# Sparse Adaptive Channel Estimation Based on Discrete Fourier Transform

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***ABSTRACT:*** *In communications, compressive sensing is largely accepted for sparse channel estimation and its variants. The key advantage of compressed sensing lies in its reconstruction algorithm recovering the original high-dimensional sparse data from low dimensional data. However, the algorithms will be in week effectiveness when the sparsity is unknown. In this article, a novel sparse adaptive channel estimation algorithm based on discrete Fourier transform (DFT) was proposed. It performed preliminary estimation by using discrete Fourier transform matrix as the observation matrix and inverse discrete Fourier transform to the received data at the receiver. It is found that the local peak indexes of this preliminary estimation results correspond with the non-zero taps of the sparse channel. After the non-zero taps were located, least square (LS) algorithm was used to estimate the channel impulse response (CIR). Mathematical analysis and simulation results have shown that the proposed algorithm (discrete Fourier transform based-least square, DFT-LS) outperform than existed multiple tracking algorithms such as sparsity adaptive matching pursuit (SAMP) and orthogonal matching pursuit (OMP), and are with lower complexity as well.*

**Keywords:** Sparse Channel, Sparse Adaptive, Discrete Fourier Transform, Least Square

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## 1. Introduction

Compressed sensing theory is a signal processing theory which makes full use of sparse signal [1]. After compressed sensing theory has been proposed, there have emerged a lot of reconstruction algorithms. These algorithms can be divided into two categories: one is the linear programming method which is based on L1-norm minimization such as basis pursuit (Basis Pursuit, BP) algorithm [2] and another is greedy algorithm which is based on L0-norm minimization such as matching pursuit (Matching Pursuit, MP) algorithm [3]. The greedy algorithm is applied widely for its fast iterative speed and the algorithm complexity is much lower than the linear programming method. But it still needs large iterative processes. DFT-LS algorithm can reconstruct the signal in high accuracy without any iterative processes, thus reduce the complexity of channel reconstruction.

In addition, many greedy algorithms such as subspace pursuit (SP) [4] and compressive sampling matching pursuit (CoSaMP) [5] assume that the sparsity is known, whereas sparsity may not be available in many practical applications. When the sparsity is unknown, we need to find other stopping criterion instead of sparsity such as sparse adaptive matching pursuit (sparsity adaptive matching pursuit (SAMP)) algorithm [6]. A stopping criterion is given in literature [7]. However, the stopping criterion in [7] is not universal. In many cases a threshold is effective only in some scope. DFT-LS algorithm does not need to design complex threshold.

The rest of the paper is organized as follows. The second part describes the system model. The third part is devoted to detail descriptions of DFT-LS algorithm. The fourth part provides computer simulations results. Finally, conclusions are drawn in the sixth part.

## 2. System model

OFDM has been widely applied in wireless communication systems because it transmits at a high rate, achieves high bandwidth efficiency, and is relatively robust to multipath fading and delay. Suppose the channel between transmitter and receiver is a frequency-selective block fading channel whose impulse response is characterized by

$$h(\tau) = \sum_{i=0}^{L-1} h_i \delta(\tau - \tau_i) \tag{1}$$

Where the channel length is L. $h_i$ and $\tau_i$ are the complex amplitude gain and the delay in the $i$ th multipath [8]. If the number of non-zero taps in $h(\tau)$ is $s$ ($s << L$), then $h(\tau)$ is called s-sparse. The received signal in the frequency domain over a fading channel can be described as follows:

$$Y = XH + W = XFh + W \tag{2}$$

Where the received signal $Y$ is $[Y_0, Y_1, ..., Y_{K-1}]^T$ and the transmitted signal $X$ is diag $(X_0, X_1, ..., X_{K-1})$; $W$ is additive white Gaussian noise; let $F$ denote the discrete Fourier transform. The number of pilot subcarriers is $N_p$ and $P$ denotes a set of pilot subcarrier positions. Then on pilot subcarriers, (2) can be written as follows:

$$Y_p = X_p F_p h + W_p = \Phi h + W_p \tag{3}$$

Where $\Phi = X_p F_p$. Since the receiving end $Y_p$ and $\Phi$ are known signals, the receiving end use reconstruction algorithms to recover CIR.

## 3. Channel Estimation Algorithm Based on DFT-LS

### 3.1 Algorithm Description

Equation (3) is underdetermined because of $N_p < L$. $\Phi$ is actually a $N_p \times L$ discrete Fourier transform matrix. The singular value decomposition (SVD) of $\Phi$ is $\Phi = U\Sigma V^H$, where $U$ and $V$ are orthogonal, and, $\Sigma_{i,j}$ = diag $(\sigma_1,...,\sigma_R)$, $R = \min(N_p, L)$ with $\sigma_1 \geq ... \geq \sigma_R \geq 0$.

The $\sigma_i$ are called singular values. The $N_p$ columns of $U$ and the $L$ columns of $V$ are called the left-singular vectors and right-singular vectors of $\Phi$, respectively. Let $\Phi^\dagger = V\Sigma^\dagger_{i,j} U^H$, where $\Sigma^\dagger_{i,j}$ is defined by

$$\Sigma^\dagger_{i,j} = \begin{cases} \dfrac{1}{\Sigma_{i,j}} & \text{if } i = j \text{ and } \Sigma_{i,j} \neq 0 \\ 0 & otherwise \end{cases}$$

Matrix $\Phi^\dagger$ satisfies the conditions:

$$\Phi\Phi^\dagger\Phi = \Phi$$
$$\Phi^\dagger\Phi\Phi^\dagger = \Phi^\dagger$$
$$(\Phi^\dagger\Phi) = \Phi^\dagger\Phi$$

So $\Phi^\dagger$ is minimal norm generalized inverse of $\Phi$. We use equation (3) left-multiplied by, then we can get

$$\overline{h} = \Phi^\dagger (Y_P - W_P) \tag{4}$$

Equation (4) is preliminary estimate of $\overline{h}(\tau)$ .

The result $\overline{h}$ of equation (3) is shown in figure 1(b) and figure 1 (a) is CIR of. It can be seen from figure 1 that equation (3) cannot reconstruct $h(\tau)$ but the local higher peak position of $h(\tau)$ and $\bar{h}$ is the emergence of correspondence. That because $\bar{h}$ is the minimal norm solution of equation (3). So we can find the non-zero taps index of $h(\tau)$ by $\bar{h}$. Then equation (3) becomes an over determined problem. Finally we can use LS algorithm to recover $h(\tau)$. The proposed DFT-LS algorithm can be divided into the following steps:

| |
|---|
| Input: Sampling matrix $\Phi$, Sampled vector $Y_P$;<br>Output: A K-sparse approximation $\hat{h}$ of the input signal; |
| Initialization:<br>$\hat{h} = 0$ {Trivial initialization}<br>$S = \varnothing$ {Empty finalist}<br>$h = \Phi^{\dagger}(Y_P - W_P)$ {Preliminary Estimate}<br>Identify the local peak index of $h$ and store in a vector $S$.<br>Output: $\hat{h}_s = (\Phi_S^T \Phi_S) - 1\, \Phi^T Y_P$ {find the channel non-zero taps coefficients} |

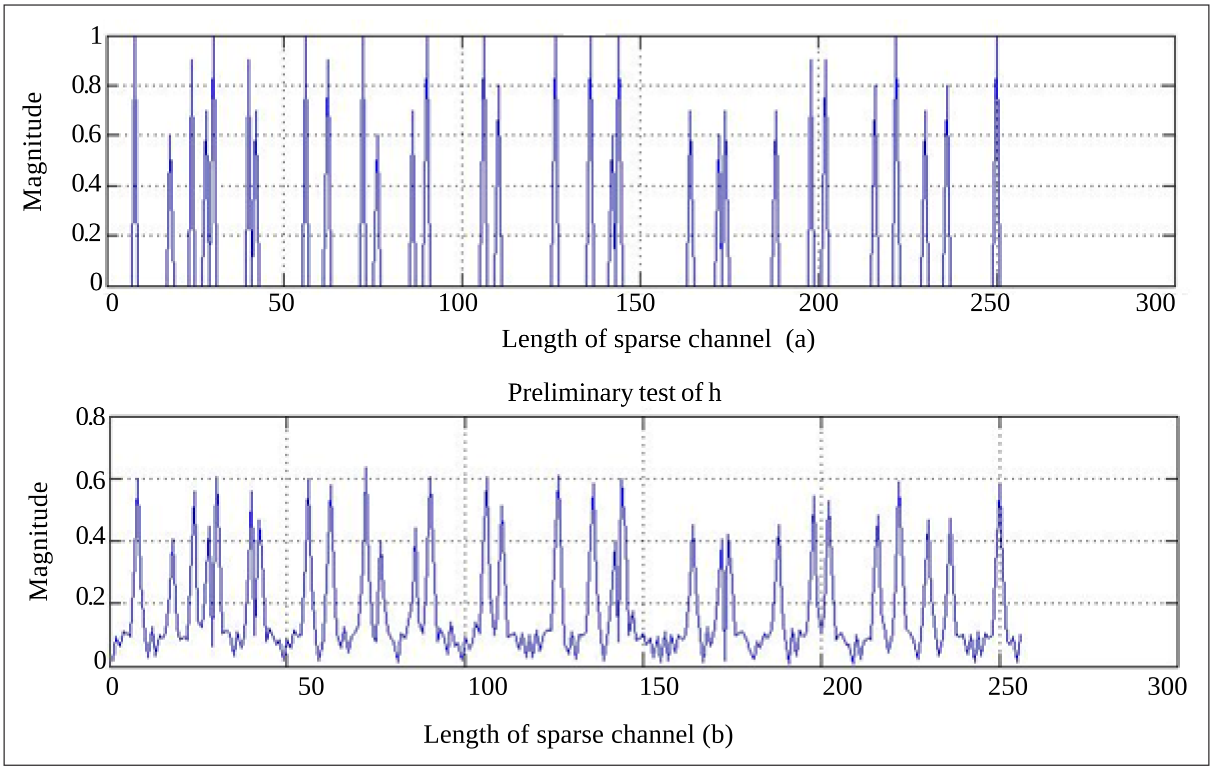


Figure 1. Preliminary estimate of $h$

Pay attention that when we identify the local peak index of $h(\tau)$ by $\bar{h}$, only the higher local peak represents the channel non-zero taps, so we can set a threshold to remove the smaller local peaks index.

**2.3 Algorithm analysis**

Reconstruction algorithm is the core content of compressed sensing. Compressive sensing-based SAMP algorithm has been considered as an effective method [6]. Here we want to point out that SAMP provides a generalized framework for OMP and SP. Note that when step size is 1, SAMP can be roughly regarded as the (generalized) OMP associated with refinement feature that can remove bad coordinates during iterations. In this case, the SAMP is always more accurate than the OMP although it may require a few more iterations to achieve that accuracy. In addition, when step size is $s$, SAMP becomes exactly SP if the restricted isometry property (RIP) condition of measurement matrix is satisfied. Reconstruction complexity of these algorithms is around $O\ (sN_p L)$, but reconstruction complexity of DFT-LS algorithm is around $O\ (N_p L)$. DFT-LS algorithm does not need iterative process and require the sparsity $S$ as prior information.

### 4. Simulation Results

In order to evaluate the channel estimation performance of the proposed DFT-LS method, we adopt the normalized mean square error (NMSE) to quantize the channel estimation error. The NMSE performance of the SAMP method and the OMP method will also be evaluated as references. The NMSE is expressed as

$$NMSE = 10 * \log \frac{\| h - h \|_2}{\| h \|_2} (dB)$$

Assume that the system parameters are constant within an OFDM block. The parameters are as follows.

| | |
|---|---|
| Channel estimation | OMP |
| | SAMP |
| | DFT-LS (proposed) |
| Channel fading | Frequency-selection block fading |
| Channel length | L= 256 |
| Non-zero taps | RandomGaussian independent variable |
| Subcarriers | N = 1024 |

Table 1. The system common parameters

A. In this experiment, the number of pilot $N_p$ is 130; $S$ is unknown to DFT-LS and SAMP. The simulation result is shown in figure 2.

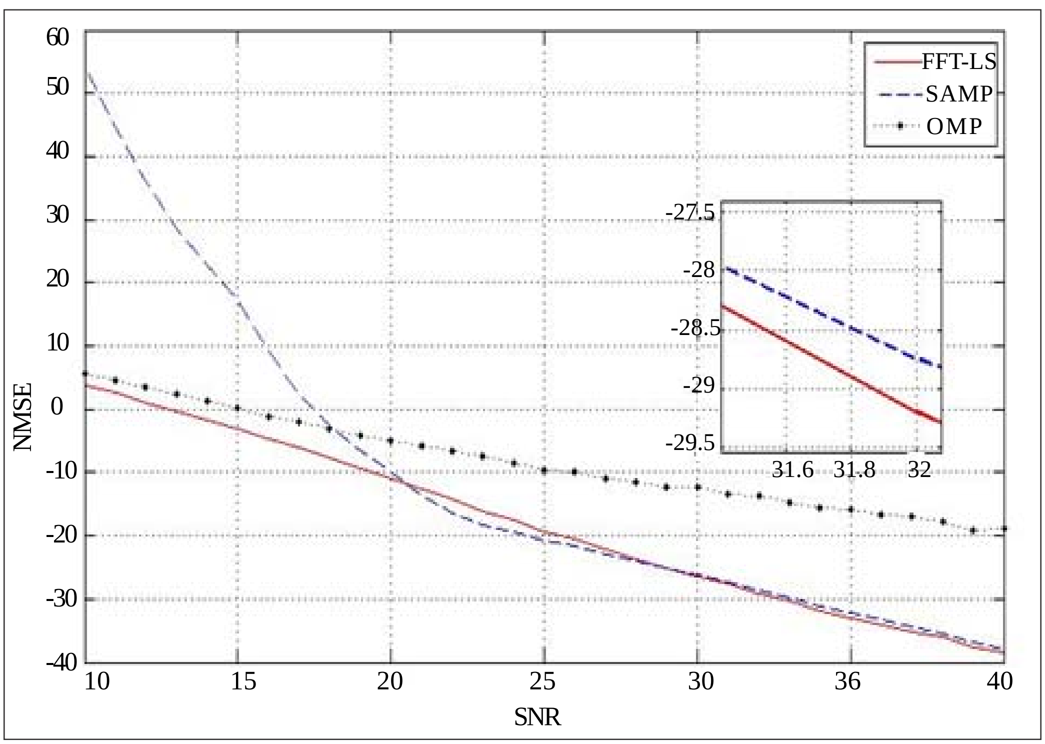


Figure 2. NMSE performance versus SNR

The above results show that the NMSE performance of DFT-LS and SAMP is very close at high SNR and the proposed DFT-LS algorithm is superior to SAMP algorithm at low SNR. SAMP halts when the relative change of reconstructed signal's energy between two consecutive stages is smaller than a certain threshold. However, as we have mentioned before, the threshold is not universal.

B. In this experiment, SNR is 30dB and $N_p$ ranges from 120 to 180. The simulation result is shown in figure 3:

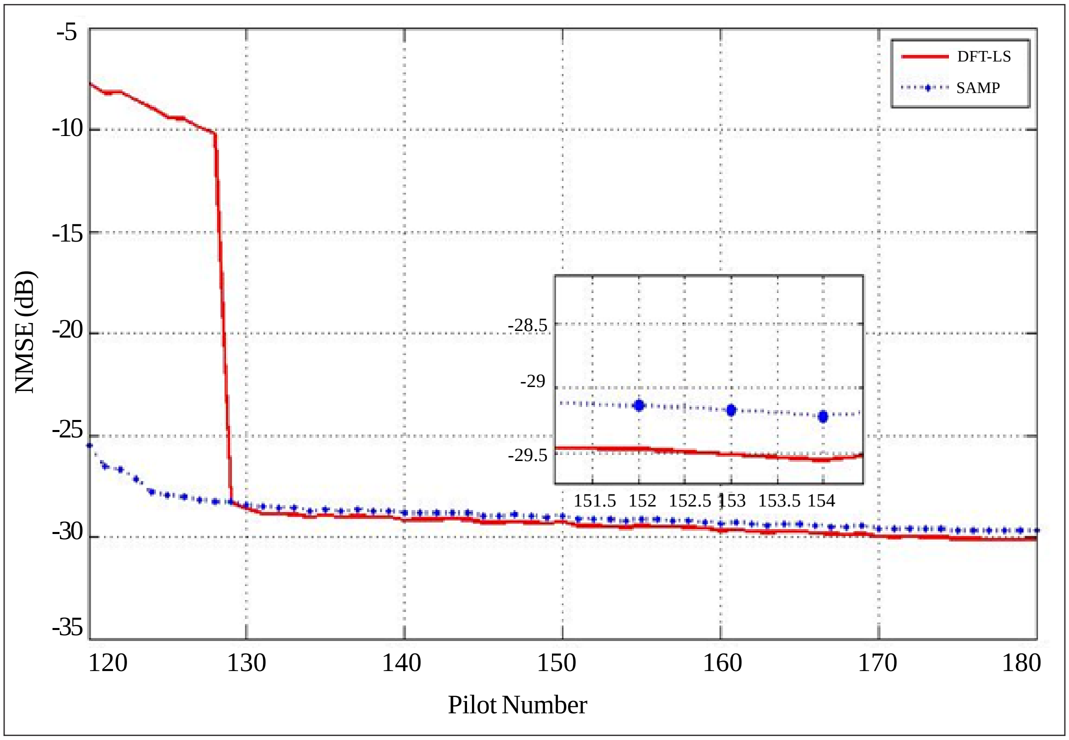


Figure 3. NMSE performance versus pilot number

Figure 3 shows that the NMSE performance of DFT-LS and SAMP is very close and nearly becomes constant when pilot number exceeds 130. DFT-LS needs enough pilots to distinguish non-zero taps index. Once the pilot number exceeds a certain value, both of two algorithms will be able to distinguish all of non-zero taps index.

**5. Conclusion**

In this paper, we proposed a novel sparse channel estimation method called DFT-LS. The propose method has a better estimation performance than the SAMP and OMP. Furthermore, the proposed method does not require the sparsity as prior information and the complexity is low. Thus, the proposed method is entirely appropriate for the practical applications.

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