Modeling and Analysis of Discrete-Time DLL Tracking for Quadrature-Spread CDMA Signals over Rayleigh Fading Channels

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ABSTRACT: *The paper addresses the modeling and performance analysis of discrete delay-locked loop (DLL) code tracking for quadrature-spread direct-sequence code division multiple access (CDMA) signals (used in 3G UMTS) over Rayleigh fading channels. A discrete-time stochastic difference equation is derived to model the DLL tracking error dynamics as a first-order Markovian process. The stationary probability density function of the tracking error is derivediteratively by solving a Kolmogorov-Chapman integral equation. Numerical examples with different system loading assumptions show that the proposed technique, while having a moderate computational complexity, produces much more accurate results when compared to simplified approximations based on Gaussian models for the DLL tracking error.*

Keywords: Wideband CDMA, DLL Code Tracking, Markovian Process, Kolmogorov-Chapman Equation, Rayleigh Fading

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1. Introduction

Code division multiple access (CDMA) employing directsequence spread-spectrum (DS-SS) modulation is a viable radio transmission technique that offers high capacity and bandwidth efficiency, with good link quality in multipath and interference-limited environments [1,2]. CDMA systems also provide high deployment versatility and are widely used in current mobile cellular systems, such as 3G UMTS [12]. However, these advantages are limited by the ability of the receiver to synchronize its pseudo-noise (PN) sequence to the one generated at the transmitter [3]. Code synchronization is commonly implemented in two steps: coarse timing acquisition followed by fine tracking. The delay-locked loop(DLL) [1, 2, 3] is extensively used for PN code tracking in spread spectrum systems. The DLL is based on early-late correlation branches operating within a feedback control loop designed to achieve accurate code phase tracking. DLL system modeling and performance analysis over noisy channels has been addressed in previous works. In particular, the impact of fading channels on DLL code tracking performance is an important aspect that is quite relevant to the design and performance optimization of mobile cellular systems, and this has been addressed in several previous works [4, 5, 7, 9]. Other issues affecting DLL performancesuch as multiple-access interference have been considered in [8]. Different DLL structures with feedback and adaptation have also been discussed in [6,10,11]. In [13, 14, 15] an extension of the modeling in [5] is carried by considering various fading conditions. In [16], DLL performance is characterized for Nakagami, frequency selective fading channel with a RAKE receiver structure.

In this paper, we consider a discrete-time DLL system model suitable for digital receiver implementation, and present a detailed performance analysis of PN code tracking performance with QPSK-spread CDMA signals under multipath fading conditions (applicable to 3G-UMTS wideband CDMA systems [12]). More specifically, the workextends the DLL modeling undertaken in [5] and considers the more complex QPSK signal spreading relevant to WCDMA-UMTS systems, combined with Rayleigh fading. Analytical expressions for the DLL discriminator output are derived as a function of the loop S-curve [1,2], and a stochastic difference equation (SDE) that describes the dynamics of the discrete-time tracking error process is developed. The SDE formulation shows that the tracking process is first-order Markovian, and is completely describedby its transition probability density function (PDF). A closed form expression for the transition PDF is obtained underRayleigh fading conditions. The steady-state PDF of the DLL tracking error is then iteratively computed from the initialerror PDF and state transition PDF using a Kolmogorov- Chapman integral equation [5].

The rest of the paper is organized as follows. Section 2 introduces the signal and system models used for the performance analysis of the digital DLL under Rayleigh fading presented in Section 3. Numerical results that illustrate the merits of the DLL performance analysis approach are given in Section 4, and final conclusions are presented in Section 5.

2. Signal and System Model Models

We consider a DS-SS signal model employing phase-shift keying (QPSK) and complex spreading UMTS mobile cellular systems. The complex baseband signal is expressed as:

$$\tilde{S}(t) = \sqrt{\beta/2} \sum_{m=-\infty}^{\infty} d_{\lfloor m \rfloor_L} C_m (mod\, N)\, h\,(t - m\, T_c) \quad (1)$$

where $\lfloor m \rfloor_L \overset{def}{=} int\left[\frac{m}{L}\right]$ with L and N denoting the spreading factor and the period of the spreading code, respectively. β is the signal power, $\{d_m\}$the complex data symbol (which for pilot channels may be set to 1), $\{c_m\}$ is the complex scrambling code sequence, ’ is the impulse response of a pulse shaping filter and $h(t)$ is the chip duration. fading channels, the received signal at the receiver front can be written as

$$r\,(t) = \sqrt{\beta/2}\,\Re\mathfrak{e}\left[\sum_{i=1}^{M} \tilde{s}\,(t - \tau_i\,(t))\, a_i(t)\, e^{\,j2\pi f_c t}\right] + n\,(t) \quad (2)$$

where M is the number of resolvable multipath signals, $a_i(t)$ and $\tau_i\,(t)$ are the complex channel amplitude and propagation delay of the i^{th} path signal, and f_c is the carrier frequency. The noise $n(t)$ is modeled by a bandpass process given by

$$n\,(t) = n_c(t)\cos\,(2\pi f_c\, t) - n_s(t)\sin\,(2\pi f_c\, t) \quad (3)$$

where $n_c(t)$ and $n_s(t)$ are zero-mean independent Gaussian random processes with power N_0W, where N_0 and W denote the spectral density and signal bandwidth respectively. It is assumed that other multipath signals are separated from the signal being tracked by more than one chip period T_c. In this case, other path signals can be regarded as the inte interference that increases the interference-plus-noise spectral density.

Figure 1 depicts a block diagram showing the different processing stages of a non-coherent DLL code tracking loop coherent [1]. In the figure, after carrier down-conversion, chip pulse conversion, matched filtering and signal sampling, the variables Y_{I-} and Y_{Q-} (Y_{I+} and Y_{Q+}) represent the I- and Q-channel outputs of Q the early (late) correlators, respectively. The sample pectively. correlator unit for QPSK-spread signals is depicted in Figure 2. spread The squaring blocks are used in this non-coherent DLL non architecture to support timing synchronization prior to RF carrier phase lock. The early & late sample correlator outputs are used for generating the loop discriminator error signal ΔZ which is lowpass-filtered with a smoothing filter, thus giving an output proportional to the timing error estimate. This error estima signal is then used to drive and fine-tune the local receiver fine PN code clock generated by the numerically-controlled numerically oscillator (NCO) unit.

Prior to receiver synchronization, in the presence of a timing eceiver offset error τ, the I- and Q-channel correlator outputs are channel cor expressed by:

$$Y_I = AN\sqrt{E_{c,p}}\; R(\tau)\cos\varphi + n_I$$
$$Y_Q = AN\sqrt{E_{c,p}}\; R(\tau)\cos\varphi + n_Q \quad (4)$$

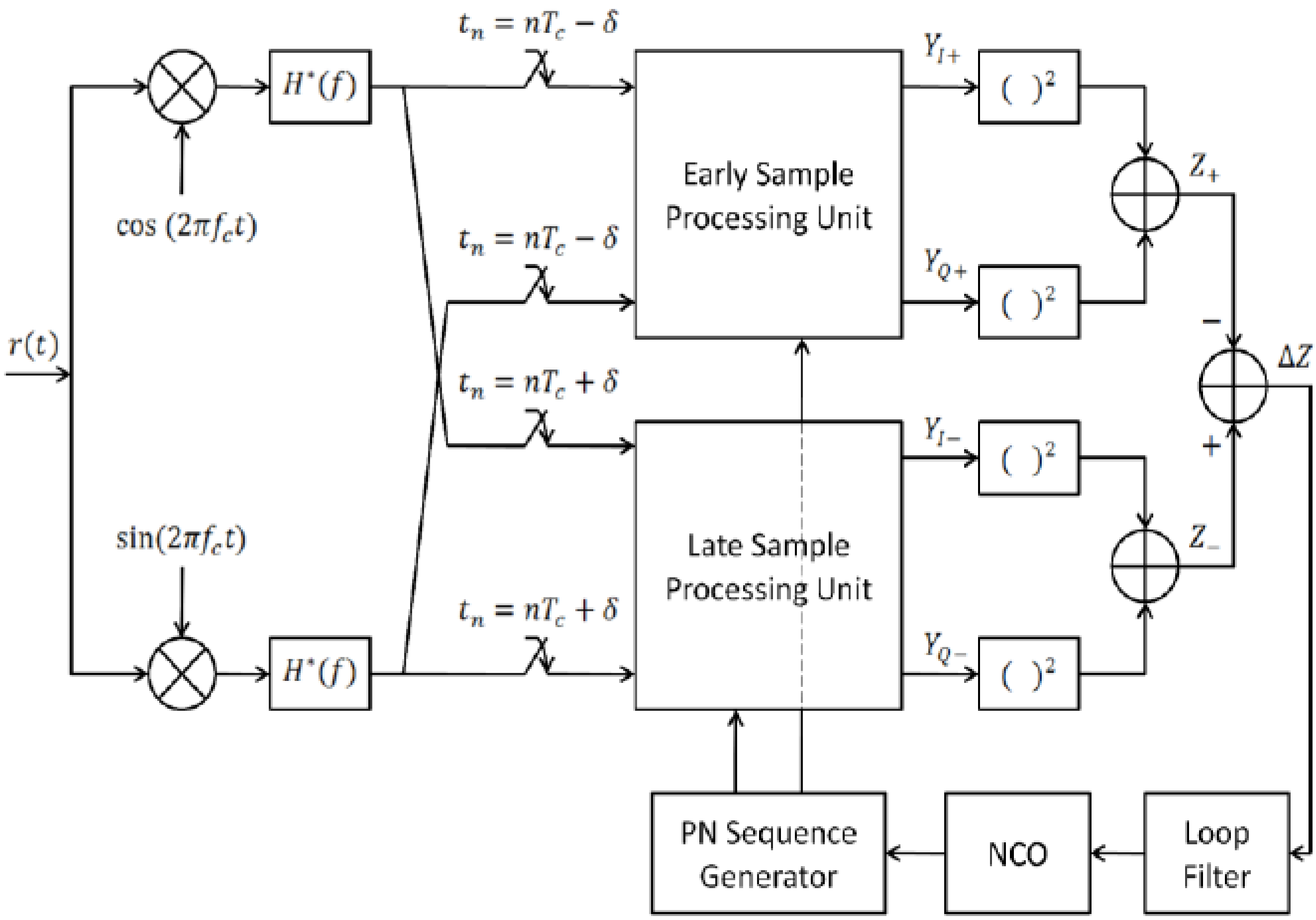

Figure 1. Processing Blocks of the DLL Tracking Loop

where A and φ are the signal envelope and phase, respectively, $E_{c,p}$ the PN chip energy, and N the number of energy accumulated PN chips. $R(\tau)$ is the convolution of the conv impulse response of the chip-shaping pulse and its matched shaping filter, given by

$$R(\tau) = h(\tau) * h(-\tau) = \int_{-\infty}^{\infty} |H(f)|^2 \cos(2\pi f_c \tau)\, df \tag{5}$$

The pulse-shaping filter is normalized so that $R(0) = 1$. The I and Q noise terms n_I and n_Q each having a variance

$$E[n_I^2] = E[n_Q^2] = I_0 / 2 \tag{6}$$

where I_0 is given by

$$I_0 = N_0 \int_{-\infty}^{\infty} |H(f)|^2\, df \tag{7}$$

with N_0 denoting the power spectral density of the ing background noise. We can model the early sample and late sample correlator outputs for a flat-fading Rayleigh channel as follows [3]

$$\begin{aligned} Y_{I\pm} &= AN\sqrt{E_{c,p}}\, R(\tau \pm \delta) \cos\varphi + n_{I\pm} \\ Y_{Q\pm} &= AN\sqrt{E_{c,p}}\, R(\tau \pm \delta) \cos\varphi + n_{Q\pm} \end{aligned} \tag{8}$$

with δ denoting the DLL discriminator timing offset typically set to half a chip. The variables $Z\_$ and Z_+ are given by:

$$Z\_ = Y_{I-}^2 + Y_{Q-}^2 \tag{9}$$

$$Z_+ = Y_{I+}^2 + Y_{Q+}^2 \tag{10}$$

The discriminator output is formed by the difference:

$$\Delta Z = Z_- - Z_+ \tag{11}$$

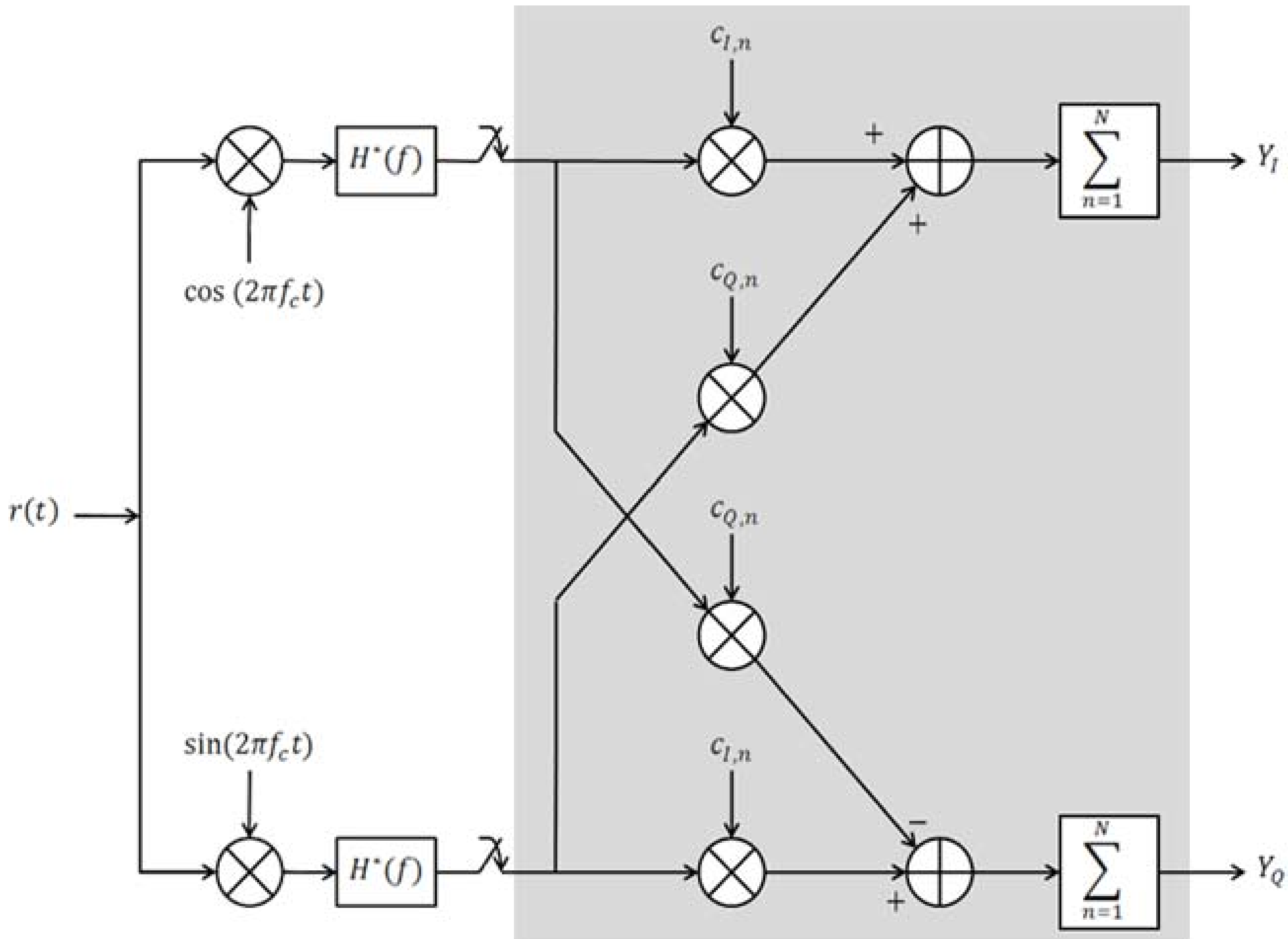

Figure 2. Early-Late Sample Correlator Unit

For the discrete-time model considered, the discriminator is discretized, and with timing normalization by the chip interval T_c, this yields:

$$\eta_k = \Delta Z_k = \beta A_k^2 S(\epsilon_k) + N_k \tag{12}$$

where the normalized loop S-curve is given by:

$$S(\epsilon_k) = N^2 E_{c,p}(R^2(\epsilon_k - \delta) - R^2(\epsilon_k + \delta)) \tag{13}$$

and

$$N_k = 2N\sqrt{E_{c,p}}(R^2(\epsilon_k - \delta)(\cos\varphi\, n_{I-} + \sin\varphi + n_{Q-}) - R(\epsilon_k + \delta)(\cos\varphi\, n_{I+} + \sin\varphi\, n_{Q+})) + W_{IQ} \tag{14}$$

with the noise-related cross-term

$$W_{IQ} = n_{I-}^2 + n_{Q-}^2 - n_{I+}^2 - n_{Q+}^2 \tag{15}$$

In Equation (12), A_k is a Rayleigh random variable. N_k is a combined noise term that includes the effects of thermal noise as well as interference from other CDMA users in the mobile network. The factor β is a power reduction factor (less than one) and is related to the cellular system environment. $S(\epsilon_k)$ is the S-curve for the loop and $\epsilon_k = (\tau_k - \hat{\tau}_k) / T_C$ is the normalized tracking error in the interval $t_k \le t_{k+1}$

3. DLL Analysis in Rayleigh Fading Channels

In this section, we present a detailed analysis of the statistics of the DLL timing error, extending the modeling first introduced in [5] for BPSK-spread signals. For mathematical tractability, we make the reasonable assumptions that the random variables A_k and N_k are mutually independent, and Rayleigh and Gaussian distributions, respectively. With the digital loop filter $h(k)$ and the NCO gain K_{NCO}, the normalized loop timing estimate can be expressed as:

$$\frac{\hat{\tau}_k}{T_C} = \frac{\hat{\tau}_k - 1}{T_C} + K_{NCO}[h(k-1) * \eta_{k-1}] \tag{16}$$

Since $\frac{\hat{\tau}_k}{T_C} = \frac{\hat{\tau}_k}{T_C} - \epsilon_k$, the normalized timing error ϵ_k in the interval $t_k \le t \le t_k + 1$ can be expressed as

$$\epsilon_k = \epsilon_{k-1} + \frac{\tau_k - \tau_k - 1}{T_C} - K_{NCO}[h(k-1) * \eta_{k-1}] \tag{17}$$

For slowly time-varying channels, $\tau_k \approx \tau_{k-1}$ and Equation (17) becomes

$$\epsilon_k = \epsilon_{k-1} - K_{NCO}\Delta t(\beta A_{k-1}^2 S(\epsilon_{k-1}) + N_{k-1}) \tag{18}$$

As stated previously, in order to ease theoretical analysis we model the noise N_k as Gaussian (a valid assumption supported by the central limit theorem for a large number of interferers [2]). From Equation (19). The timing error sampled process can be modeled as a discrete-time continuous variable Markov process [5]. The statistics of this process is completely characterized by the one-step transition probabilities $f_k(x' | x)$, $x', x \in S'$ and the first order density $p_0(x)$, $x \in S'$ where S' is the total state space. The PDF of ϵ_k, conditioned on the initial timing error, satisfies the Chapman-Kolmogrov (K-C) equation [5]:

$$p_k(\epsilon | \epsilon_0) = \int_{-\infty}^{\infty} f_{k-1}(\epsilon | x)\, p_{k-1}(x | \epsilon_0)\, dx \tag{19}$$

where ϵ_0 is the initial timing error value, $p_k(.|\epsilon_0)$ is the PDF of ϵ_k given ϵ_0, and $f_{k-1}(.|x)$ is the transition PDF of ϵ_k given $\epsilon_{k-1} = x$.

Since A_k^2 and N_{k-1} are independent, the transition PDF $f_{k-1}(\epsilon | x)$ can be derived using the fact that ϵ_k is a PDF function of two independent random variables A_k^2 and N_{k-1}

$$\epsilon_k = x - K_{NCO}\Delta t(\beta A_{k-1}^2 S(\epsilon_{k-1}) + N_{k-1}) \tag{20}$$

Letting $z \triangleq x - K_{NCO}\Delta t N_{k-1}$, it is seen that is a Gaussian random variable with mean and variance $(K_{NCO}\Delta t \sigma_n)^2$ where σ_n^2 is the variance of N_{k-1}. Assuming that the pulse- shaping filter can be approximated by an ideal lowpass filter with $H(f) = 1/\sqrt{W}$ for $-W/2 \le f < W/2$ and $W = 1/T_C$ and that ϵ_k is small, some mathematical manipulation yields:

$$\sigma_n^2 = 2N^2 I_0^2\{1 - R^2(2\delta)\} + 4N^3 I_0 E_{c,p} R^2(\delta)\{1 - R^2(2\delta)\} \tag{21}$$

After deriving the variance of N_{k-1} we can obtain the PDF of $z \triangleq x - K_{NCO}\Delta t N_{k-1}$ as follows [5]:

$$\begin{aligned} f_z(z) &= \frac{1}{\sqrt{2\pi} K_{NCO}\Delta t \sigma_n} \exp\left[-\frac{(z-x)^2}{2(K_{NCO}\Delta t \sigma_n)^2}\right] \\ &= \frac{1}{\sqrt{2\pi} K_2 \sigma_n} \exp\left[-\frac{(z-x)^2}{2(K_2\sigma_n)^2}\right] \end{aligned} \tag{22}$$

where $K_2 \triangleq K_{NCO} \Delta t$.

On the other hand, the PDF of A_{k-1}^2 is exponentially distributed since the channel gain is Rayleigh-distributed. Letting $K_2 \triangleq K_{NCO} \Delta t \, \beta \, S(x) \, A_{k-1}^2$, the PDF of v is shown to be (when is $s(x)$positive) [5]

$$f_v(v) = \begin{cases} \frac{1}{K_1 S(x)} \exp\left[\frac{v}{K_1 S(x)}\right] & \text{for } v < 0 \\ 0 & \text{elsewhere} \end{cases} \tag{23}$$

where $K_1 = \beta P K_{NCO} \Delta t$ and $P = E[A_{k-1}^2]$ is the average signal power. When $S(x)$ and is negative, the PDF of v becomes:

$$f_v(v) = \begin{cases} \frac{-1}{K_1 S(x)} \exp\left[\frac{v}{K_1 S(x)}\right] & \text{for } v > 0 \\ 0 & \text{elsewhere} \end{cases} \tag{24}$$

The transition PDF can then be obtained as the convolution of the PDF's of the random variables s and v, giving [5]:

$$f_{k-1}(\epsilon \mid x) = \int_{-\infty}^{\infty} f_z(\varepsilon - v) f_v(v)$$

$$= \begin{cases} \frac{1}{K_1 S(x)} \exp(\Gamma_1).\, Q(\Lambda_1) & \text{if } S(x) > 0 \\ \frac{-1}{K_1 S(x)} \exp(\Gamma_1).\, Q(-\Lambda_1) & \text{if } S(x) < 0 \\ \frac{1}{\sqrt{2\pi} K_2 \sigma_n} \exp\left[-\frac{(z-x)^2}{2(K_2\sigma_n)^2}\right] & \text{if } S(x) = 0 \end{cases} \tag{25}$$

where

$$Q(y) \triangleq \frac{1}{\sqrt{2\pi}} \int_y^{\infty} \exp\left(\frac{-x^2}{2}\right) dx$$

$$\Gamma_1 = \frac{K_2^2 \sigma_n^2}{2K_1^2 S^2(x)} + \frac{\epsilon - x}{K_1 S(x)}$$

$$\Lambda_1 = \frac{\epsilon - x + \frac{K_2^2 \sigma_n^2}{K_1 S(x)}}{K_2 \sigma_n}$$

It is observed that $f_{k-1}(\epsilon \mid x)$ is independent of k. The PDF of the normalized tracking error, $p(\epsilon \mid \epsilon_0)$ can finally be calculated by iterating the Chapman-Kolmogrov equation given in (19).

In summary, it is seen that the previous DLL analysis led to the derivation of a closed form expression for the transition PDF by computing the convolution of the PDF's for Gaussian and Rayleigh random variables. It should be noted that, in the absence of the detailed modeling of multipath, multi-user fading signal and system models described above, the customary approach presented in most previous works relies on a Gaussian assumption of the DLL tracking error, and simply attempts to estimate its variance. The accuracy of he proposed method in this paper and its comparison to the Gaussian approximation will be presented next in the numerical results.

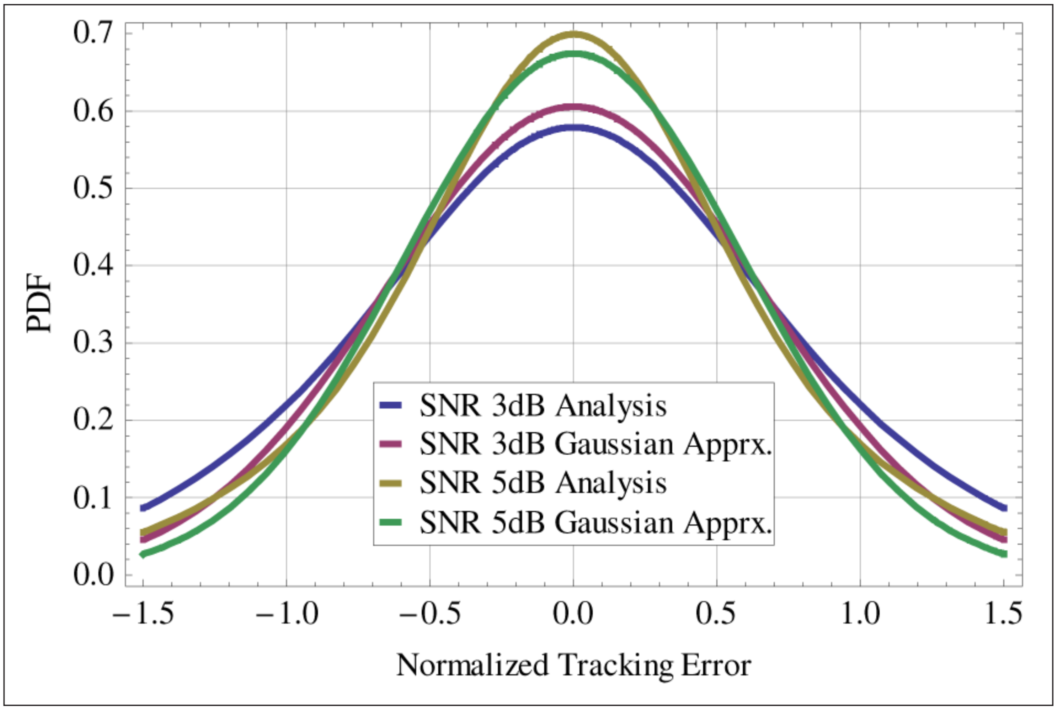

Figure 3.Comparison of DLL tracking error PDF's for direct analysis and Gaussian approximation at low-to-moderate SNR values (under Rayleigh fading channel conditions).

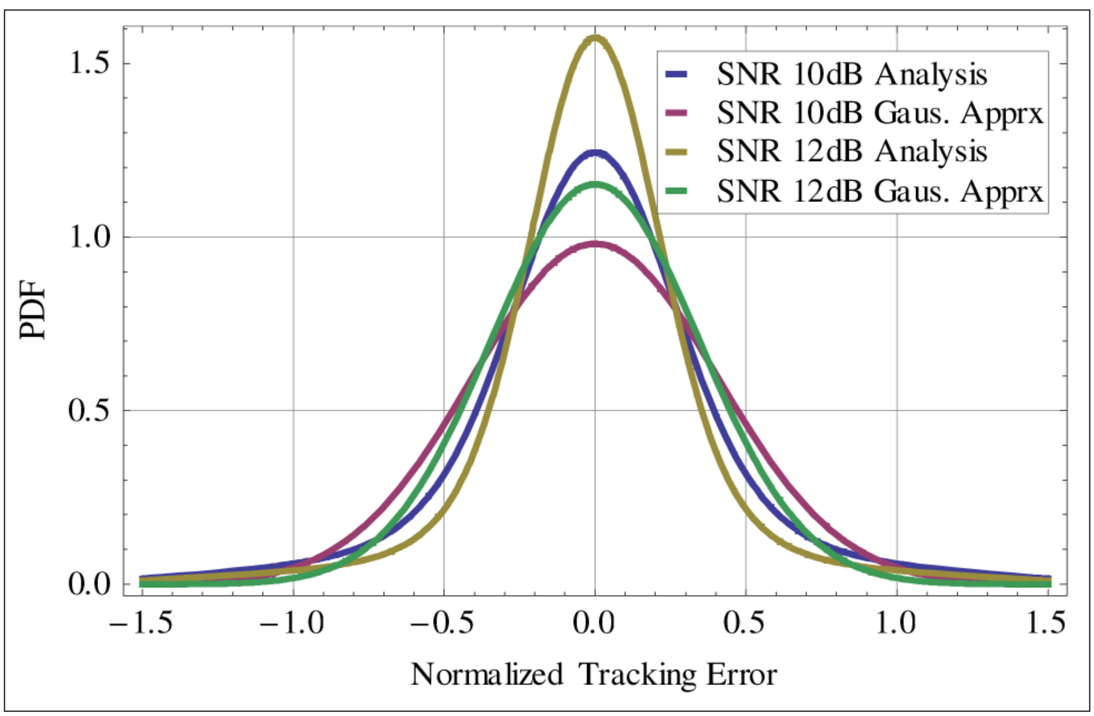

Figure 4. DLL tracking error PDF's with direct analysis and Gaussian approximation at intermediate SNR values

4. Numerical Results

This section presents different numerical results to illustrate the DLL performance analysis discussed in the previous sections.

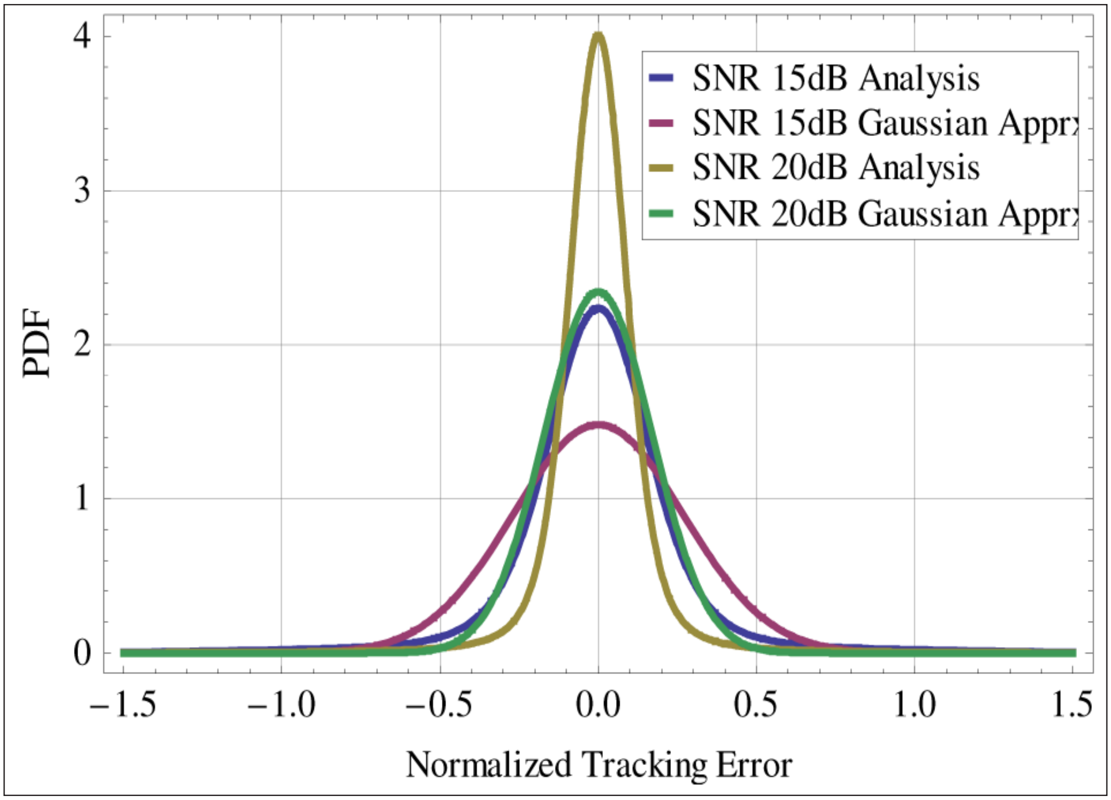

Figure 5. DLL tracking error PDF's with direct analysis and Gaussian approximation at high SNR values

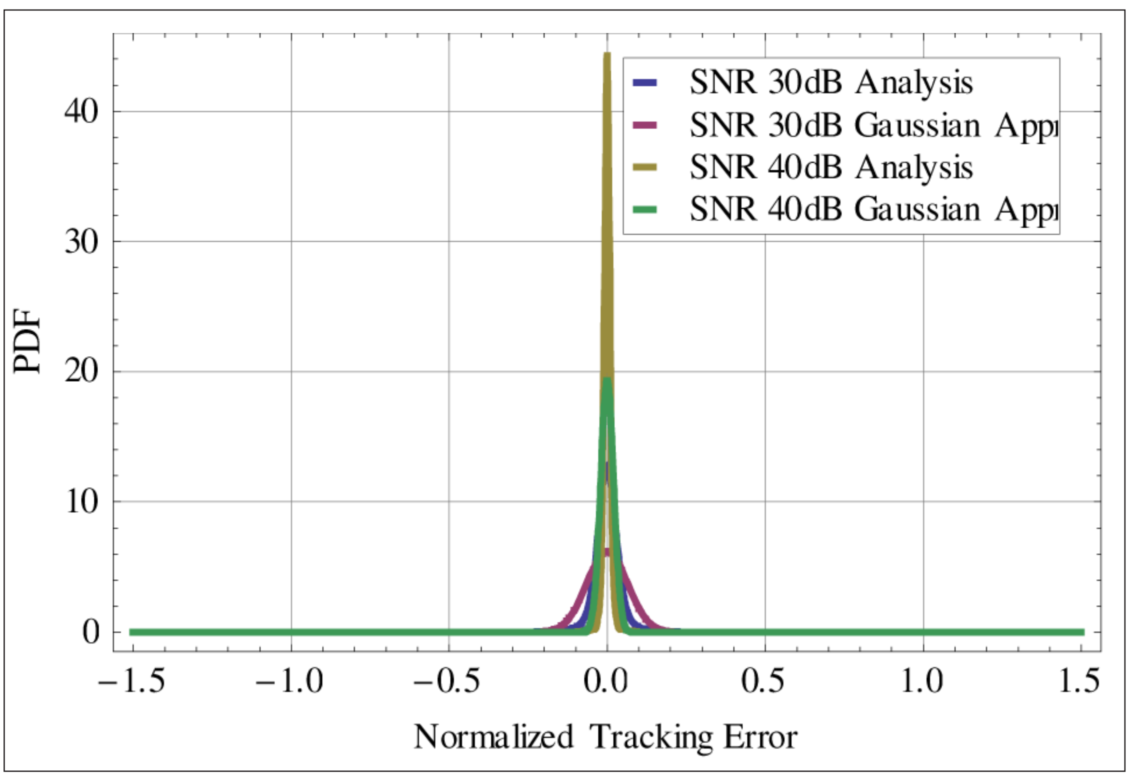

Figure 6. DLL tracking error PDF's with direct analysis and Gaussian approximation at very high SNR values

As discussed, the numerical algorithm used in this paper attempts to get the stationary PDFs by iterating the Kolmogrov-Chapman equation for $k = 0, 1, \ldots$ up until the PDF $p_k = (\epsilon \mid \epsilon_0)$ stops changing appreciably. For the initial distribution, it is assumed that $P(\epsilon \mid \epsilon_0) = \delta(\epsilon - \epsilon_0) = \delta(\epsilon)$, where we let $\epsilon = 0$. The computation of the successive PDF's $p_k(\epsilon \mid \epsilon_0)$ is obtained by simple numerical integration based on Equation (19). The information needed for this step is the conditional transition PDF $f_k(\epsilon \mid \epsilon_{k-1})$, found using Equation (25).

In the following examples, different cases are considered to model different levels of cellular network loading: heavy to normal interference load (resulting in low-moderate SNR of 3-5dB), and medium to light load (with SNR in the range of 15-20dB). The results of Figure 3 show the PDF of the DLL tracking error obtained by direct analysis as illustrated in the previous sections and a normal PDF for a Gaussian random variable with the same variance. It is seen that in the low SNR case, the Gaussian approximation is reasonably accurate, as the Central Limit Theorem (for large number of users creating high interference and low SNR) justifies the Gaussian approximation model. It is also noted that the tracking performance of the DLL is rather poor as the PDFs show heavy tails with normalized residual error is in excess of 0.5 (i.e., the local PN code clock is offset by more than half a chip $T_c / 2$). On the other hand, the results of Figure 5 and Figure 6 show very good performance for the DLL PN residual code tracking error with high SNRs in the 15-20dB and 30-40dB range respectively. It is interesting to note that the commonly used Gaussian error assumption is no longer accurate in this case as it tends to be overly pessimistic by producing a flatter DLL error PDF (This is true also for Figure 4 which illustrates the case of intermediate SNR levels). The merits of the proposed K-C based SDE analysis in accurately estimating the DLL performance are therefore clearly demonstrated in this case.

5. Conclusion

This paper presented a detailed analysis of the PN code tracking performance achieved by delay-locked loops used in the synchronization of CDMA receivers operating with QPSK-spread direct-sequence spread-spectrum signals employed in 3G mobile wireless networks.

A detailed signal and system analysis that took into account multipath and multi-user interference was developed, and a first-order discrete-time Markovian model was derived to capture the dynamics of the DLL tracking error described by a discrete stochastic differential equation. Subsequently, an iterative solution algorithm based on a Chapman- Kolmogorov integral equation was presented, leading to a mathematically tractable closed-form expression for the probability density function of the DLL tracking error.

Simulations and numerical results were presented to illustrate the accuracy of the analysis method in comparison with simplified Gaussian error models for the tracking error, and it was shown that the latter approximation is only accurate forhigh SNR ranges.

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