Power and Transmission Duration Control in Un-Slotted Cognitive Radio Networks

Marwa Ali[1], Ahmed Ahmedin[2], Ahmed Sultan[3], Mohammed Nafie[4]
[1]Texas A&M University, USA
[2]University of California, Davis, USA
[3]Alexandria University, Egypt
[4]Nile University, Egypt
marwa.ali@tamu.edu, ahmedin@ucdavis.edu, salatino@alumni.stanford.edu, mnafie@nileuniversity.edu.eg

ABSTRACT: *We consider an unslotted primary channel with alternating on/off activity and provide a solution to the problem of finding the optimal secondary transmission power and duration given the sensing outcome. The goal is to maximize a weighted sum of the primary and secondary throughput where the weight is determined by the degree of protection and the minimum rate required by the primary terminals. Two sensing schemes are considered: perfect sensing in which the actual state of the primary channel is revealed, and soft sensing in which the secondary transmission power and time are determined based on the sensing metric directly. We use an upper bound for the secondary throughput assuming that the secondary receiver tracks the instantaneous secondary channel state information. We justify the upper bound on information-theoretic grounds and also provide a lower bound on secondary throughput. The weighted sum throughput objective function is non-convex and, hence, the optimal solution is obtained via exhaustive search. Our results show enhanced throughput by allowing the secondary to transmit even when the channel is found to be busy. The enhancement increases as the channel gain between the secondary transmitter and primary receiver decreases. For the examined system parameter values, the throughput gain from soft sensing depends on the “distance” between the likelihood functions of the received primary signal at the secondary transmitter. Further investigation is needed to quantify the potential of soft sensing.*

Keywords: Cognitive Radios, Spectrum Sensing, MAC Protocols

Received: 12 October 2013, Revised 26 November 2013, Accepted 4 December 2013

1. Introduction

Static spectrum allocation has been the major approach to limit the interference between different wireless systems and support their coexistence. Most of the licensed spectrum resources are under-utilized, however [1]. This observation has encouraged the emergence of cognitive radio technology and opportunistic spectrum access concepts. In cognitive radio networks, two classes of users coexist. The primary users are the classical licensed users, whereas the cognitive users, also known as the secondary or unlicensed users, attempt to utilize the resources unused by the primary users following schemes and protocols designed to protect the primary network from interference and service disruption. There are two main scenarios for the primary-secondary coexistence. The first is the overlay scenario where the secondary transmitter checks for primary activity before transmitting. The secondary user utilizes a certain resource, such as a frequency channel, only when it is unused by the primary network. The second scenario is the underlay system where simultaneous transmission is allowed to occur so long as the interference caused by secondary transmission on the primary receiving terminals is limited below a certain level determined by the required primary quality of service. There is a significant amount of research that pertains to the determination of the optimal secondary transmission parameters to meet certain objectives and constraints. The research in this area has two main flavors. The first takes a physical layer perspective and focuses on the secondary power control problem given the channel gains between the primary and secondary transmitters and receivers. In [2], for instance, the focus is on maximizing a weighted sum rate of secondary users with constraints on the maximum secondary transmitted powers and the maximum tolerable interference level at primary terminals. The traffic pattern on the primary channel is typically not included in this approach save for a primary activity factor such as in [3] and [4] .

The second line of research concentrates on primary traffic and seeks to obtain the optimal time between secondary sensing activities in an unslotted system, or the optimal decision, whether to sense or transmit, in a slotted system. Usually under this approach the physical layer is abstracted and the assumption is made that any two packets transmitted in the same time/ frequency slot are incorrectly received (e.g., [5], [6], [7], and [8]).

In this paper, we combine aspects of both the overlay and underlay schemes. As in the overlay systems, the secondary transmitter carries out sensing to detect primary activity. However, we adopt a potentially more efficient cognitive transmission model and allow for secondary transmission **even** when the channel is perfectly sensed to be busy. The rationale behind this is clear from the extreme case of having a very small channel gain between the secondary transmitter and primary receiver enabling the transmitter to work at maximum power without hurting the primary link. We assume that the primary system operates in an unslotted fashion with the primary terminal switching its state of activity at random times. Unslotted primary systems are studied in [9–11].

Our objective is to find the optimal sensing-dependent powers and transmission durations in order to maximize a weighted sum of primary and secondary rates. The weight used is specified according to the minimum guaranteed primary rate and the degree of protection needed by the primary link. Though in actual systems, the primary network would have higher priority (reflected in a weight close to unity in our formulation detailed below), we present the general case to account for other possible operation scenarios involving networks with no clear priority structure, such as all-secondary networks. Note that the weighted sum-rate concept has already been employed in the cognitive radio context, such as in [12–15]. However, it mainly concerns the sum-rate of multiple secondary users with constraints on the inference level inflicted on the primary receiver. The interference power level constraint does not take into account the primary link quality as it focuses only on the interference power at the primary receiver. It is more reasonable, albeit more difficult, to impose the constraint on the primary signal-to-interference-plus-noise-ratio (SINR) or primary rate because a high-quality primary link can withstand more interference power given some guaranteed primary SINR, thereby allowing more secondary throughput gains relative to the "*interference temperature*" or power constraint.

We consider two sensing schemes in this work: (a) perfect sensing and (b) soft sensing, introduced in [3], where secondary transmission parameters are determined directly from some sensing metric. We jointly optimize the transmission time and power. This is in contrast to previous works [4, 18] and [19]. In [4], although the secondary is allowed to transmit even if the primary channel is busy, there is no optimization of the transmission or inter-sensing time because the authors assume that the primary network follows a slotted manner of operation. Also, the notion of soft sensing is not investigated. In [18], sensing is carried out periodically and the secondary transmitter remains silent if the channel is sensed to be busy. In [19], only the transmission time is optimized.

We make the following contributions in this paper. We obtain the optimal power and transmission time for operation with an unslotted primary network given the sensing metric. In the case of conventional sensing, if the channel is sensed to be free, a certain transmit power is used and the channel is re-sensed after a specific time. A possibly different power and transmission time are used if the channel is busy. Optimizing the transmit power and transmission periods makes use of primary traffic parameters in addition to the physical channels between the transmitters and the receivers. We extend the power and transmission duration control to the soft sensing case. Moreover, we investigate the scenario of continuous transmission by the secondary user. This means that the secondary transmitter performs no sensing and its optimization parameter is the transmit power only as in underlay networks. The objective is to determine the situations in which a pure underlay strategy performs almost the same as the sensing-dependent power and transmission time scheme. (Our results are presented in bullet form in Subsection 5.2.)

In addition, we provide an information-theoretic justification for the expression we employ for the secondary throughput. In the literature, the ergodic capacity is often used without justification (see, for example, [19, 20] and [21]). In this paper we derive an upper and lower bounds on the mutual information between the input and output of the channel between the secondary terminals. This mutual information is then used to obtain an expression for secondary link throughput. We also provide a lower bound on the capacity of the secondary link which is below the upper bound by a maximum of one bit.

The paper is organized as follows: in Section 2 the system model is introduced. The optimization problem of maximizing the weighted sum rates is provided in Section 3. We present an information-theoretic analysis for the formulas used for secondary throughtput in Section 4, in addition to a lower bound on secondary capacity. In Section 5, we provide simulation results. Section 6 concludes the paper.

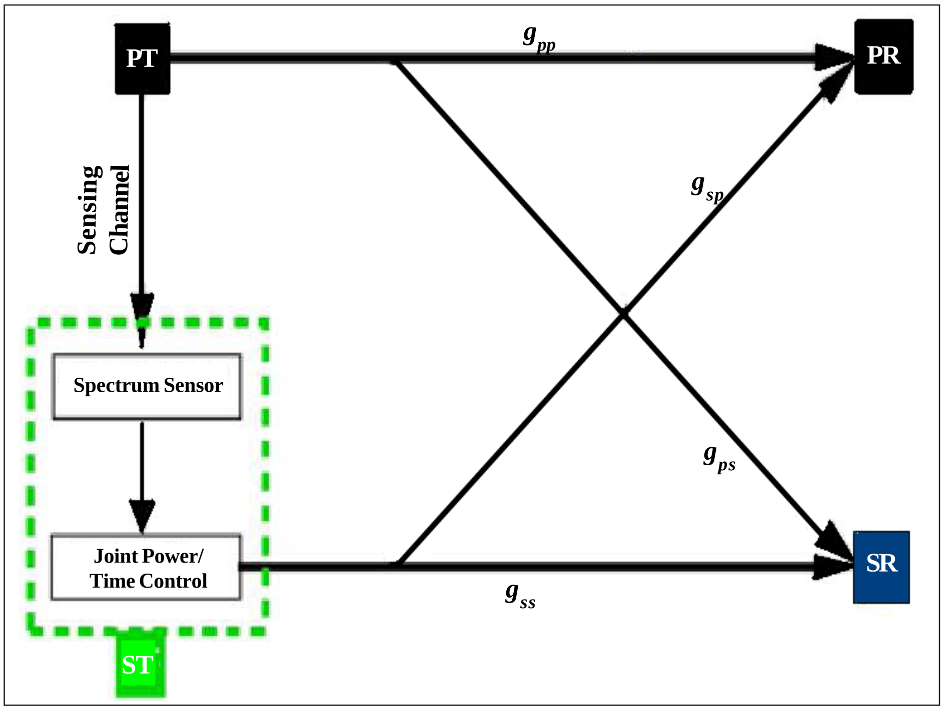


Figure 1. System model involving one primary and one secondary links, where PT denotes the primary transmitter, PR: primary receiver, SR: secondary receiver and ST: secondary transmitter

2. System Model

We consider a system composed of one primary and one secondary links assuming an unslotted primary channel with alternating on/off primary activity similar to the model employed in [6]. This implies that the primary transmitter switches between the active and inactive states at random times. For simplicity, we assume that the probability density function (pdf) of the duration of the on period is exponential and is given by:

$$f_{on}(t) = \lambda_{on} \exp(-\lambda_{on} t), \; t \geq 0 \tag{1}$$

where λ_{on} is the reciprocal of the mean on duration T_{on}. Similarly, the pdf of the off duration is:

$$f_{off}(t) = \lambda_{off} \exp(-\lambda_{off} t), \; t \geq 0 \tag{2}$$

and $\lambda_{off} = 1/T_{off}$, where T_{off} is the mean of the off duration. The channel utilization factor u is given by

$$u = \frac{T_{on}}{T_{on} + T_{off}} \tag{3}$$

Based on results from renewal theory [22], the transition probability that the primary channel is free at time $t' + t$ given that it is free at time t', is given by:

$$P^{00}(t) = (1-u) + u \exp(-[\lambda_{on} + \lambda_{off}]\, t) \tag{4}$$

Given that the channel is busy at time t', the transition probability of being free at $t' + t$, is given by:

$$P^{10}(t) = (1-u) - (1-u) \exp(-[\lambda_{on} + \lambda_{off}]\, t) \tag{5}$$

We assume no cooperation between the primary and secondary terminals. It is the responsibility of the cognitive users to estimate the primary traffic parameters and to adjust their transmission parameters taking into account the primary average rate that should not be disrupted by secondary operation.

The traffic parameters of the primary network can be learned by probing the channel for a specified learning period while keeping silent. The sensing outcome can be used to estimate the unknown parameters. In the case of perfect sensing, a maximum likelihood estimator can be employed [6]. The parameters λ_{on} and λ_{off} are obtained via maximizing the likelihood function

$$f(S_1, S_2, S_3, \ldots S_L \mid \lambda_{on}, \lambda_{off}) \tag{6}$$

where L is the number of sensing outcomes obtained during the learning phase, and S_i is the i^{th} sensing outcome which has one of two values: $S_i = 0$ if the channel is sensed to be free, and $S_i = 1$ for a busy sensing outcome. Using the Markovian property, the likelihood function (6) can be written as

$$f(S_1)\, f(S_2 \mid S_1)\, f(S_3 | S_2) \ldots f(S_L \mid S_{L-1}) \tag{7}$$

where $f(S_i = v \mid S_{i-1} = w)$ is the transition probability $P^{wv}(\tau_L)$ defined above with $v \in \{0, 1\}$, $w \in \{0, 1\}$, and τ_L is the time between two sensing events. In the simulation section, we compare the throughput when the learned rather than the true primary traffic parameters are used to optimize the secondary transmission parameters. When sensing is not perfect, which is the real world situation, the true state of the channel becomes hidden. Under such situation, a hidden Markov model (HMM) can be employed for learning the traffic parameters. Please refer to [23] and the references therein for more information about learning in the context of cognitive radio networks. It is important to mention that parameter learning is not the main focus of this work.

The primary transmitter sends with a fixed power P_p and at a fixed rate r_o. A secondary pair tries to communicate over the same channel utilized by the primary terminals. As seen in Figure 1, we denote the gain between primary transmitter and primary receiver as g_{pp}, the gain between secondary transmitter and secondary receiver as g_{ss}, the gain between primary transmitter and secondary receiver as g_{ps}, and finally the gain between secondary transmitter and primary receiver as g_{sp}. We assume Rayleigh fading channels and, hence, the channel gains are exponentially distributed with mean values: $\overline{g}_{sp}$, $\overline{g}_{ss}$, $\overline{g}_{ps}$ and $\overline{g}_{pp}$. The channel gains are independent of one another, and the primary and secondary receivers are assumed to know their instantaneous values. In practice, the channels need to be estimated. This can be done through conventional channel training methods, or via exploiting channel reciprocity in systems operating in time-division duplex (TDD) mode. More sophisticated techniques are required by the secondary user to estimate the primary link channel state information utilizing the widely used automatic repeat request (ARQ) feedback from the primary receiver to the primary transmitter [16] and [24], or through cooperation between secondary nodes that could be present close enough to the primary receiver [25].

The secondary transmitter is equipped with a single antenna and does not transmit while sensing the channel. It senses the channel for a constant time t_s assumed to be much smaller than transmission times T_{on} and T_{off}. This assumption guarantees that the primary is highly unlikely to change state during the sensing period. Based on the sensing outcome, the secondary transmitter determines its own transmit power and the duration of transmission after which it has to sense the primary channel again.

3. Optimal power level and transmission time

We formulate the cognitive power and transmission time control problem as an optimization problem with the objective of maximizing a weighted sum of the primary, R_p, and secondary, R_s, rates. Specifically, we seek to find the transmission powers and durations that maximize,

$$\mathbb{E}\{(1-\alpha)\, R_s + \alpha R_p\}$$

where $\mathbb{E}$ {.} denotes the expectation operation over the sensing outcome and primary activity. The constant $\alpha \in [0, 1]$ is chosen on the basis of the required primary throughput. In order to protect the primary user from interference and service interruption, parameter should be close to one. In the sequel, however, we study the full range of α so that our results account for other cases where there is no clear priority among the users.

The constraints of the optimization problem are that the secondary power lies in the interval $[0, P_{max}]$, where P_{max} is the maximum power level available to the secondary transmitter, and that the time between sensing operations exceeds t_s. The problem is generally non-convex and, consequently , we resort to exhaustive search to obtain the solution when the number of optimization parameters is small.

In this paper, we consider two sensing scenarios: 1) perfect sensing, and 2) soft sensing where the cognitive transmitter uses some sensing metric γ, say the output of an energy detector, to determine its transmission parameters. Under the soft sensing mode of operation, the range of values of γ is divided into intervals and the transmission power and time are determined based on the interval on which the actual sensing metric γ lies. The optimization parameters are the transmission powers and times corresponding to each interval, as well as the boundaries between the intervals.

We investigate as well a no-sensing scenario with a constant transmitted secondary power in order to examine when the secondary user can make the decision not to sense and to use only one power level when it transmits. The no-sensing optimization problem is significantly easier to solve. Note that the no-sensing outcome is a special case of the sensing scenarios. If we solve the optimization problem and get the same optimal transmit power regardless of the sensed channel state, then sensing is superfluous and the optimal strategy is continuous transmission using one power without the need for channel probing. In reality, of course, the traffic and channel parameters are time-varying and, hence, the no-sensing situation may be transient (for example, at periods when the link between the secondary transmitter and the primary receiver is in deep fade).

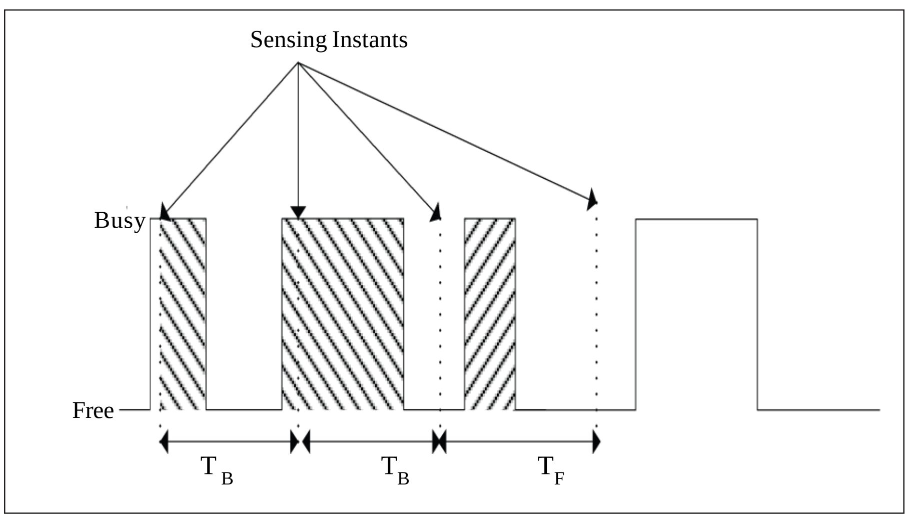


Figure 2. System operation in time. The two levels indicate primary activity. Due to the unslotted nature of the primary network, there are times with concurrent primary and secondary transmission (shaded intervals). The cognitive terminal transmits with power P_F for a duration T_F if the channel is found free. If busy, the transmit power is P_B for T_B units of time. In either case the channel is re-sensed after transmission ceases.

We assume that the primary link is in outage whenever the primary rate r_o exceeds the capacity of the primary channel.

The primary outage probability when the secondary transmitter emits power p is given by:

$$P_o(p) = \Pr\left\{ r_o > \log\left(1 + \frac{P_p\, g_{pp}}{p\, g_{sp} + \sigma_p^2}\right)\right\} \tag{8}$$

where σ_p^2 is the noise variance at the primary receiver. The expression of $P_o(p)$ for Rayleigh fading channels is given in Appendix *A*. We assume that the channel gains vary slowly over time and are almost constant over several epochs of primary and secondary transmission.

For the secondary rate, we assume that the secondary receiver tracks the instantaneous capacity of the channel and, hence, the maximum achievable rate is obtained by averaging over the channel gains and interference levels [26, equation 7]. The ergodic capacity of the secondary channel when the cognitive transmitter emits power *p* and the primary transmitter is off is expressed as

$$C_o(p) = \mathbb{E}_{gss}\left\{\log\left(1 + \frac{pg_{ss}}{\sigma_s^2}\right)\right\} \tag{9}$$

where σ_s^2 is the noise variance at the secondary receiver. When there is simultaneous primary and secondary transmissions, the ergodic capacity of the secondary channel becomes

$$C_1(p) = \mathbb{E}_{g_{ss},g_{ps}}\left\{\log\left(1 + \frac{pg_{ss}}{P_p\, g_{ps} + \sigma_s^2}\right)\right\} \tag{10}$$

We provide expressions for $C_o(p)$ and $C_1(p)$ in Appendix *A*. In Section 4, we provide an information-theoretic analysis for secondary throughput justifying the employed formulas and also presenting a lower bound on secondary link capacity. Now we present the formulation of the problem for the three cases of perfect sensing, soft sensing, and no sensing.

3.1 Perfect Sensing

We mean by perfect sensing that the state of the channel, whether vacant or occupied, is known without error after the channel is sensed. The four parameters used to maximize the weighted sum throughput are P_F and T_F defined as the power and transmission time when the primary channel is free, and P_B and T_B corresponding to the busy primary state. Refer to Figure 2 for an illustration of system operation over time. Before formulating the optimization problem under perfect sensing, we need to introduce several parameters that pertain to the primary traffic. The probability, π_m, that the *m*th observation of the channel occurs when the channel is free can be calculated using Markovian property of the traffic model.

$$\pi_m = \pi_{m-1} P^{00}(t_s + T_F) + (1 - \pi_{m-1}) P^{10}(t_s + T_B) \tag{11}$$

Another parameter is P^{ss} which is the steady state fraction of time the channel is free when sensed according to some scheme. In the perfect sensing scheme, the channel, when sensed free, is sensed again after $t_s + T_F$. When sensed busy, it is sensed again after $t_s + T_B$. Parameter P^{ss} can be obtained by setting $\pi_m = \pi_{m-1} = P^{ss}$ in (11) to get

$$P^{ss} = \frac{P^{10}(t_s + T_B)}{1 - P^{00}(t_s + T_F) + P^{10}(t_s + T_B)} \tag{12}$$

The average time between sensing times is given by:

$$\mu = P^{ss}(t_s + T_F) + (1 - P^{ss})(t_s + T_B) \tag{13}$$

Finally, we also need the average time the channel is free during a period of *t* units of time if sensed to be free. We denote this quantity by $\delta^0(t)$ and is given by (from [8])

$$\delta^0(t) = t - u\left(t + \frac{exp\,[-(\lambda_{on}+\lambda_{off})\,t]-1}{\lambda_{on}+\lambda_{off}}\right) \tag{14}$$

On the other hand, if the channel is sensed to be busy, the average time the channel is free during a period of t units of time is given by

$$\delta^1(t) = (1-u)\left(t + \frac{exp\,[-(\lambda_{on}+\lambda_{off})\,t]-1}{\lambda_{on}+\lambda_{off}}\right) \tag{15}$$

The secondary throughput averaged over primary activity is given by

$$\bar{R}_s = P_{ss}\frac{\delta^0(T_F)}{\mu}C_o(P_F) + P_{ss}\frac{T_F-\delta^0(T_F)}{\mu}C_1(P_F) + (1-P_{ss})\frac{\delta^1(T_B)}{\mu}C_o(P_B) + (1-P_{ss})\frac{T_B-\delta^1(T_B)}{\mu}C_1(P_B) \tag{16}$$

The first two terms in the above expression are the secondary throughput obtained if the primary is inactive when the channel is sensed. When the sensing outcome is that the channel is free, the secondary emits power P_F for a duration T_F. During the secondary transmission period, the primary transmitter may resume activity. The average amount of time the primary remains idle during a period of length TF after the channel is sensed to be free is obtained by using $t = T_F$ in (14). This is the duration of secondary transmission free from interference from the primary transmitter. On the other hand, the primary transmits during secondary operation for an average period of $T_F - \delta^0(T_F)$. The last two terms in (16) are the same as the first two but when the channel is sensed to be busy. In this case, the transmit secondary power is P_B and the transmission time is T_B, of which a duration of $\delta^1(T_B)$ is free, on average, from primary interference. The primary throughput is given by

$$\bar{R}_p = r_0 P^{ss}\frac{T_F-\delta^0(T_F)}{\mu}[1-P_o(P_F)] + r_0(1-P_{ss})\frac{T_B-\delta^1(T_B)}{\mu}[1-P_o(P_B)] \tag{17}$$

We ignore the primary throughput that may be achieved during the sensing period because t_s is assumed to be much smaller than T_{on} and T_{off}. The two terms of (17) correspond to the sensing outcomes of the channel being free and busy, respectively. The optimization problem can then be written as

Find: T_F, T_B, P_F and P_B

That maximize: $(1-\alpha)\bar{R}_s(T_F, T_B, P_F, P_B) + \alpha\bar{R}_p(T_F, T_B, P_F, P_B)$

Subject to: $T_F \geq 0$, $T_B \geq 0$, $0 \leq P_F \leq P_{max}$ and $0 \leq P_B \leq P_{max}$ (P1)

3.2 Soft Sensing

We now re-formulate the weighted sum throughput optimization problem assuming quantized soft sensing, where the sensing metric, from a matched filter or an energy detector for instance, is quantized before determining the power and duration of transmission. Let γ be the sensing metric with the known conditional pdf's: $f_o(\gamma)$ given that the primary is in the idle state and $f_1(\gamma)$ conditioned on the primary transmitter being active. We assume that the number of quantization levels is $S+1$. The kth level extends from threshold γ_{k-1}^{th} to γ_k^{th} assuming that $\gamma_0^{th} = 0$ and $\gamma_{S+1}^{th} = \infty$. The probability that the metric γ is between γ_{k-1}^{th} and γ_k^{th} when the primary channel is free is given by

$$\begin{aligned}\epsilon_k &= \Pr\{\gamma_{k-1}^{th} \leq \gamma \leq \gamma_k^{th} \mid \text{channel is free}\} \\ &= \int_{\gamma_{k-1}^{th}}^{\gamma_k^{th}} f_o(\gamma)\,d\gamma\end{aligned} \tag{18}$$

where k = 1, 2, ... (S + 1). On the other hand, The probability that γ is between γ_{k-1}^{th} and γ_k^{th} when the primary channel is busy is given by

$$\upsilon_k = \Pr \{ \gamma_{k-1}^{th} \leq \gamma \leq \gamma_k^{th} \mid \text{channel is busy} \}$$

$$= \int_{\gamma_{k-1}^{th}}^{\gamma_k^{th}} f_1(\gamma)\, d\gamma \quad (19)$$

When γ is between γ_{k-1}^{th} and γ_k^{th}, the secondary transmitted power is P_k and the duration of transmission is T_k. The case of one threshold corresponds to the imperfect sensing case where the primary is assumed to be active when γ exceeds some threshold and inactive otherwise. The false alarm probability in this case is given by ϵ_2, whereas the miss detection probability is ϑ_1.

As in the perfect sensing case, the probability that the mth observation of the channel happens when the channel is free, denoted by π_m, can be calculated using Markovian property of the channel model.

$$\pi_m = \pi_m - 1 \sum_{k=1}^{S\pm1} \epsilon_k P^{00}(t_s + T_k) + (1 - \pi_m - 1) \sum_{k=1}^{S+1} \vartheta_k P^{00}(t_s + T_k) \quad (20)$$

At steady state, $\pi_m - 1 = \pi_m$ and the steady state probability of sensing the channel while it is free becomes

$$P^{ss} = \frac{\sum_{k=1}^{S+1} \vartheta_k P^{10}(t_s + T_k)}{1 - \sum_{k=1}^{S+1} \epsilon_k P^{00}(t_s + T_k) + \sum_{k=1}^{S+1} \vartheta_k P^{10}(t_s + T_k)} \quad (21)$$

The average time between sensing events is given by

$$\mu = P^{ss} \sum_{k=1}^{S+1} \epsilon_k (t_s + T_k) + (1 - P^{ss}) \sum_{k=1}^{S+1} \vartheta_k (t_s + T_k) \quad (22)$$

The mean secondary throughput averaged over the primary activity and the sensing metric is given by

$$\begin{aligned} \bar{R}_s = {} & P^{ss} \sum_{k=1}^{S+1} \epsilon_k \left[\frac{\delta^0(T_k)}{\mu} C_o(P_k) + \frac{T_k - \delta^0(T_k)}{\mu} C_1(P_k) \right] \\ & + (1 - P_{ss}) \sum_{k=1}^{S+1} \vartheta_k \left[\frac{\delta^1(T_k)}{\mu} C_o(P_k) + \frac{T_k - \delta^1(T_k)}{\mu} C_1(P_k) \right] \end{aligned} \quad (23)$$

The mean primary throughput is

$$rCl\bar{R}_p = r_o P^{ss} \sum_{k=1}^{S+1} \epsilon_k \frac{T_k - \delta^0(T_k)}{\mu} [1 - P_0(P_k)] + r_o (1 - P_{ss}) \sum_{k=1}^{S+1} \vartheta_k \frac{T_k - \delta^1(T_k)}{\mu} [1 - P_0(P_k)] \quad (24)$$

In this case, the optimization problem can then be written as

(P2)

Find: T_k and P_k, $k = 1,..., S + 1$

That maximize: $(1 - \alpha) \bar{R}_s(T_1, P_1, ..T_{S+1}, P_{S+1}) + \alpha \bar{R}_p(T_1, P_1, ..T_{S+1}, P_{S+1})$

Subject to: $T_k \geq 0, \forall k = 1,..., S + 1$

$0 \leq P_F \leq P_{max}, \forall k = 1,..., S + 1$

3.3 No Sensing

In the previous subsections, we investigated the problem of using potentially different transmit powers and transmission times based on the sensing outcome. The question arises as how much performance is lost if the secondary transmitter, given the channel gains, adopts a pure underlay strategy transmitting with only one power level and doing no sensing. Note that the optimization problem here is considerably more manageable than that corresponding to the perfect and soft sensing cases because we just have one optimization parameter, $\hat{p}$. The scheme of no sensing and continuous transmission is indeed important to demonstrate the gain in weighted sum throughput as a result of solving the more complex problem detailed in Subsections 3.1 and 3.2. In this case, the secondary throughput is given by

$$\bar{R}_s = uC_o(\hat{p}) + (1-u)\,C_1(\hat{p}) \tag{25}$$

which is proved in Appendix B. The primary throughput is given by

$$\bar{R}_p = r_o u\,[1 - P_o(\hat{p})] \tag{26}$$

The optimization problem can be formulated as

$$\begin{aligned}&\text{Find: } \hat{p}\\ &\text{That maximize: } (1-\alpha)\,R_s(\hat{p}) + \alpha R_p(\hat{p})\\ &\text{Subject to: } 0 \le \hat{p} \le P_{max}\end{aligned} \tag{P3}$$

We emphasize again that the no-sensing policy is a special case of perfect and soft sensing that emerges when the solution to the weighted sum throughput optimization problem yields power levels that are the same regardless of the sensing metric or the hardened sensing outcome. Take, for instance, the case of perfect sensing. If when the channel is sensed to be free, the optimal transmission power $P_F = \hat{p}$, and if when the channel is sensed to be busy, the optimal transmission power $P_B = \hat{p}$, then the optimal strategy for the secondary transmitter is to use $\hat{p}$ and to stop sensing. In time-varying environments, the situations that render the no-sensing scheme optimal will be transient. This actually applies to all sensing scenarios where the optimization problem to maximize weighted sum throughout should be re-solved when the channel and traffic parameters change significantly.

4. Secondary Link Capacity

For the secondary throughput we have used equations (9) and (10). In this section we derive an upper bound on the mutual information between the input and output of the channel between the secondary terminals, and show that it is equal to our expressions for secondary link throughput. We also derive a lower bound that is a maximum of one bit below the upper bound. The focus here is on the perfect sensing case. It is important to note that all mutual information expressions below are conditioned on the channel gains, which are assumed to be perfectly known at the secondary receiver.

4.1 Upper Bound

We consider a genie-aided secondary receiver with knowledge of the exact pattern of primary activity. We assume that the transmitter sends two codewords, both interleaved in time. One codeword is sent successively when the channel is sensed to be free, whereas the other is sent when the channel is sensed to be busy. The analysis of the mutual information between secondary channel input and output is the same for the two codewords with appropriate use of traffic and transmission parameters. Hence, we focus here on the codeword sent successively when the channel is sensed to be free. We further assume that the time parameters, such as T_F and $\delta^0(T_F)$ are all integer multiples of codeword inter-sample duration, τ_s, which is assumed to be very small relative to T_{on}, T_{off}, T_F *and* T_B. Assuming the codeword is sent over m blocks each composed of T_F/τ_s samples, the total number of codeword samples, n, is equal to mT_F/τ_s.

The average number of interference-free samples within a transmission block is equal to $\delta^0(T_F)/\tau_s$. Applying the law of large numbers as m goes to infinity, the number of samples in the codeword that suffer from primary user interference is $n\dfrac{T_F - \delta^0(T_F)}{T_F}$. Let $\bar{\delta}^0 = \delta^0(T_F)/T_F$. The number of different possible patterns for the primary activity is

$$S = \left(\frac{n}{n\,(1-\bar{\delta}^0)} \right)$$

where the notation $\binom{n}{k}$ means "*n choose k*."

Let $I\,(X_F^n\,;\,Y_F^n)$ be the mutual information between the input sequence of length n, X_F^n, to the secondary channel when the primary is sensed to be inactive, and the output Y_F^n. Mutual information $I\,(X_F^n\,;\,Y_F^n)$ is bounded by the mutual information conditioned on primary activity pattern $I\,(X_F^n\,;\,Y_F^n\,|\,s)$ [27]. That is,

$$I\,(X_F^n\,;\,Y_F^n) \le I\,(X_F^n\,;\,Y_F^n\,|\,s) \tag{27}$$

$$I\,(X_F^n\,;\,Y_F^n\,|\,s) = \sum_l P\,(s=l)\,I\,(X_F^n\,;\,Y_F^n\,|\,s=l) \tag{28}$$

where the summation is over the possible interference patterns. Since the number of samples that suffer from primary interference as m goes to infinity is the same for all possible activity patterns, the term $I\,(X_F^n\,;\,Y_F^n\,|\,s=l)$ is constant $\forall l$. Assuming Gaussian inputs,

$$I\,(X_F^n\,;\,Y_F^n\,|\,s=l) = \log \frac{\det(\sigma_s^2\,I_n + P_F\,g_{ss}\,I_n + P_p\,g_{ps}\,A_l)}{\det(\sigma_s^2\,I_n + P_p\,g_{ps}\,A_l)} \tag{29}$$

Matrix I_n is the $n \times n$ identity matrix, whereas A_l is an $n \times n$ diagonal matrix with ones in places corresponding to received samples during primary activity and zeros elsewhere. Recall that the number of zeros on the diagonal of A_l is $n\,\frac{\delta^0(T_F)}{T_F} = n\bar{\delta}^0$. Combining (27) and (29), we obtain

$$I\,(X_F^n\,;\,Y_F^n) \le n\delta^0\,\log\left(1+\frac{P_F\,g_{ss}}{\sigma_s^2}\right) + n\,(1-\bar{\delta}^0)\,\log\left(1+\frac{P_F\,g_{ss}}{P_p\,g_{ps}A+\sigma_s^2}\right) \tag{30}$$

4.2 Lower Bound

In this subsection we obtain a lower bound on the mutual information $I\,(X_F^n;\,Y_F^n)$ following the analysis in [27].

$$I\,(X_F^n\,;\,Y_F^n\,|\,s) - I\,(X_F^n\,;\,Y_F^n) = h\,(X_F^n\,|\,s) - h\,(X_F^n\,;\,Y_F^n\,|\,s) - h\,(X_F^n) + h\,(X_F^n\,|\,Y_F^n) \tag{31}$$

where $h\,(z)$ denotes the entropy of the random variable z. Given that the input is independent of the primary interference pattern, $h\,(X_F^n\,|\,s) = h\,(X_F^n)$ and

$$I\,(X_F^n\,;\,Y_F^n\,|\,s) - I\,(X_F^n\,;\,Y_F^n) = h\,(X_F^n\,;\,Y_F^n) - h\,(X_F^n\,;\,Y_F^n,\,s) = I\,(X_F^n\,;\,s\,|Y_F^n) = h\,(s\,|\,Y_F^n) - h\,(s\,|\,X_F^n\,|\,Y_F^n) \tag{32}$$

Since $h\,(s\,|\,X_F^n\,;\,Y_F^n) \ge 0$,

$$I\,(X_F^n\,;\,Y_F^n\,|\,s) - I\,(X_F^n\,;\,Y_F^n) \le h\,(s\,|Y_F^n) \le h\,(s) \tag{33}$$

$$I\,(X_F^n\,;\,Y_F^n) \ge I\,(X_F^n\,;\,Y_F^n|\,s) - h\,(s) \tag{34}$$

Note that the term $I\,(X_F^n\,;\,Y_F^n\,|\,s)$ is the upper bound on the mutual information, which is the right-hand-side of inequality (30). The term $h\,(s)$ represents the gap between the upper and lower bounds.

4.3 Link Capacity

When the channel is sensed to be free and codeword X_F^n is transmitted, the capacity conditioned on the channel gains is given by $I\,(X_F^n\,;\,Y_F^n)\,/\,n$ as $n \to \infty$. The upper bound on ergodic capacity, C_F^U, is obtained from (30) by averaging over the channel gains

$$C_F^U = \bar{\delta}^0\,C_o\,(P_F) + (1-\bar{\delta}^0)\,C_1\,(P_F) \tag{35}$$

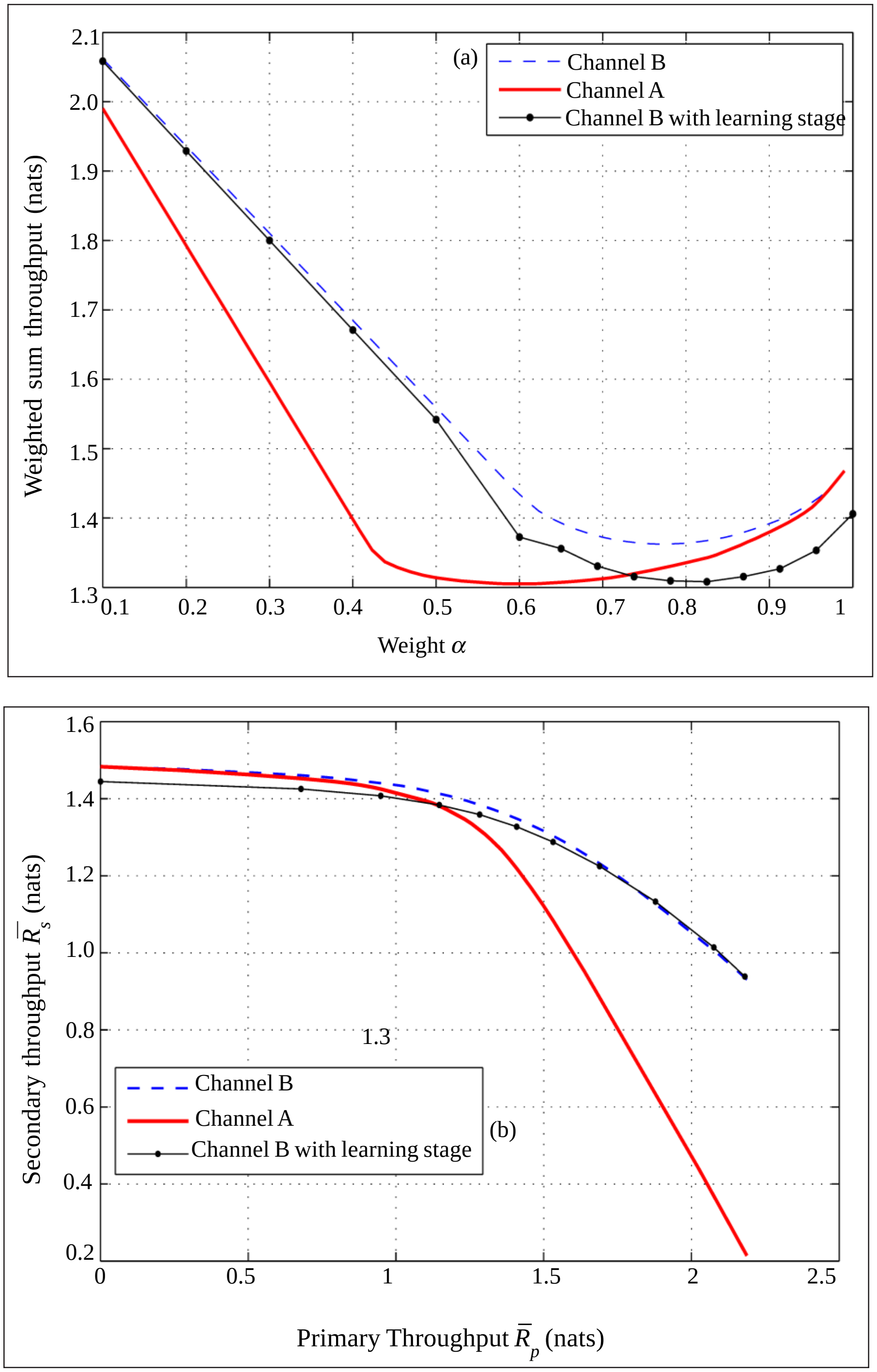


Figure 3. (a) Perfect sensing weighted sum throughput versus weight α for channels **A** (g_{sp}= 2) and **B** (g_{sp}= 0.2). (b) Rate region, which is the secondary throughput $\bar{R}_s$ vs. primary throughput $\bar{R}_p$ for channels **A** and **B**. Both plots also include the weighted sum throughout and rate region for channel **B** when the traffic parameters are learned.

where $C_o(P_F)$ and $C_1(P_F)$ are given by (9) and (10), respectively.

The lower bound on capacity depends on the entropy of the interference pattern $h(s)$. From (34), the capacity of the channel when sensed to be free and X_F^n is transmitted is

$$C_F \geq C_F^U - D_F \tag{36}$$

where D_F represents the gap between the upper and lower bound of capacity of the secondary link when the channel is sensed to be free. The least lower bound can be obtained by maximizing $h(s)$ assuming that all interference patterns are equally likely. Hence D_F is given by

$$D_F = \lim_{n \to \infty} \frac{h(s)}{n} = \lim_{n \to \infty} \frac{1}{n} \log \left(\frac{n}{n(1-\overline{\delta^0})} \right) \tag{37}$$

Applying Stirling's approximation to the previous equation and taking the limit, we obtain

$$D_F = H(\overline{\delta^0}) \tag{38}$$

where $H(z) = -z \log z - (1-z) \log(1-z)$. The maximum value of D_F occurs when $\overline{\delta^0} = 0.5$ and is equal to one bit. Doing the same steps when the channel is sensed to be busy and the second codeword X_B^n is transmitted with power P_B, the upper bound on the capacity, denoted by C_B^U, has the exact expression as (35) replacing P_F by P_B, and $\overline{\delta^0}$ by $\overline{\delta^1} = \delta^1(T_B)/T_B$. If the output of the channel at the secondary receiver is Y_B^n when X_B^n is sent, the capacity of the channel, C_B^n, when the channel is sensed to be busy is given by

$$C_B \geq C_B^U - D_B \tag{39}$$

where D_B, assuming equally likely interference profiles, is equal to $H(\overline{\delta^1})$.

The channel capacity C is then

$$\begin{aligned} C &= P^{ss} \frac{T_F}{\mu} C_F + (1-P^{ss}) \frac{T_B}{\mu} C_B \\ &\geq P^{ss} \frac{T_F}{\mu} C_F^U + (1-P^{ss}) \frac{T_B}{\mu} C_B^U - D \end{aligned} \tag{40}$$

where the gap D between the upper and lower bounds on ergodic capacity is given by

$$D = P^{ss} \frac{T_F}{\mu} D_F + (1-P^{ss}) \frac{T_B}{\mu} D_B \tag{41}$$

We provide in Section 5 a graph showing the dependence of the gap D on the utilization factor u. Note that the upper bound $P^{ss} \frac{T_F}{\mu} C_F^U + (1-P^{ss}) \frac{T_B}{\mu} C_B^U$, after minor manipulation, is the expression (16) used for secondary throughput in the case of perfect sensing.

5. Numerical Results

In this section we present simulation results for the perfect, soft sensing and no sensing schemes. The weighted sum rate maximization problem is non-convex, hence, we do exhaustive search to obtain the optimal parameters. The parameter range of values is finely discretized and the global optimal is obtained via exhaustive numerical search. Since the exhaustive search is infeasible for a large number of parameters, a gradient descent algorithm can be employed to find the solution. However, there would be no guarantees that the obtained solution is the global optimum. All the results presented in this section are obtained via exhaustive numerical search.

In addition, we try to elaborate the impact of this joint optimization for the powers and transmission times as well as the reward in terms of rate from allowing simultaneous transmission of the primary and the secondary. In the simulations the parameters used are: $T_{on} = 4$, $T_{off} = 5$, $t_s = 0.05$, $r_o = 4.5$ nats, $\sigma_s^2 = \sigma_p^2 = 1$, $P_p = 100$, $P_{max} = 10$, $\overline{g_{ss}} = 2$, $\overline{g_{pp}} = 3$, and $\overline{g_{ps}} = .03$. In order to do the exhaustive search, we have imposed an artificial upper bound on transmission time equal to 20 units of time. The parameters for channels **A** and **B** used in the analysis are the same except for average channel gain between secondary transmitter and primary receiver $\overline{g_{sp}}$. We assume $\overline{g_{sp}}$ is equal to 2 for channel **A**, 0.2 for channel **B**. Finally, we present results pertaining to the capacity gap explained in Section 4.

5.1 Sensing-based

5.1.1 Perfect Sensing

We report here some results for the perfect sensing case. In Figure 3a, the weighted sum throughput versus α is shown for channels **A** and **B**. Though in the cognitive context α would be closer to one, we present the results for small α's for completeness. The rate region depicting the variation of secondary with primary throughput is provided in Figure 3b. Note the enhanced throughput of channel **B** relative to channel **A** as a result of the lower $\overline{g_{sp}}$ value. We also include here the curve for the weighted sum throughput for channel **B** when the traffic parameters λ_{on} and λ_{off} are estimated during a learning phase. For this curve, the learning parameters (explained in Section 2) are $L = 25$ and $\tau_L = 0.5$. The weighted sum throughput curve for the learning case is obtained via averaging over 100 simulation runs. It is clear from the figure that there is a degradation in weighted sum throughput due to the uncertainty regarding the traffic parameters. As we have emphasized earlier, learning is not the main focus of this paper, but will be the subject of future investigation.

The optimal transmission power and time parameters for channel **A** are given in Figure 4. For small α values, the secondary transmitter transmits at full power P_{max} whether the channel is sensed to be free or busy. The transmission time for both sensing outcomes are the maximum possible. Recall that this maximum is artificial and is imposed by the exhaustive search solution. As we have previously explained regarding the no-sensing scenario, if the optimal $P_F = P_B$, then sensing becomes superfluous because the exact same power would be used to transmit regardless of the sensing outcome. In this situation we get the no-sensing solution even if we solve the full optimization problem to obtain the sensing-dependent parameters. As α increases, the power transmitted when the channel is sensed to be busy is reduced below Pmax. In addition, the transmission times are reduced for more frequent checking of primary activity. As α approaches unity indicating exclusive emphasis on primary throughput, the secondary transmitter is turned off and the channel is not sensed. Figure 5 gives the optimal transmission parameters for channel **B**. It is evident from the figure that as the level of interference from secondary transmitter to primary receiver is decreased, PB becomes lower than Pmax at a higher α compared to channel **A**.

In Figure 6 we compare our scheme, giving the secondary user the ability to transmit even if the channel is sensed to be in busy state, with the traditional overlay scenario where the secondary transmitter remains silent when the primary occupies the channel. The figure highlights the gain in the weighted sum throughput for small values of α and for both channels **A** and **B**. It is evident that the mean value of the cross channel gain $\overline{g_{sp}}$ is the key parameter controlling the amount of throughput gain. Figure 6 illustrates that, for limited interference to the primary receiver represented by channel **B**, the objective function shows enhanced weighted sum throughput when the secondary terminal is allowed to transmit during the busy primary state. For higher values of interference to the primary receiver represented by channel **A**, the optimal power policy when the channel is sensed to be busy approaches the conventional overlay model with PB = 0. Note that if the secondary transmitter is given the freedom to transmit at a higher maximum level, the throughput gap between the two models increases in favor of ours.

5.1.2 Soft Sensing

For the soft sensing case, the optimization parameters, mentioned in details in Section 3.2, are $2(S + 1)$ transmission powers and times corresponding to each quantization level. There are also S thresholds defining the boundaries of the quantization levels, which are optimization parameters as well. This makes a total of $3S + 2$ optimization parameters. If the primary is inactive, the signal at the secondary transmitter/sensor is due to noise only. Without loss of generality, we use the following chi-square distribution for likelihood function of the received signal at the secondary transmitter given that the primary is off

$$f_o(\gamma) = \frac{M^M \gamma^{M-1} \exp(-M\gamma)}{(M-1)!} \tag{42}$$

where γ here is the average energy of M samples. This distribution results fromuse of a conventional in-phase/quadrature (I/Q) receiver [28] with the noise variance normalized to unity. For the likelihood function given that the primary is active, we examine two cases. The first case is that of a fixed sensing channel gain between the primary transmitter and the secondary transmitter. The likelihood in this case is given by the following non-central chi-square distribution

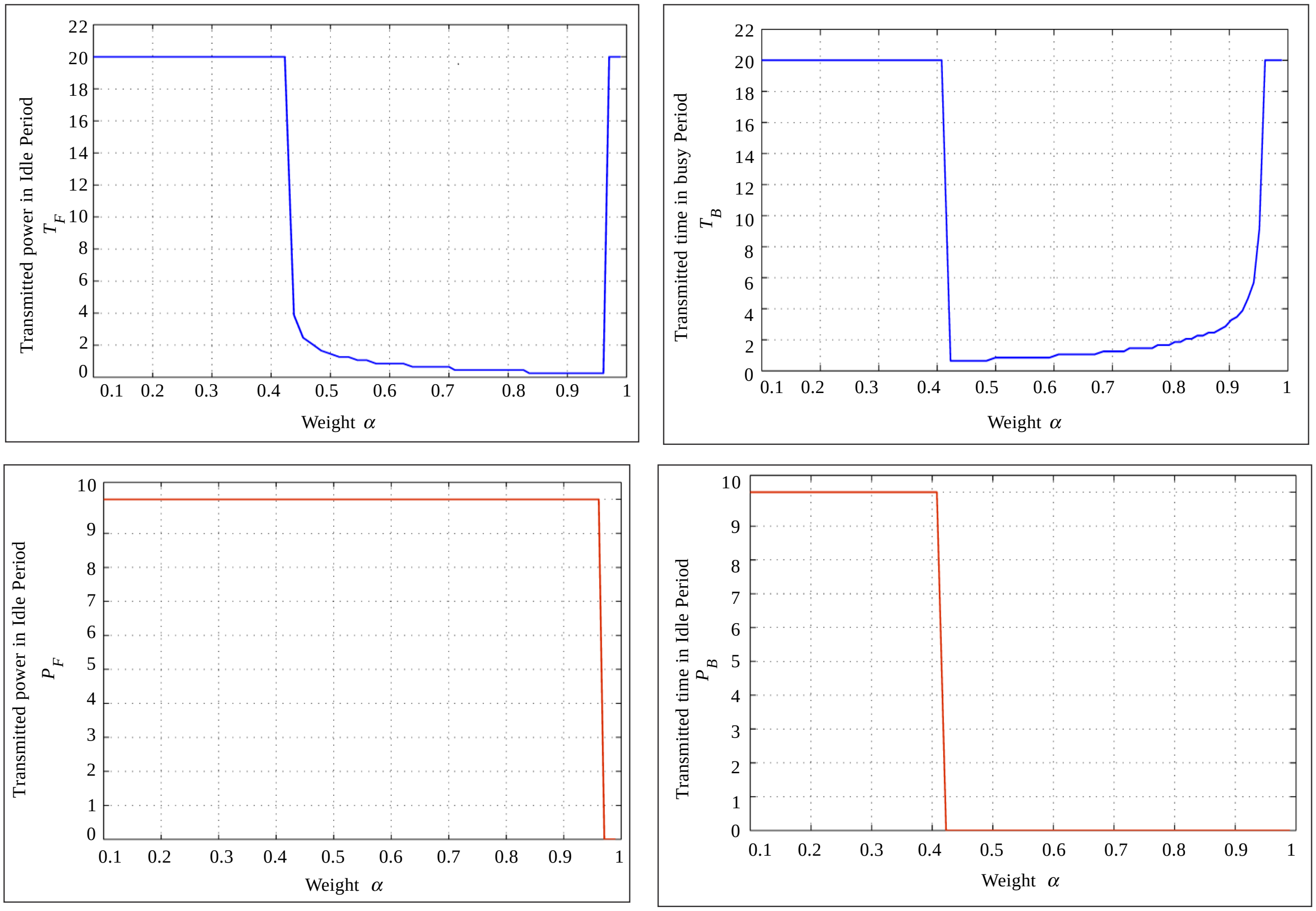


Figure 4. Perfect sensing power and transmission time results for channel **A** ($\overline{g_{sp}} = 2$)

$$f_1(\gamma) = M\left(\frac{\gamma}{\gamma_o}\right)^{\frac{M-1}{2}} exp\,(-M[\,\gamma + \gamma_o\,])\, I_{M-1}(2M\sqrt{\gamma\gamma_o}) \tag{43}$$

where I_{M-1} is the $(M-1)th$ order modified Bessel function, and γ_o is the average signal energy. We also consider a Rayleigh fading sensing channel. The likelihood function of the average energy of the received M samples can then be obtained by averaging (43) over the fading gain pdf. In this case, the conditional distribution is

$$f_1(\gamma) = \frac{M^M \gamma^{M-1} \exp\left(\dfrac{M\gamma}{1+\gamma_o h}\right)}{(1+\gamma_o \bar{h})^M (M-1)\,!} \tag{44}$$

where $\bar{h}$ is the average sensing channel gain. The results for one and two thresholds with $M = 1$ are presented in Figure 7 which shows the weighted sum throughput using one and two thresholds for channel **B**. For the fixed gain sensing channel, parameter $\gamma_o = 3$, whereas for Rayleigh fading sensing channel, $\eta = 1/3$ where $\eta = \dfrac{M}{1+\gamma_o \bar{h}}$. The two-threshold scheme shows slight improvement for the weighted sum rates over a range of α values above $\alpha = 0.6$. The figure also shows that a lower weighted sum throughput is achieved when the sensing channel is Rayleigh. This is expected as a fixed gain sensing channel is more reliable than a fading channel with the same average received energy. Figure **??** gives the optimal threshold when using two quantization intervals as a function of α and for $\gamma_o = 3$. As is evident from the figure, the optimal threshold decreases with α. Recall from Subsection 3.2 that the one threshold case corresponds to imperfect sensing with a false alarm probability of ϵ_2 and a miss

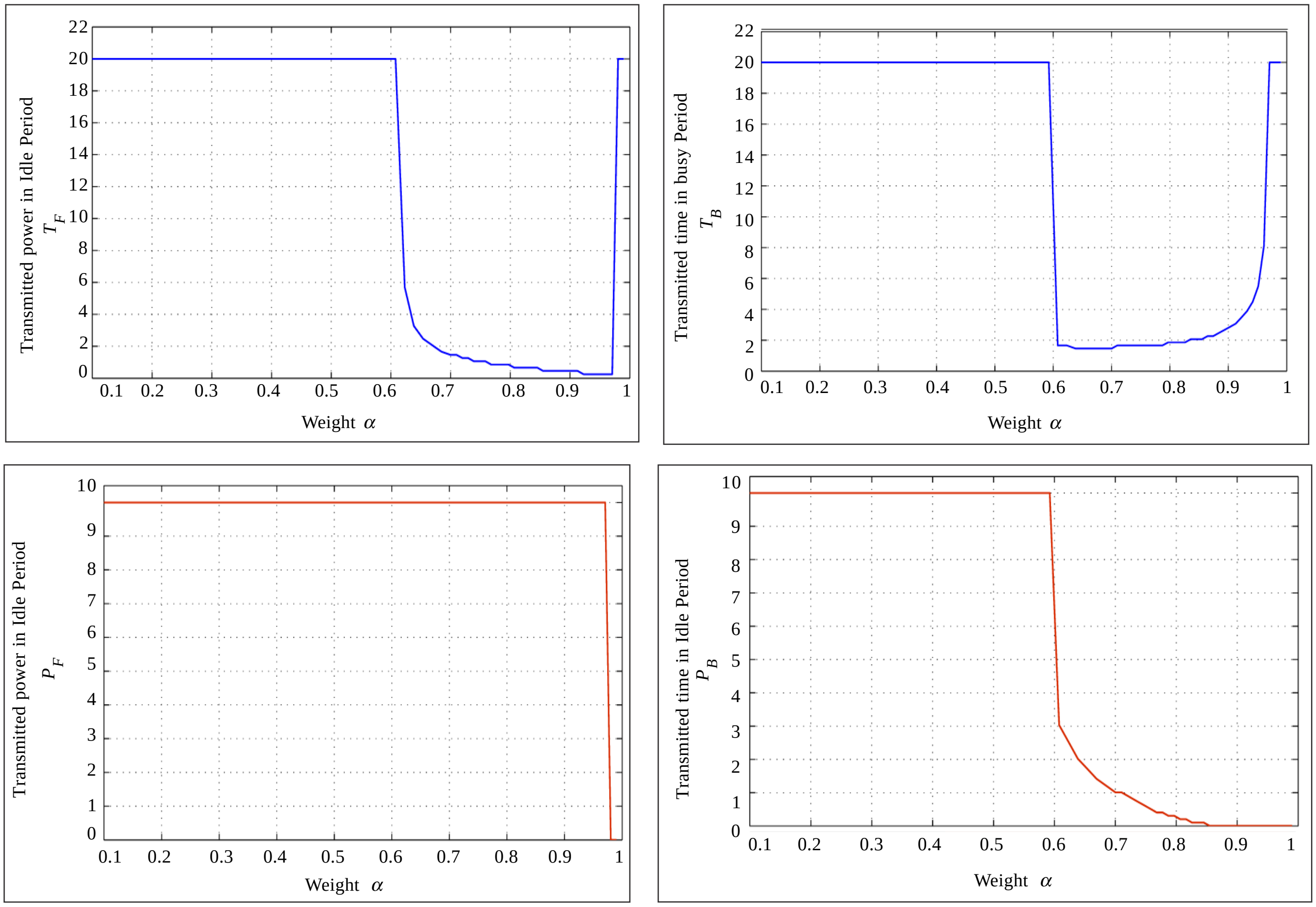


Figure 5. Perfect sensing power and transmission time results for channel **B** ($\bar{g}_{sp} = 0.2$)

detection probability of υ_1. Under this imperfect sensing interpretation, the trend of the threshold versus weight α can be explained as follows. When α increases putting more emphasis on the primary rate, the required false alarm probability is increased while the miss detection probability is decreased to reduce the chance of collision with the primary user.

Another question arises as to how much performance is lost if we solve the weighted sum throughput optimization problem assuming we have no uncertainty about the sensing decision while in fact sensing is not perfect. The motivation here is that if the loss in throughput is tolerable, the secondary terminal may solve the easier perfect sensing problem that has only four parameters. Figure **??** provides the weighted sum throughput for channel **B** when the soft sensing optimization problem with one threshold is solved versus the throughput obtained by solving the perfect sensing problem and using its optimal parameters in the throughput formula for soft sensing with one threshold. Note that since the soft sensing problem has an additional parameter, which is the threshold, we optimize the throughput for this parameter given the power and transmission time results from perfect sensing problem. The figure shows a loss in performance above about $\alpha = 0.6$. This demonstrates the tradeoff between throughput performance and computational complexity needed to obtain the optimal transmission policy. As is evident from the figure, this tradeoff depends on the value of α at which the system is operated in order to guarantee a certain quality of service for primary link. From the above investigation, we can state the following. When the sensing channel gain is low in value, sensing becomes highly unreliable. The low gain means that the "*distance*" between the likelihood functions corresponding to the primary on and off states is small. (The distance between two probability distributions can be quantified using Kullback ÝU-Leibler divergence, Kolmogorov ÝU-Smirnov statistic, deflection coefficient, etc.) Whether one or more thresholds are used in this case, the performance in terms of weighted sum throughput is almost the same. On the other hand, if the sensing quality is very high and the likelihood functions given that the primary is on and off are "*far apart*", then sensing is needed and yields reliable results. Since in this case the distance between the likelihood functions is large, the threshold position is not very critical

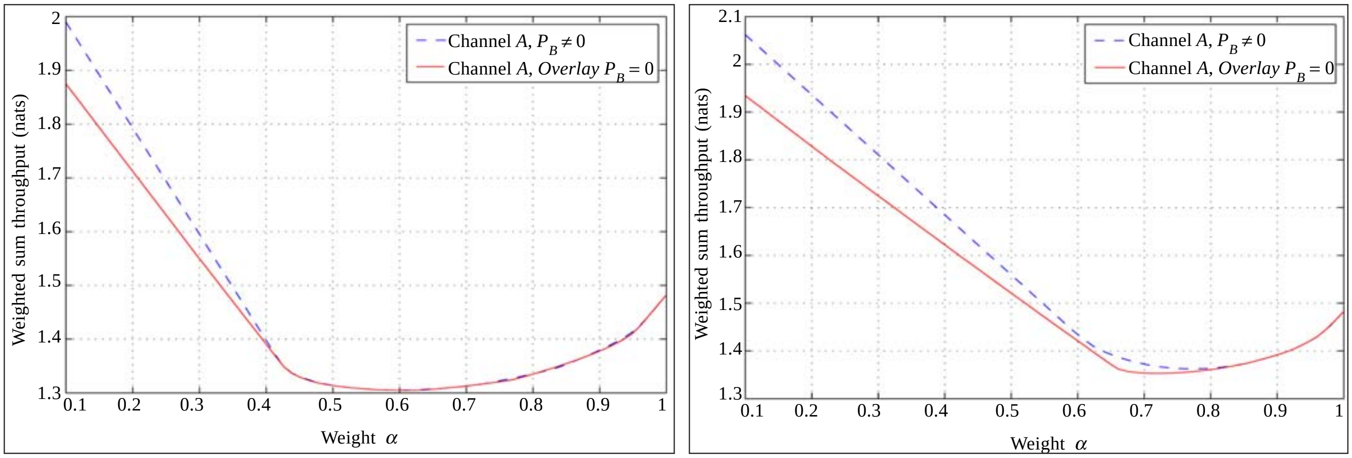


Figure 6. Comparing perfect sensing weighted sum throughput and rate regions using conventional overlay scheme with $P_B = 0$ with that achieved by allowing simultaneous primary and secondary transmissions. Comparison is done for channel **A** and **B**.

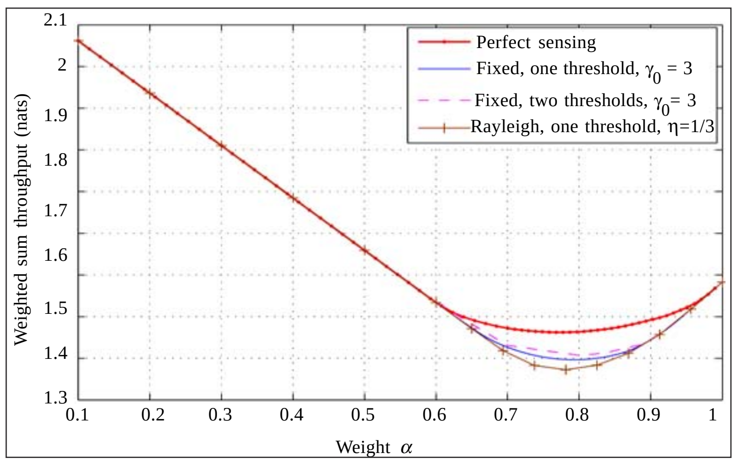


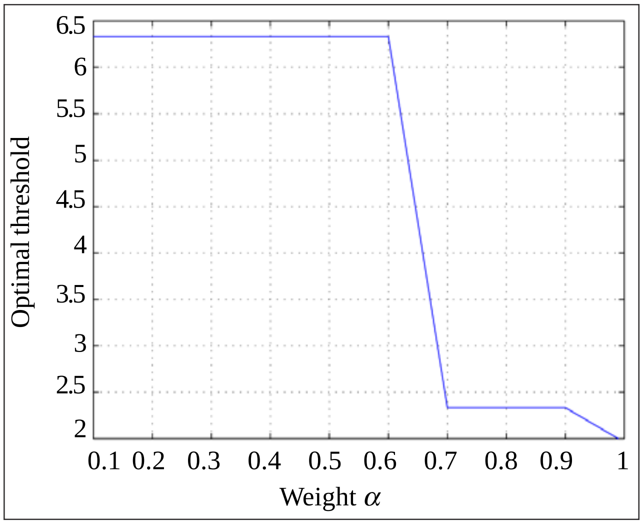
Figure 7. Soft sensing weighted sum throughput versus α using one and two thresholds for channel **B**. The result from perfect sensing is provided for comparison. The weighted sum throughput for channel **B** using one threshold and assuming a Rayleigh fading sensing channel is also included.

in determining the performance of the system. That is, the threshold can take a range of values that all produce almost the same weighted sum throughput. Our preliminary conclusion is that soft sensing may be important and beneficial in the cases between these two extremes. More investigation is needed to specify in quantitative terms the region where soft sensing complexity pays off as a significant improvement in the weighted sum throughput.

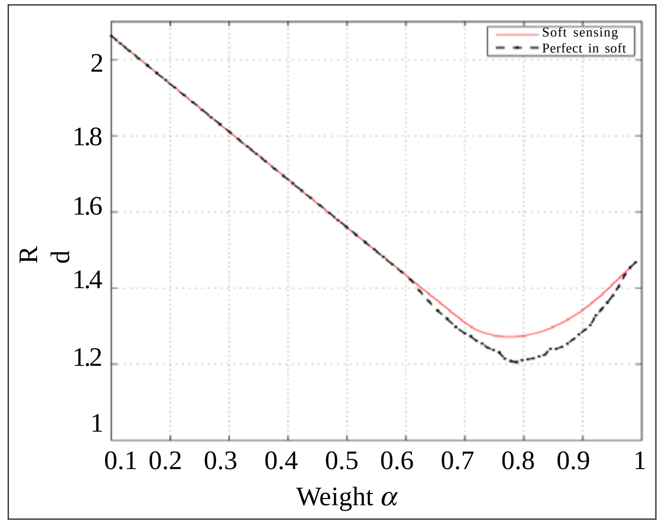
5.2 Sensing vs. no-sensing

Figure 8a shows the weighted sum throughput versus α and the corresponding rate regions for channel **B**. The result from perfect sensing acts as an upper bound on the throughput. It is expected that the no sensing scheme has a worse performance relative to the case when sensing is employed. However, it is interesting to point out that their exist situations where no sensing behaves almost the same as revisiting the channel to sense it. This is clear from both the weighted sum throughput of the network in Figure 8a and the rate region in Figure 8b. This case is usually for very limited interference from secondary to primary receiver when there is no need to waste time to sense unless the channel conditions have changed. Combining the results from this and the preceding subsections:

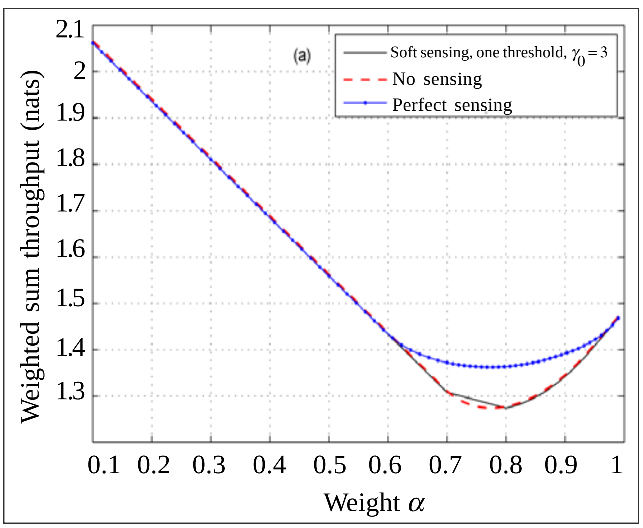
- When the mean channel gain between the secondary transmitter and primary receiver, $\overline{g_{sp}}$, is high, sensing is important. The



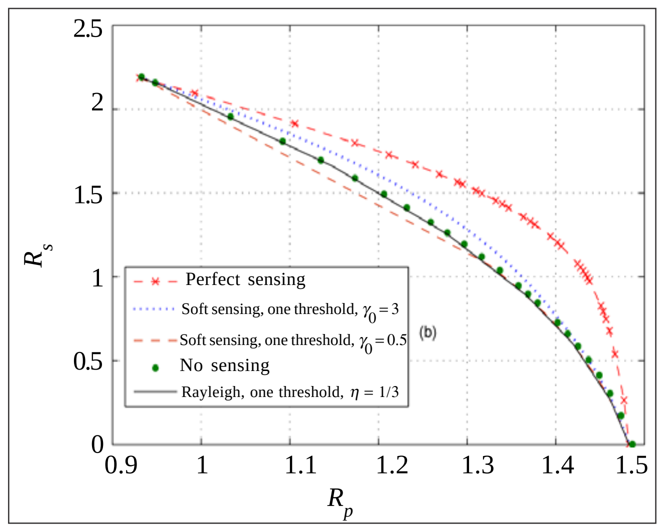
(a) Soft sensing optimal threshold when using two quantization intervals for channel **B**. The non-smooth nature of the curve results from the coarse discretization of α and threshold values.



(b) Comparison between the weighted sum throughput when soft sensing with one threshold is used and when the transmission parameters are obtained via solving the perfect sensing problem. The latter case is dubbed "*perfect in soft*" in the legend. These results are for channel **B**.



(a)Weighted sum throughput versus comparing sensing versus no sensing schemes for channel **B**.



(b) Secondary throughput $\overline{R}_s$ vs. primary throughput $\overline{R}_p$ comparing sensing versus no sensing schemes for channel **B**.

Figure 8. Sensing versus no-sensing weighted sum throughput and rate regions for channel **B**

perfect sensing acts as an upper bound on the throughput. It is expected that the no sensing scheme has a worse performance power transmitted when the channel is found busy approaches zero as in the conventional overlay schemes. Soft sensing may provide throughput gains depending on the sensing channel between the primary transmitter and the secondary transmitter/ sensor.

• As $\overline{g_{sp}}$ decreases, it is better, from a weighted sum throughput point of view, to allow the secondary terminal to transmit even when the channel is sensed to be busy. Again, soft sensing may be employed depending on the sensing channel.

• If $\overline{g_{sp}}$ is very low, the optimal transmit power when the channel is sensed to be busy approaches that when the channel is found

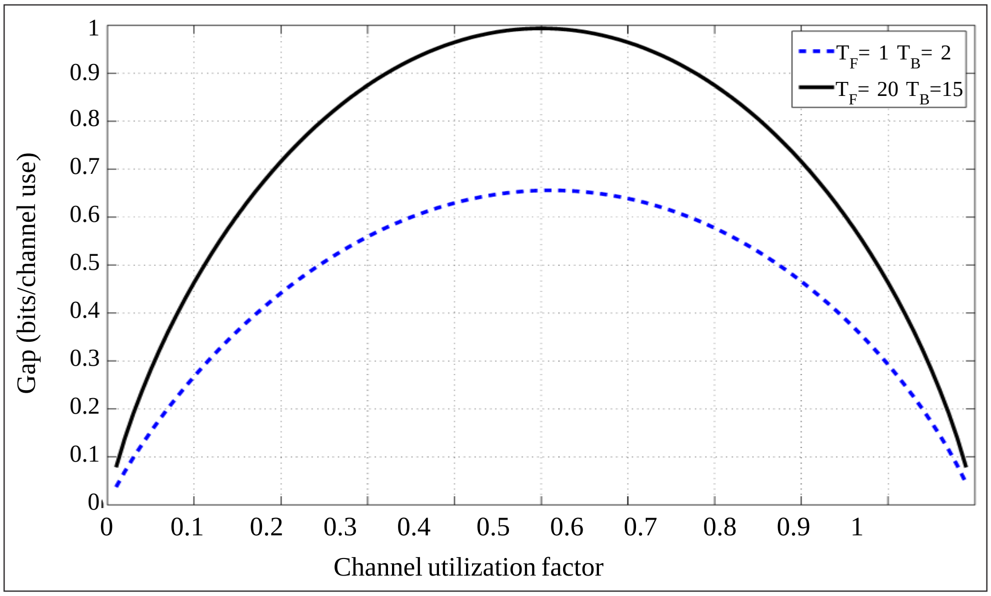


Figure 9. Gap D between lower and upper bounds as a function of u. For the case T_F = 20 and T_B = 15, the gap is almost $H(u)$. For T_F = 1 and T_B = 2, the gap is smaller.

free. This indicates that a no sensing strategy, which is equivalent to the conventional underlay scheme, would be optimal. Our weighted sum throughput objective function transmit power with a constraint on the minimum acceptable primary rate.

Note that Figure 8b can be used to obtain the weight needed for a certain constraint on the primary throughput. The rate region is obtained by sweeping the weight from 0 to 1 obtaining for each weight the achievable primary and secondary throughputs. If the primary throughout is required to exceed 1.4 nats and soft sensing with one threshold is employed, then the secondary system operates with the weight corresponding to the point on the curve of the implemented sensing mode with the desired primary throughput.

5.3 Secondary link capacity

We provide here some numerical results for the gap between the upper and lower bounds on the capacity derived in Section 4. We use T_{off} = 5 units of time and sweep the value of u from 0 to 1. We present here two cases corresponding to small and large T_F and T_B, namely, T_F = 1 and T_B = 2 for first case, and T_F = 20 and T_B = 15 for the second. It is shown in Figure 9 that the gap is lower than $H(u)$ for small values of transmission times. On the other hand, when T_F and T_B are large in value, the gap is exactly equal to $H(u)$. It can be easily shown that both $\overline{\delta^0}$ and $\overline{\delta^1}$ converge to $1 - u$ as T_F and T_B go to infinity. The maximum of the gap in this case is 1 bit per channel use when $u = 0.5$.

6. Conclusion

We have investigated the problem of specifying transmission power and duration for a cognitive terminal operating with a primary link that follows an unslotted mode of operation. We have used an upper bound for the secondary throughput, justified it on information-theoretic grounds, and provided also a lower bound. The optimal secondary transmission power and duration that maximize a weighted sum of the primary and secondary throughputs are obtained numerically. Our results have shown that an increase in the overall weighted throughput can be obtained by allowing the secondary to transmit even when the channel is found to be busy.

We have extended our formulation to the soft sensing case where the decision of the secondary transmission power and duration depends on the quantized value of the sensing metric, rather than on the binary decision of whether the channel is free or not. Our preliminary results, however, show that the gain of using this scheme, and for the range of parameters we have simulated, is marginal. The throughput gain from soft sensing depends on the "*distance*" between the likelihood functions of the received primary signal at the secondary transmitter. If the distance is small, sensing becomes unreliable and indeed a no-sensing strategy may perform almost as good as soft sensing. If the distance is large, sensing is beneficial for the system but using more than one threshold yields marginal throughput gains.

The benefit of soft sensing is discernible in the cases between these two extremes. Further investigation is required to identify the range of system parameters for which soft sensing produces considerable gains in throughput. Future work may also address the exact evaluation of $h(s)$, discussed in Section 4, given the considered renewal model for primary activity. This may lead to more technically sound expressions for secondary throughput with proven achievability.

Appendix

∫ We provide here expressions for primary outage probability in the presence of secondary transmission, and the ergodic capacities of the secondary in the absence and presence of primary transmission. Assuming the channel gains g_{pp} and g_{sp} are independent and exponentially distributed with means $\overline{g_{pp}}$ and $\overline{g_{sp}}$, the outage probability (8) can be written as

$$P_o(p) = \Pr\left\{ r_o > \log\left(1 + \frac{a\, g_{pp}}{b\, g_{sp} + 1}\right)\right\}$$

$$= 1 - \frac{P_p\, \overline{g_{pp}}}{P_p\, \overline{g_{pp}} + Pc\, \overline{g_{sp}}} \exp\left(-\frac{c\sigma_p^2}{P_p\, \overline{g_{pp}}}\right) \quad (45)$$

where $a = P_p/\sigma_p^2$, $b = p/\sigma_s^2$ and $c = \exp(r_o) - 1$. Given an exponential distribution for g_{ss} with mean $\overline{g}_{ss}$, the ergodic capacity when the primary is off, (9) becomes

$$C_o(p) = \int_0^\infty \log\left(1 + \frac{pg_{ss}}{\sigma_s^2}\right)\frac{1}{\overline{g}_{ss}} \exp\left(-\frac{g_{ss}}{\overline{g}_{ss}}\right) dg_{ss} \quad (46)$$

Defining $\Psi(x) = \int_x^\infty \exp(-\mu)/\mu \, d\mu$, it is straightforward to show that

$$C_o(p) = \exp\left(\frac{\sigma_s^2}{p\overline{g}_{ss}}\right)\Psi\left(\frac{\sigma_s^2}{p\overline{g}_{ss}}\right) \quad (47)$$

Assuming that g_{ss} and g_{ps} are independent and have means $\overline{g}_{ss}$ and $\overline{g}_{ps}$, respectively, when $p\overline{g}_{ss} \neq P_p \overline{g}_{ps}$, the ergodic capacity when the primary is off, (10) can be expressed as

$$C_1(p) = \frac{p\overline{g}_{ss}}{p\overline{g}_{ss} - P_p\overline{g}_{ps}}\left[\exp\left(\frac{\sigma_s^2}{p\overline{g}_{ss}}\right)\Psi\left(\frac{\sigma_s^2}{p\overline{g}_{ss}}\right) - \exp\left(\frac{\sigma_s^2}{P_p\overline{g}_{ss}}\right)\Psi\left(\frac{\sigma_s^2}{P_p\overline{g}_{ss}}\right)\right] \quad (48)$$

In the case $p\overline{g}_{ss} = P_p\overline{g}_{ps}$,

$$C_1(p) = 1 - \frac{\sigma_s^2}{p\overline{g}_{ss}} \exp\left(\frac{\sigma_s^2}{p\overline{g}_{ss}}\right)\Psi\left(\frac{\sigma_s^2}{p\overline{g}_{ss}}\right) \quad (49)$$

B

We provide here a proof for the expressions (25) and (26) of primary and secondary throughput when no sensing is employed and the secondary terminal transmits continuously using a certain power level. This scheme can be viewed as the perfect sensing scheme with $P_F = P_B = \hat{p}$ and $T_F = T_B = \infty$. From equations (4) and (5), for large t

$$P^{00}(t) = P^{10}(t) = 1 - u \quad (50)$$

Using this in the expression (12) for P^{ss}, we get $P^{ss} = 1 - u$. Similarly, for large t, using expression (14) and (15) we get $\delta^1(t) = \delta^0(t) = (1 - u)\, t$. Expressions (25) and (26) can then be readily obtained. For example, the factor multiplied by $C_o(\hat{p})$ in (25) is

$$P^{ss} \frac{\delta^0(t)}{t} + (1 - P^{ss}) \frac{\delta^1(t)}{t} = 1 - u \quad (51)$$

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