Statistical Modeling of Novel Coherent Algorithms of Reception of Signals QAM for Wireless and Satellite Communications Systems

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ABSTRACT: *This paper presents the results of a study of Quadrature Amplitude Modulation (QAM) coherent reception bit by bit. The research was conducted using statistical modeling methods; the results are compared with the statistical modeling calculation of analytical expression given in previously published documents. In the last year, the discrete signals became widely employed; and with more perspective in the digital systems for transmission of information, the QAM signals became more beneficial due to its high spectral efficiency. The satellite communications systems are evolving very quickly, so the purpose of developing this kind of research is to maximize the efficiency in the communication rate with the minimal error, during the limited communications time while satellite transits the user´s horizon.*

Keywords: Statistical, Coherent, Discrete signal, Satellite, Communications, Quadrature Amplitude Modulation

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1. Introduction

Actually, the interest for using wireless and satellites communications systems for data reception and transmission is rapidly increasing. In the present work, the research was carried out for the signals 2QAM (PSK), 4QAM (QPSK) and 16QAM [1]. Next, the methodology and results of the statistical modeling for signal 16QAM are presented. Quadrature amplitude modulation is widely employed due to its high spectral efficiency and is a standard for coherent modulation technique. As data rates increase, and operating with Additive White Gaussian Noise (AWGN), minimum of channel bandwidth, etc., more sources become sources of error appear in the system. Recent satellite systems give promise of spectral efficiencies of more than 2 bits/s/Hz. During the study it was corroborated that non-coherent modulators have simpler structure, as they do not require a coherent carrier recovery circuit, because their performance is not as good as that of coherent modems; for example, for a given value of probability of error (p_e), noncoherent modems require a higher carrier to noise ratio than coherent modems. QAM is an excellent technique to achieve high rate transmission without increasing the bandwidth [2]. A great deal of recent attention has been devoted to the study of *bit error rate* (BER) performance of QAM, and approximate expressions for the bit error probability of QAM have been developed in many technical articles around the world.

This modulation scheme e.g. is already being used in the framework for channel modeling in the Intelsat SCPC Systems and currently this technique, already used in satellite mission is being considered for applications in small microsatellite mission such as Condor and Quetzal [3],[4]. Even, 64QAM and 256QAM systems found wide applications in terrestrial microwave systems.

The coherent reception is being considered because noncoherent demodulators have a simpler structure as they do not require a coherent carrier recovery circuit and their performance is not as good as that of coherent modems; for a specific BER, noncoherent modems require a higher carrier to noise ratio C/N than coherent modems [5].

The objective of this paper is to present the Analytical method to calculate the BER of QAM coherent signals reception bit by bit. The technique is based on the obtained expressions from investigation to evaluate the error probability to distinguish two signals having a cross-correlation coefficient equal to one and different power in the interval of observation.

Expressions are obtained for assessing the quality of the reception of individual bits (code point) data message, and for the average bit-error probability of symbol reception.

Research was carried out by methods of statistical modeling on the PC in the software environment MATLAB® 7.0.1. The results were compared with the calculations of statistical modeling of the analytical expression given in previously published papers.

2. Methodology Description

2.1 QAM Modulation Technique

Signals Quadrature Amplitude Modulation may be represented by the following general Equation:

$$s\,(t) = d\,\{A_c\, cos\,[\omega_0 t + \varphi_0] + A_s \sin\,[\omega_0 t + \varphi_0]\} \tag{1}$$

Where: d — Energy base of signal $s(t)$; $cos\,[\omega_0 t + \varphi_0]$, $\sin\,[\omega_0 t + \varphi_0]$— Quadrature signal components of $s(t)$; ω_0, ω_0— Nominal frequency and phase of the signal. Information parameter QAM signal is amplitude, cosine and sine quadrature components which, $A_c \equiv A_c\,(\boldsymbol{\alpha})$; $A_s \equiv A_s\,(\boldsymbol{\beta})$ depending on the vectors of information and discrete parameters $\boldsymbol{\alpha} = [\alpha_1, \alpha_2, \alpha_3 ...]$, $\boldsymbol{\beta} = [\beta_1, \beta_2, \beta_3 ...]$.

Discrete parameters $[\alpha_1, \alpha_2, \alpha_3 ...]$ and $[\beta_1, \beta_2, \beta_3 ...]$ are independent random variables with two possible values [+1; –1]. Equation (1), describes the entire spectrum of the signals related to the class of signals QAM, including the special case of a simple binary phase shift keying signals (PSK"phase shift keying) for which $[A_c = \alpha_1 = \pm 1;\ A_s = \beta_1 = \pm 1]$and the signal twice PSK (QPSK – quadrature phase shift keying), for which $[A_c = \alpha_1 = \pm 1;\ A_s = \beta_1 = \pm 1]$.

Based on the general equation (1):

4QAM (QPSK)

$$s(t) = d\,\{\alpha_1\, cos\,[\omega_0 t + \varphi_0] + \beta_1 \sin\,[\omega_0 t + \varphi_0]\} \tag{2}$$

16QAM

$$s(t) = d\,\{(2\alpha_1 + \alpha_2)\, cos\,[\omega_0 t + \varphi_0] + (2\beta_1 + \beta_2) \sin\,[\omega_0 t + \varphi_0]\} \tag{3}$$

64QAM

$$s(t) = d\,\{(4\alpha_1 + 2\alpha_2 + \alpha_{.3})\, cos\,[\omega_0 t + \varphi_0] + (4\beta_1 + \beta_2 + \beta_3) \sin\,[\omega_0 t + \varphi_0]\} \tag{4}$$

From (2) to (4) it follows that the signal 4QAM (QPSK) in the interval duration $[0, T]$ carries 2 bits of information (α_1, β_1), 16QAM carriers 4 bits ($\alpha_1, \alpha_2, \beta_1, \beta_2$) and the signal 64QAM carriers 6 bits of information ($\alpha_1, \alpha_2, \alpha_3, \beta_1, \beta_2, \beta_3$).

QAM signals can also be represented as a vector diagram in a cartesian coordinate system on the quadrature plane [I, Q], where the coordinates of I and Q corresponds to the amplitude of the **cosine** and **sine** components of the signal (1).

Figure 1, illustrates a 16QAM signal constellation that is obtained by amplitude modulating each quadrature carrier by M = 4QAM.

In general, rectangular signal constellations result when two quadrature carriers are each modulated by PAM. Actually, there is a lot of functional bloke diagram of modulators for QAM.

More generally, QAM may be viewed as a form combined digital-amplitude and digital-phase modulation [6].

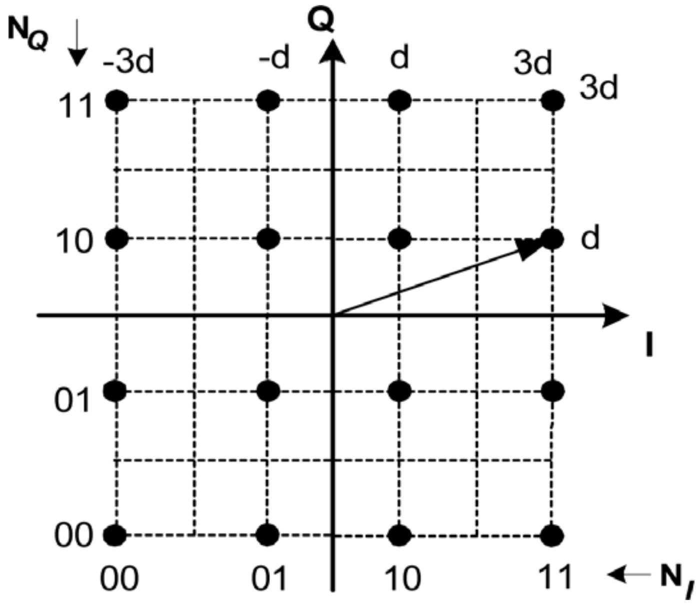

Figure 1. Rectangular signal-space constellation for 16QAM

For the expressions (2), (3), (4) the value of “1” corresponds in code combinations to the value “+1” in components of the discrete parameters, and the “0” to the values of discrete parameters equal “–1”.

For further research about immunity QAM reception, in the literature was selected in the literature optimal coherent algorithms bitwise reception. In Figure 2, as an example, it shows the corresponding optimal algorithm structure signal demodulator 16QAM.

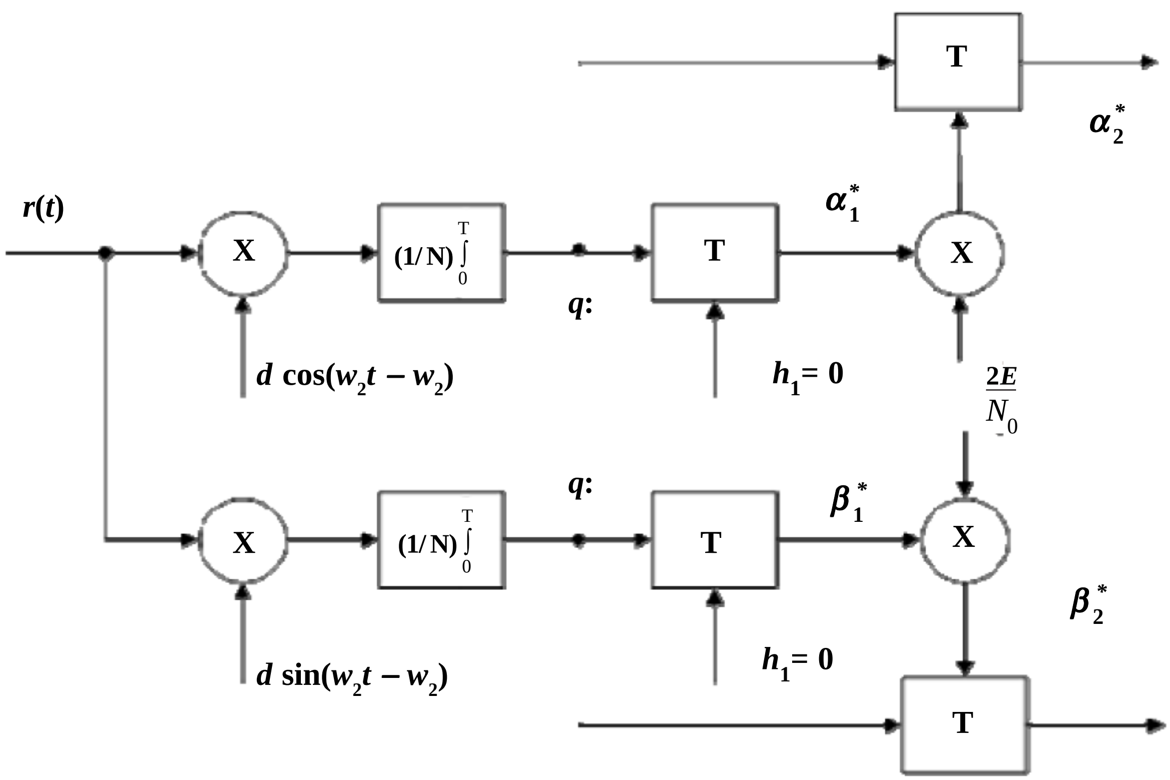

Figure 2. Structure 16QAM signal demodulator

2.2 Probability of error (BER) for PSK and QPSK

Expression for the error probability BER of symbol reception PSK and QPSK structurally are identical and differ only by energy:

$$BER_{PSK}(\alpha) = 1 - \Phi\left(\sqrt{\frac{2E}{N_0}}\right) = \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{E}{N_0}}\right) \tag{5}$$

$$BER_{QPSK}(\alpha) = BER_{QPSK}(\beta) = 1 - \Phi\left(\sqrt{\frac{E}{N_0}}\right) = \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{E}{2N_0}}\right) \tag{6}$$

We have the following equation:

$$r(t) = s_I(t; \boldsymbol{\lambda}, \boldsymbol{\alpha}) + n(t) \tag{7}$$

$$= d\,A_C(\alpha)\cos(\omega_0 t + \varphi_0) + n(t)$$

$$r(t) = s_Q(t; \boldsymbol{\lambda}, \boldsymbol{\beta}) + n(t) \tag{8}$$

$$= d\,A_S(\beta)\sin(\omega_0 t + \varphi_0) + n(t)$$

Where:

E – Energy of signals PSK and QPSK; $E/2$ – Energy of the quadrants of the signal (7) and (8). N_o – Noise power spectral density (noise in 1-Hz bandwidth). Note that quadrature components) $s_I(t; \boldsymbol{\lambda}, \boldsymbol{\alpha})$ and $s_Q(t; \boldsymbol{\lambda}, \boldsymbol{\beta})$ of the signal in (1) are not correlated with each other.

We know that:

$$\mathrm{erfc}(z) = 1 - \frac{2}{\sqrt{\pi}} \int_z^{\infty} \exp(-x^2)\,dx \tag{9}$$

and

$$\Phi(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} \exp\left(-\frac{x^2}{2}\right) dx \tag{10}$$

Where

erfc(z)– Error function; $\Phi(z)$– integral of probability. Consequently, the problem of signal reception (1) can be transformed into the problem of separate coherent reception uncorrelated with each quadrature signal components QAM.i.e. two unrelated problems receiving signals:

$$s_I(t; \boldsymbol{\lambda}, \boldsymbol{\alpha}) = d\,A_C(\alpha)\cos(\omega_0 t + \varphi_0) \tag{11}$$

and

$$s_Q(t; \boldsymbol{\lambda}, \boldsymbol{\beta}) = d\,A_S(\alpha)\cos(\omega_0 t + \varphi_0) \tag{12}$$

Equations (11) and (12) can be classified as signals pulse amplitude modulation (PAM) signals, differing only in the nature of the quadrature of the carrier signal. Obviously, the algorithms of optimal coherent reception of the signal (11) and (12) are identical, which makes it possible to calculate the BER for any of the quadrature signal QAM. The expressionfor the error probability (BER) of symbol reception PSK and QPSK, structurally, are identical and differ only by the energy.

2.3 Probability of error (BER) for QAM

Vector diagrams of 4PAM (16QAM) is shown in Figure 3.

The symbol (0) shows the start of the vectors of all possible combination of signals PAM and black points (•) show the location of the end of these vectors.

Considered for in-phase quadrature signal 16QAM based in (3) specify all possible variants of the received signal:

$$s_{00}(t) = s(t; \alpha_1 = -1, \alpha_2 = -1) = -3d\cos(\omega_0 t + \varphi_0)$$

$$s_{01}(t) = s(t; \alpha_1 = -1, \alpha_2 = +1) = -d\cos(\omega_0 t + \varphi_0) \tag{13}$$

$$s_{11}(t) = s(t; \alpha_1 = +1, \alpha_2 = +1) = 3d\cos(\omega_0 t + \varphi_0)$$

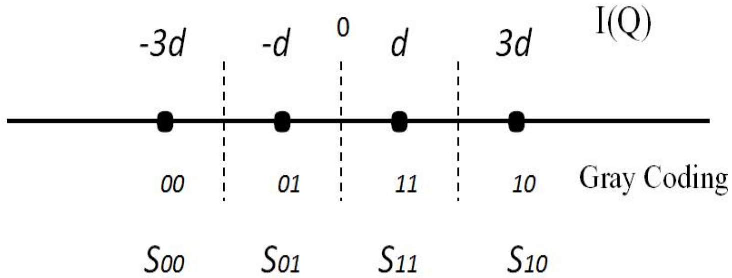

Figure 3. Possible Vectors M = 4 level I-channel 4PAM

$$s_{10}(t) = s(t;\alpha_1 = +1, \alpha_2 = -1) = 3d\cos(\omega_0 t + \varphi_0)$$

Given the structure of the vector diagram for 4PAM Figure 3 and that the signals are equally probable; the expression for the BER of 16QAM signal has the following form:

$$p_{pr}(s_{00}) = p_{pr}(s_{01}) = p_{pr}(s_{11}) = p_{pr}(s_{10}) = 1/4 \qquad (14)$$

Then, the expression for the probability of error (bit - $\boldsymbol{\alpha}_1$; $\boldsymbol{\beta}_1$) from 16QAM signal has the form:

$$BER_{16QAM}(\alpha_1) = BER_{16QAM}(\beta_1) = (1/4)\,[p(s_{01}/s_{11}) + p(s_{00}/s_{11}) + p(s_{01}/s_{10}) + p(s_{00}/s_{10}) + \ldots + p(s_{11}/s_{01}) + p(s_{10}/s_{01}) + p(s_{11}/s_{00}) + p(s_{10}/s_{00}) \qquad (15)$$

We obtained:

$$BER_{16QAM}(\alpha_1) = BER_{16QAM}(\beta_1) = \frac{1}{4}\left\{\mathrm{erfc}\left(\sqrt{\frac{E}{N_0}}\right) + \mathrm{erfc}\left(3\sqrt{\frac{E}{N_0}}\right)\right\} \qquad (16)$$

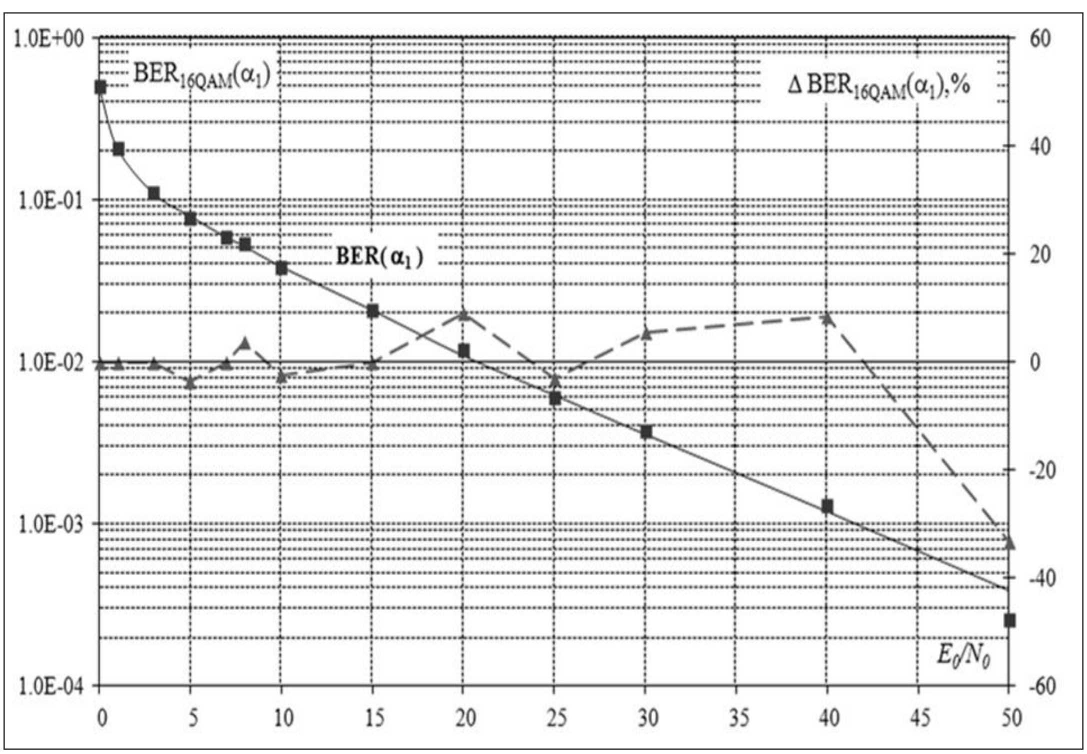

Figure 4. Probability of error for (α_1)

Finally the expression for **(bit-α_2; β_2)** has the form:

$$=\frac{1}{4}\left\{\left(2\sqrt{\frac{E}{N_0}}\right)+\operatorname{erfc}\left(3\sqrt{\frac{E}{N_0}}\right)-\operatorname{erfc}\left(5\sqrt{\frac{E}{N_0}}\right)\right\} \tag{17}$$

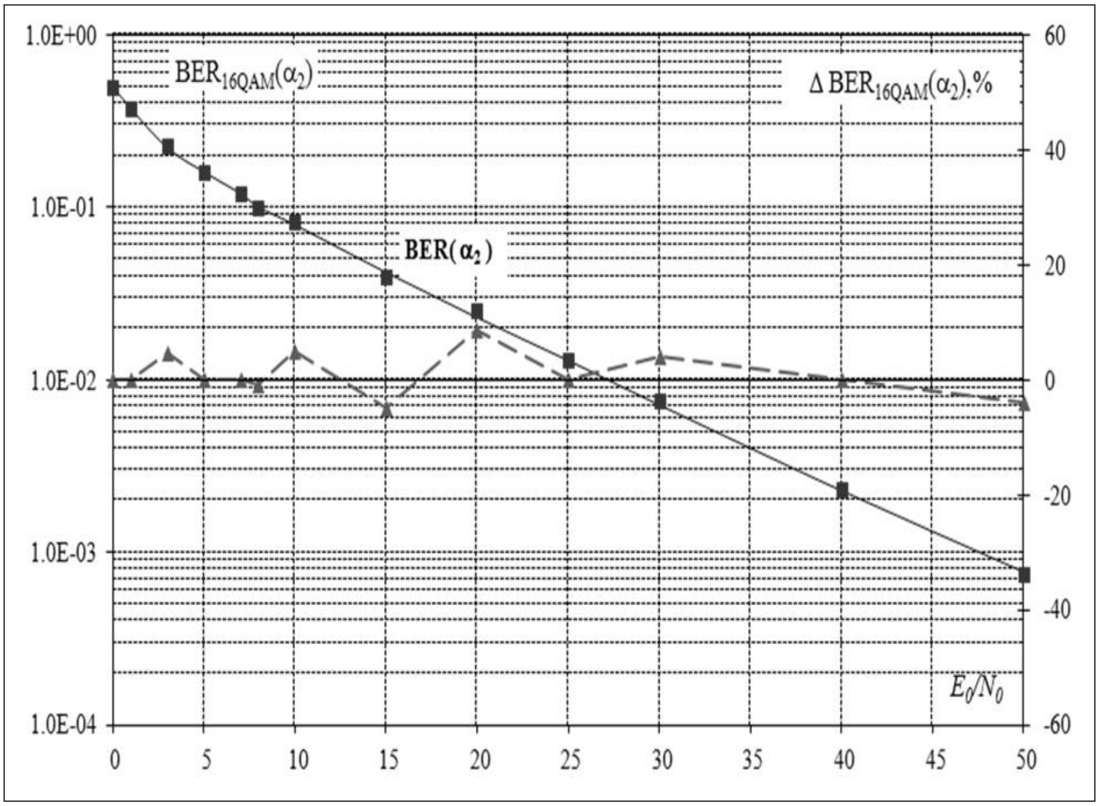

Figure 5. Probability of error for (α_2)

3. Simulation Results

In this section the performance of the analytical expression has been evaluated (16QAM). Our research was carried out by methods of statistical on the PC in the software environment MATLAB 7.0.1®.

The results of statistical simulation confirm the correct analytical expressions for the reception PSK, QPSK and 16QAM.

Figure 4 and 5, show the results of calculations and statistical modeling of the BER bit by bit for 16QAM. The solid lines show the dependences calculated from equation (15) and (16) obtained analytically. Markers show the results of statistical modeling algorithms for coherent reception.

4. Conclusion

In this paper, we have developed and evaluated statistical algorithms modelling bit by bit reception of signals QAM, including the presence of AWGN. The results confirmed the validity of the statistical model derived in comparison with works on analytical expressions developed previously.

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References

[1] Proakis, J. (2001). Digital Communications, 4rd ed., McGraw-Hill, p. 278–286.

[2] Cho, K., Yun, D. (2002). On the general BER expression of one and two dimensional amplitude modulation, *IEEE* Transactions on Communication , 50, p. 1074–1080, july.

[3] Serna, A., *et. al*. (2012). Diseño del sitema de Telemetria de abordo del satelite Quetzal para el subsistema de potencia, SOMECyTA. Sn Luis Potosi, SLP., p. 125– 128, September [2th Annual Conf. Ciencia y Tecnologia Aeroespacial, p. 368].

[4] Romo, C., *et. al*. (2012). Proyecto satelital Quetzal UNAMMIT, SOMECyTA. Sn Luis Potosi, SLP., p. 129–135, September [2th Annual Conf. Ciencia y Tecnologia Aeroespacial, p. 368].

[5] Feher, K. (1997). Digital Communications: Satellite Earth Station Engineering, 4rd ed., SciTech Publisher.

[6] Proakis. (2005). Fundamental of Communications Sistems, 1rd ed., Prentice Hall, p.567–579.