-Kortweg equation for local PWV $c = \sqrt{\frac{Eh}{\rho D}}$ where E is the Young elastic modulus, h is wall thickness, ρ is fluid density and D is the local diameter. Otherwise the pulse wave velocity can be derived from propagation coefficient [15-18].

The aim of this work is to study the effect of terminal impedance and measurements errors in the determination of pulse wave velocity using Dv_{cl} - *loop* (centerline velocity and radius). The formulas used in this work are deduced by the authors from previous works [20, 21]. We also aim to compare the results of these methods with those obtained using the f-t-f method using computational data.

2. Theoretical background

Numerical pulse waveforms were simulated using the Womersley theory [12]. We assume a laminar pulsatile blood flow through a uniform and impermeable vessel of instantaneous diameter D (x, t) and length of L, where x is the distance along the vessel and t is time. The arterial wall is assumed to be thin, homogeneous, incompressible and elastic, with each cross section deforming axisymmetrically.We also assume that blood is Newtonian and incompressible and the effect of gravity is negligible. The velocity of the fluid enclosed within the cylindrical coordinates is denoted by $V = [u\ (r, x, t),\ v\ (r, x, t)]$, where u *is* the radial velocity and v the axial velocity of blood. We also assume that the wall of the vessel only undergoes radial motions and that parietal deformations are small as compared to D, which is small as compared to the wavelength. We further assume the fluid velocity is small as compared to wave speed $v << c$. For a periodic flow, each hemodynamic parameter can be expressed as

$$\varnothing\,(x, r, t) = \sum_{-\infty}^{+\infty} \Phi(x, r, \omega)\, e^{-i\omega t}$$

Governing equations and boundary conditions are linear. We suppose that during the rise of the systolic peak before the arrival of reflected waves we can consider unidirectional waves propagation. For rapid variations of flow, one supposes that the propagation wave velocity is constant equal to c (limit of dispersion equation for high frequency) and that wave attenuation effects are negligible. Therefore we can deduce that during the systolic rise:

$$\frac{\partial p}{\partial x} = \frac{1}{c}\frac{\partial p}{\partial t} \qquad (1)$$

Therefore, we suppose that the boarding time of the peak is short in front of $d^{\,2}/\eta$ where d denotes the non stationary boundary layer thickness and η denotes the kinematic viscosity. In other word, we suppose that during the systolic rise the non stationary profiles are almost flat at the centre, since the non stationary viscous effects are concentrated on a zone close to the wall. This implies that the first and second derivative of the centre line velocity (v_{cl}) are null (the viscous effects are thus negligible in the centre for the non stationary part and during the rise of the systolic peak).

From this assumption, and for linear case, the component according to 'x' coordinate of the movement equations can be written

$$\frac{\partial v_{cl}}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial x} = -\frac{1}{\rho c}\frac{\partial p}{\partial t} \qquad (2)$$

For linear elastic behaviour of the wall

$$C^{\,2} = \frac{\mathrm{A}}{\rho}\frac{\partial p}{\partial A}$$

We can write (A is the area of the tube section)

$$\frac{\partial v_{cl}}{\partial t} = \frac{1}{\rho c}\frac{\rho c^2}{A}\frac{\partial A}{\partial t} \qquad (3)$$

We therefore have the following expression of the PWV (D is the tube diameter)

$$C = \frac{D}{2}\frac{\partial v_{cl}/\partial t}{\partial D/\partial t} \qquad (4)$$

From expression (4), it is apparent that the local pulse wave velocity can be determined from the slop of the initial linear portion of $v_{cl}(D)$ by fitting a straight line to the linear part of loop corresponding to early systole assuming the presence of only unidirectional waves.

3. Materials and Methods

We consider a uniform tube, filled with viscous fluid, of finite length of 'L' with characteristic impedance Z_0 and terminating in load impedance Z_T, the reflection coefficient is given as K. A single pulse of fundamental frequency F_0 = 1 Hz, is generated at the inlet of the tube. The number of harmonics constituting the signal was equal to 10 harmonics. To respect the physiological conditions we have chosen the amplitude of velocity < 0.8 m/s and the amplitude of diameter displacements < 0.4×10^{-3} m. We assumed the blood density $\rho = 1060$ kg/m^3; viscosity $\eta = 5 \times 10^{-6}$ m^2 s^{-1}; output location are x = 10, 20, 30, 40 cm; the wall thickness $h = 0.15 \times 10^{-3}$ m; diameter $D = 3 \times 10^{-3}$ m and length L = 0.5 m.

All simulations were carried out using MATLAB (The Mathworks, 6.5). The numerical computation can be summarised as follows:

i. Specify input signal and output location 'x'.

ii. Load arterial tree data as described above.

iii. Compute the velocity and radius signals at location of coordinate 'x' using Womersley theory [30].

iv. Plot the centre line flow velocity v_{cl} as a function of the radius $D(x, t)$ and derive PWV using equations (4).

4. Results and discussion

Numerical simulations were performed to study the effect of varying physiological and geometrical parameters on the computation of PWV. In the subsequent text the input values are the parameters values used to obtain diameter and flow data.

In a first stage; we studied the influence of a decrease of the arterial compliance (change in the mechanical properties of the wall) by simulating an increase of reflection coefficient K (K = 0.16, 0.26, 0.36, 0.46, 0.56, 0.66 and 0.76). Second, the computer program was used to study the applicability of the method to various vessel length by simulating several vessel length (L= 32, 42, 52, 62, 72, 82, and 92 cm). Then, we studied the effect time lag or spatial misalignment of the measurement equipment on the results of the Dv_{cl} – *loop* method. We assumed simultaneous and exact measurements of the diameter and the velocity at location of coordinate x. Using graphical method the slope of the linear part of the loop was determined by fitting a straight line to the appropriate portion of the loop by eye using a mouse driven graphical interface on MATLAB.

4.1 The effect of increasing reflections

Figure 1 show results when we changed the terminal impedance of the tube by simulating a range of reflection coefficients K (K = ±0.16, ±0.26, ±0.36, ±0.46, ±0.56, ±0.66 and ±0.76). PWV as computed by f-t-f and Dv_{cl} – *loop* for seven reflections coefficients values are given in Table 1.

For low reflection coefficient, the PWV computed by both methods is similar to the input value (c = 6.48 m/s). However, at high reflection coefficient, the wave speed computed by the maintains a good agreement with the input value, while the wave speed obtained by the foot-to-foot becomes higher than the input values.

K (m/s)	0.16	0.26	0.36	0.46	0.56	0.66	0.76
c_{ff} (m/s)	6.64	6.82	7.12	7.36	7.71	7.95	8.41
cDv_{cl} (m/s)	6.92	6.85	6.69	6.55	6.48	6.35	6.34

Table 1. PWV as computed by Dv_{cl} – *loop* and foot-to-foot methods for several reflection coefficients

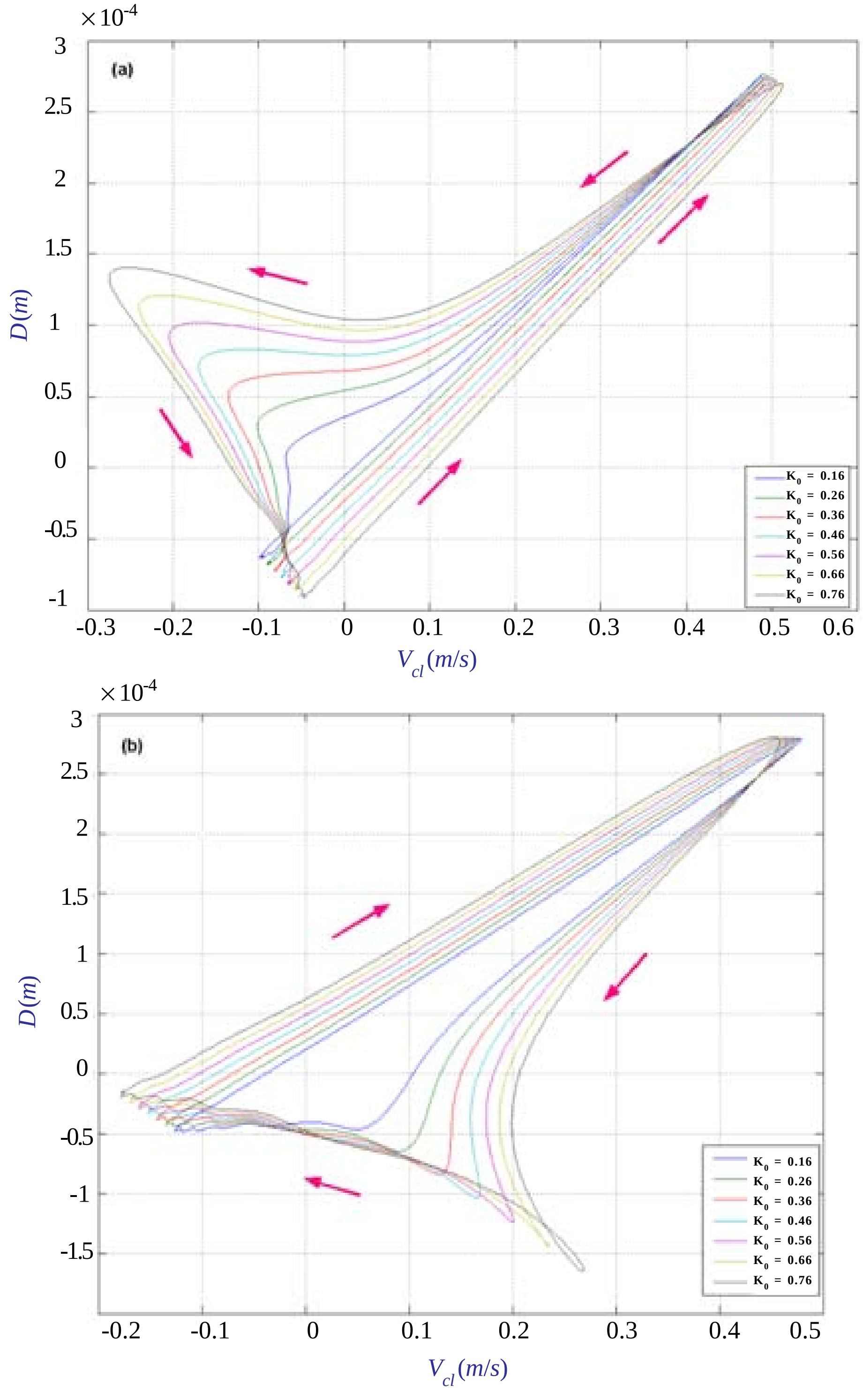

Figure 1. Results corresponding to seven reflection coefficient simulating an increase of peripheral impedance. (a): Positive reflection coefficient and (b): Negative reflection coefficient

This can be explained by the influence of the wave reflection that affects the foot of the flow velocity signals used to compute the wave speed. In fact, the foot-to-foot method is simple but has several potential sources of inaccuracy. Exact identification of the foot of pressure, or flow velocity, or diameter waveform is difficult. In order to avoid this difficulty; Franc [13] used 1/5 of the ascending slope of the systolic pressure contour as reference points. More recently Fitch used a point on the descending limb of the arterial pressure when the pressure is 1mmHg higher than diastolic pressure [14].

Another error source related to the application of the foot-to-foot method is that evaluation in vivo of the distance between measurement sites, which vary between patients, and cannot be obtained precisely. Moreover this method determines only an average PWV over the distance between the two measurement sites, which can be quite different from the local wave velocity of clinical interest.

4.2 The applicability of the $Dv_{cl} - loop$ method to various vessel lengths:

Figure 2 shows the results obtained for seven vessel lengths (L = 32, 42, 52, 62, 72, 82, and 92 cm). The values of PWV obtained agree with the input one (c = 6.48 m/s) for the seven lengths studied as shown in Table 2. However, it should be noted that the accuracy of the $Dv_{cl} - loop$ depends on the vessel length. The linear portion of the loop becomes ambiguous for small lengths. This can be explained by the early arrival of the reflected wave from the terminal site which depend on the position of the reflection site and wave speed.

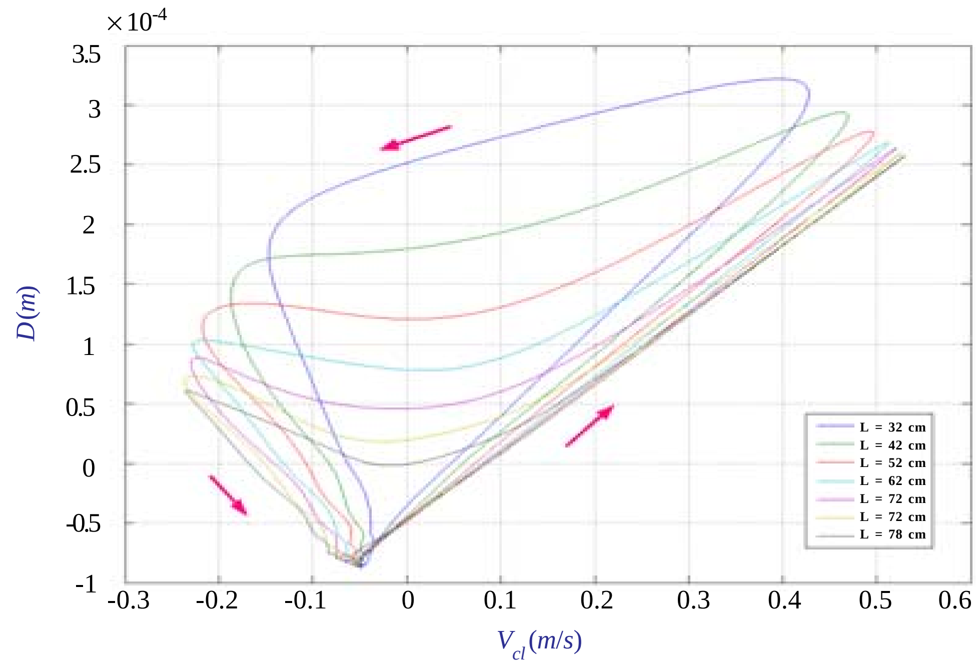

Figure 2. DU-loop results corresponding to seven vessels length

***L* (*cm*)**	92	82	72	62	52	42	32
c_{ff} (*m/s*)	6.48	6.55	6.71	7.29	7.91	8.64	12.15
cDv_{cl} (*m/s*)	6.93	6.91	6.76	6.55	6.33	5.92	5.30

Table 2. PWV as computed by $Dv_{cl} - loop$ and foot-to-foot methods for several lengths

4.3 The effect of time lag and spatial misalignment of the measurement equipment:

The influence of measurement errors and positioning measurement probes on the calculation of hemodynamic parameter have been discussed elsewhere [15, 16]. Hence, numerical simulations were used to analyse the influence of the time lag in recording data on the accuracy of the $Dv_{cl} - loop$ method in determining PWV. This time lag can be due to the different frequency response of the different measuring devices or if there is a spatial difference between measurement equipment.

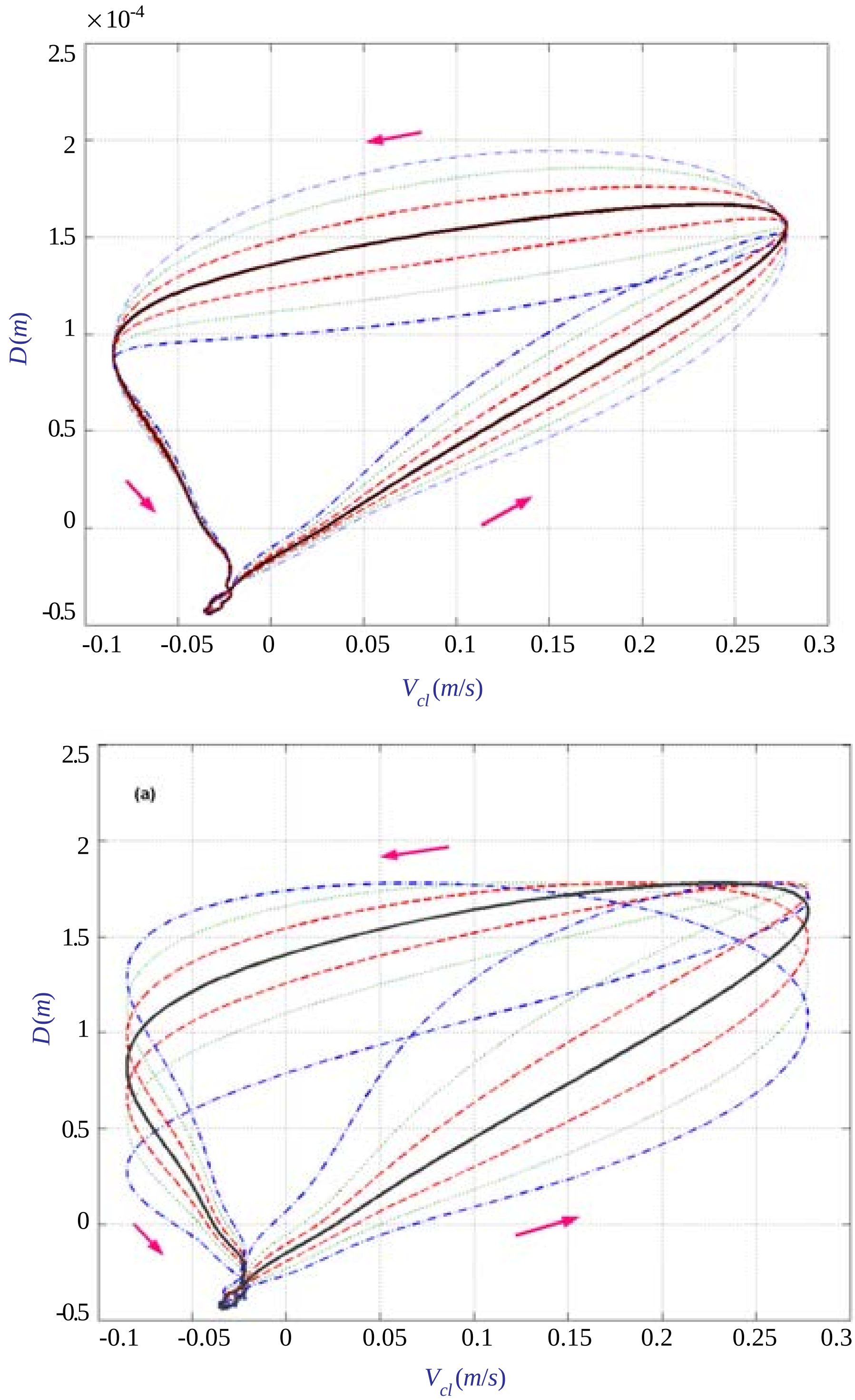

Figure 3. Top panel: $Dv_{cl} - loop$ for different distances between measurements sites of flow velocity and diameter. The centre line curve corresponds to measurement at the same position. The two three curves above and below the optimal loop when the measurement position of flow velocity is behind and in front the measurement position of diameter by 5cm (---), 10 cm (…) and 15 cm (-.-). Bottom panel: for different time lags between flow and diameter. The centre line curve corresponds to no time lag between flow velocity and diameter. The two three curves above and below the optimal loop correspond to diameter lagging and leading flow velocity by 10ms (---), 20ms (…) and 30ms (-.-)

The influence of the distance between positioning probes has been also studied. Figure 5a shows the results for three distances between recording position of the flow velocity and the radius. The central loop corresponds to data recorded in same location. The concave curves correspond to a distance respectively 5, 10 and 15cm between the measurements sites of flow velocity and the diameter where the flow velocity position is behind that of the diameter position.

The convex curves correspond to a distance of respectively 5, 10 and 15cm between measurement sites when the flow velocity measurement position is ahead of that of the diameter position. Figure 5b shows the effect of a time lag on the $Dv_{cl} - loop$ method. The central loop corresponds to the diameter and velocity data are recorded simultaneously. The concave curves correspond to the cases where the flow velocity lags the radius by 10, 20 and 30ms, and the convex curves correspond to the case where the radius lags the flow velocity by 20, 30 and 40ms. It is clearly shown that the shape of the method is significantly affected by the time lag between diameter and flow velocity. As shown in Figure 5a, the is also affected when a small distance between the measurement probes exists, although to a lesser degree than the effect of time lag as shown in Figure 5b. This finding maybe of a significant for using this method importance because it is difficult to make both velocity and diameter probes aligned at the same position, whether it is in vitro or in vivo application.

5. Conclusion

In the present work, a non invasive method for determination of the waves speed is described and evaluated. It requires only the measurement of centre line velocity and diameter in one site. This study based on numerical investigation of this method show a good agreement between computed values and theoretical values. The wave speed determined by the is as accurate as that obtained by the conventional foot-to-foot method. We believe that this method is well founded theoretically and easy to be implemented in vivo and in vitro. So, it can be automated and applied to non invasive measurements obtained by ultrasound and/or MRI. Moreover, this method has the advantage that provides a local PWV useful in assessing changes of local mechanical properties of the arterial wall. Finally it should be noted that this method is limited by the effect of wave reflection so it cannot be used when the reflection is very high and in peripheral artery. It remains to explore the application of this method for accurate assessment of local pulse wave velocity in vivo.

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