# Approximate Approaches for Estimating the Parameters of Fatigue

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***ABSTRACT:*** *A material undergoes progressive damage that creates micro-cracks and induces its breaking. To determine its life time, very costly experimental trials are achieved. To overcome this problem, different strategies have been developed using experimental results. These strategies have given different models that predict the mechanical behavior of materials with few and low costs tests. Most of the existing methods for modelling the fatigue of a material have focused on the estimation of the coefficient of fatigue strength* $\sigma_f'$ *and on the coefficient of ductility in fatigue* $\varepsilon_f'$.

*In this work, we study the performance of a dozen of methods proposed in the literature to estimate these coefficients and to predict the life time of the material. We have determined the errors committed in the theoretical models in relation to experimental data. To this end, we have compared the results obtained by these methods with the experimental results on a basis of 82 steels. We have established a protocol to compare the results, which consists in computing the error committed by each model on the estimation of the coefficients of fatigue. The analyses of the results have shown that none of the models can estimate correctly both of the coefficients. We then identified two models that estimate accurately both coefficients* $\sigma_f'$ *and* $\varepsilon_f'$ *separately.*

**Keywords:** Coefficients of Fatigue, Lifetime, Tensile, Hardness Brinell, Regularization Parameters, Regression

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## 1. Introduction

The fatigue damage of a material is characterized in large part of its life (up to 80%) through the development and growth of micro-cracks which are located on its surface [1, 2, 3]. The initiation of micro-cracks results from plastic deformation [4, 5] which is done randomly within the plastically deformed grains. The atomic force microscopy (AFM) is recently used as powerful tool for the precise study of surfaces. It has led to remarkable progress in understanding the evolution of the surface relief and its relationship to the substructure created during the cyclic loading of metallic materials [2, 6]. The growth of these microcracks is strongly related to the micro-structure of the material [7, 8], which plays a major role in the evolution of damage, especially during this early stage [12]. When the main crack propagates under cyclic loading, it is important to quickly assess the residual life of the structure cracked. Other factors such as temperature, environment (vacuum, air, aggressive media) and the amplitude of the imposed deformation (total or plastic) affect the fatigue behavior of materials [10, 11]. In recent decades, approaches based on the plastic deformation have been used successfully in solving many problems of fatigue. Indeed, it is well known that the total strain ($\Delta\varepsilon_t$) is related to plastic ($\Delta\varepsilon_p$) and elastic ($\Delta\varepsilon_e$) deformations by the equation 1 [12, 13].

$$\frac{\Delta\varepsilon_t}{2} = \frac{\Delta\varepsilon_p}{2} + \frac{\Delta\varepsilon_e}{2} \quad (1)$$

called law of Manson-Coffin resistance to plastic deformation, where $\varepsilon'_f$ is the fatigue ductility coefficient and *c* the exponent of ductility in fatigue,

$$\frac{\Delta\varepsilon_e}{2} = \frac{\sigma'_f}{E}(2Nr)^b \quad (2)$$

$$\frac{\Delta\varepsilon_p}{2} = \varepsilon'_f (2Nr)^c \quad (3)$$

called Basquin law of resistance to elastic deformation, where *b* is the fatigue strength exponent, $\sigma'_f$ is the fatigue strength coefficient and *E* is modulus of elasticity. These two empirical laws are used to connect the number of cycles to failure (*Nr*) to the amplitude of the imposed deformation.

This paper is organized as follows, in section 2; we recall some elements related to fatigue. In section 3, we build a state of the art of the different methods used to estimate the coefficients of material fatigue. Then we perform a comparative study, which allowed us to highlight the shortcomings of these methods. In section 4, we propose a model that improves significantly the estimation of these coefficients. We conclude by giving some perspectives to this work.

## 2. Background and Methods

### 2.1 Fatigue of materials

The experimental study of fatigue of materials is difficult and expensive to realize. During these experiments, tests are done. They allow to obtain the variation of the constraint on the number of cycles which determines the number of cycles to failure and estimates the parameters of fatigue (b, c, $\varepsilon'_f$, $\sigma'_f$). Many methods based on models simulated numerically have been developed to predict the lifetime of a structure [14, 15, 16, 17]. These models have been proposed to describe the relationships between various parameters and the coefficients of fatigue of a material. Among these models, we can cite those based on the experimental determination of Modulus of elasticity ($E$), elastic limit ($\sigma_e$) of the ultimate tensile strength($\sigma_u$), the reduction of area coefficient (RA) obtained in tension, and the Brinell hardness number (BHN) [18, 19]. To predict the fatigue behavior of a material and determine its life when it is subjected to cyclic loading, fatigue tests are performed. They consist on submitting a set of test pieces to repetitive cycles of stress. These models are related to the strain range applied to the number of cycles to failure (lifetime of the material), whose prediction requires to estimate the coefficients of fatigue $\sigma'_f$ and $\varepsilon'_f$ [11].

The fatigue tests show significant variability. Indeed, for the same level of load, the lifetime of a material depends on its nature and its stress [20]. Different possibilities are then generated that require many attempts to model the phenomenon of fatigue. These tests are costly in time and money. To overcome this problem, we propose various strategies to ensure optimal prediction of the fatigue behavior of materials, by using some data obtained experimentally [18, 19, 21, 22]. In practice, using tensile tests suffices to determine the material properties [23].

### 2.2 Existing models

We have studied the main models proposed in the literature to estimate the fatigue parameters [20]. These models are based on the traction coefficients and on the characteristics of the material used. The analysis of these models shows that most authors have proposed a linear model to estimate $\sigma'_f$ coefficient as a function of $\sigma_u$ parameter. For instance, see Mitchell [22], Baumel and Seeger [24], Meggiolaro and Castro [25], Manson's Universal Slopes [26] and Roessle and Fatemi [19]. The latters also provided a linear model based on BHN to estimate $\sigma'_f$ since these coefficients are highly correlated (0.98) for steels [20], i.e. $\sigma_u$ is equal approximately to 3.4 × BHN. However, some non-linear models have been proposed such as Muralidarham-Manson [27], Manson's Four Point [18] or Manson's Universal Slopes [26].

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$\overline{\sigma'_f}$ : Average, $s(\sigma'_f)$ : Standard deviation,

$Er(\sigma'_f)$ : Standard deviation error,

$\overline{Er(\sigma'_f)}$ : The average of errors

$\overline{s(Er(\varepsilon'_f))}$ : Standard deviation errors

For the fatigue ductility coefficient $\varepsilon'_f$, Mitchell [22] and Four-Point Ong [28] have proposed to estimate the value by $\varepsilon'_f$, while Meggiolaro and Castro [25] have used a constant equal to 0.45. Other authors have used other parameters such as BHN, $\sigma_u$, $E$, etc.

## 3. Comparative Study of Existing Models

### 3.1 Simulations and protocol comparison

To analyze the characteristics of these models, we define a comparison protocol:

a- We start by extracting data from our database composed of 82 experimental tests of tensile, fatigue and hardness, made on several steels [18, 19, 21, 25].

b- We then compute $\sigma'_f$ and $\varepsilon'_f$ by the proposed models,

c- Finally, we compute the mean absolute error for each model i.e.

$$RE = \frac{\sum_{i=1}^{n} \frac{|o_i - c_i|}{o_i}}{n} \quad (4)$$

Where $o_i$ is the experimental parameter, $c_i$ is the parameter estimated by the model and $n$ is the size of the sample [23].

### 3.2 Results and Interpretation

The analysis of the different models shows that the linear models for computing the coefficient of fatigue strength $\sigma'_f$ is the most relevant and give the best results, in particular, Mitchell model [22] for which the error RE = 0.1088 and Roessle & Fatemi model [19] for which RE = 0.114.

| Methode | $\sigma'_f$ | s ($\sigma'_f$) | Er ($\sigma'_f$) | Er ($\sigma'_f$) | s (Er ($\varepsilon'_f$)) | $\varepsilon'_f$ | s ($\varepsilon'_f$) | Er ($\varepsilon'_f$) | Er ($\varepsilon'_f$) | s (Er ($\varepsilon'_f$)) |
|---|---|---|---|---|---|---|---|---|---|---|
| Experim | 1455,92 | 562,79 | | | | 0,48 | 0,33 | | | |
| Mitchell | 1445,36 | **531,39** | **0.1088** | 10,56 | 181,7957 | 0,71 | 0,32 | 1.4312 | -0,23 | 0,4218 |
| Slope | 2092,66 | 1010,60 | 0.4166 | -623,56 | 503,0638 | 0,60 | 0,17 | 1.1212 | -0,12 | 0,3445 |
| MU-Slope | 1620,85 | 647,65 | 0.1655 | -164,72 | 217,3041 | 0,32 | **0,11** | **0.6545** | 0,16 | 0,3417 |
| Fpoint_Ong | 1790,90 | 750,56 | 0.2943 | -334,80 | 408,2855 | 0,71 | 0,32 | 1.4312 | -0,23 | 0,4218 |
| FLaw | 1650,53 | 797,09 | 0.1903 | -197,28 | 322,4443 | 0,41 | 0,20 | 0.8176 | 0,07 | 0,3594 |
| Fatemi($\sigma_u$) | 1489,36 | **552,64** | **0.1142** | -33,06 | 183,2510 | | | | | |
| Fatemi(BHN) | 1494,12 | **554,49** | **0.1249** | -36,40 | 198,8492 | 0,39 | 0,17 | 0.7404 | 0,09 | 0,3647 |
| M four-point | 1854,03 | 785,17 | 0.3151 | -412,74 | 422,3137 | 3,15 | 1,88 | 2.6979 | -2,68 | 1,8629 |
| Medians | 1650,53 | 797,08 | 0.1903 | -197,28 | 322,4443 | 0,45 | 5E-16 | 0.8588 | 0,03 | 0,3309 |

Table 1. Comparative Table Of Means And Standard Deard Deviation
Of The Coefficients And Errors Commtted By The Models Studied

The regression curves allowed us to obtain the following results: 89.56% of the variability of the coefficient $\sigma'_f$ is explained by $\sigma_u$ (Figure 1-a) and 87.65% is explained by BHN (Figure 1-b). Using the same approach, we note that none of the existing models could estimate correctly the fatigue ductility coefficient $\varepsilon'_f$ (65% error at least).

Indeed, the use of true ductility factor ($\varepsilon'_f$) to estimate $\varepsilon'_f$ coefficient [22, 28] may cause a significant error (RE = 1.431). Moreover, estimating $\varepsilon'_f$ coefficient by a constant [25] is not accurate to predict the lifetime of the material: RE = 0.8588. In fact, all materials cannot have the same fatigue ductility coefficient $\varepsilon'_f$ (0.45). The analysis of results (Table 1) shows that Modified Universal Slope model [26, 27] gives the best results.

We can explain the poor estimation of $\varepsilon'_f$ coefficient by its low correlation (0.19) with the parameters of traction [20]. We conclude that there is no model that correctly estimates both coefficients $\sigma_f'$ and $\varepsilon_f'$.

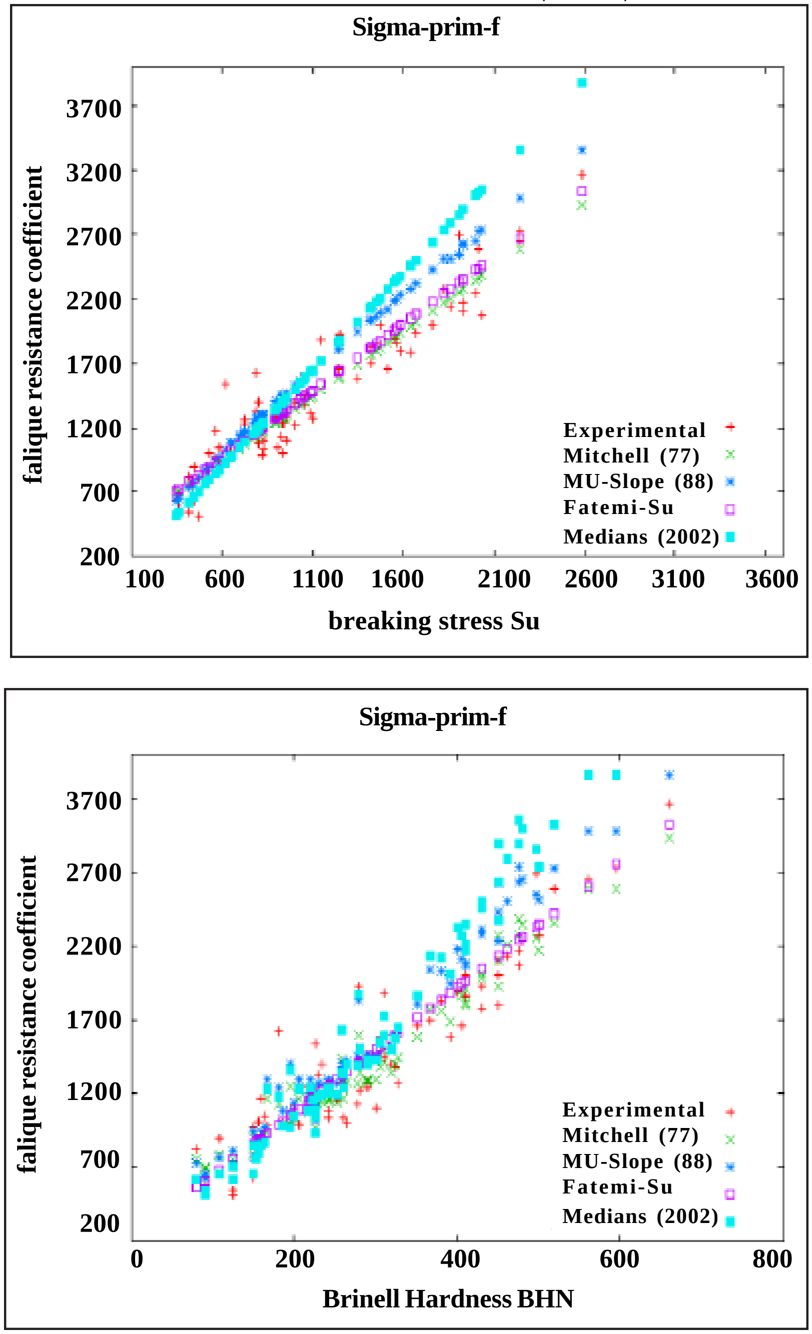


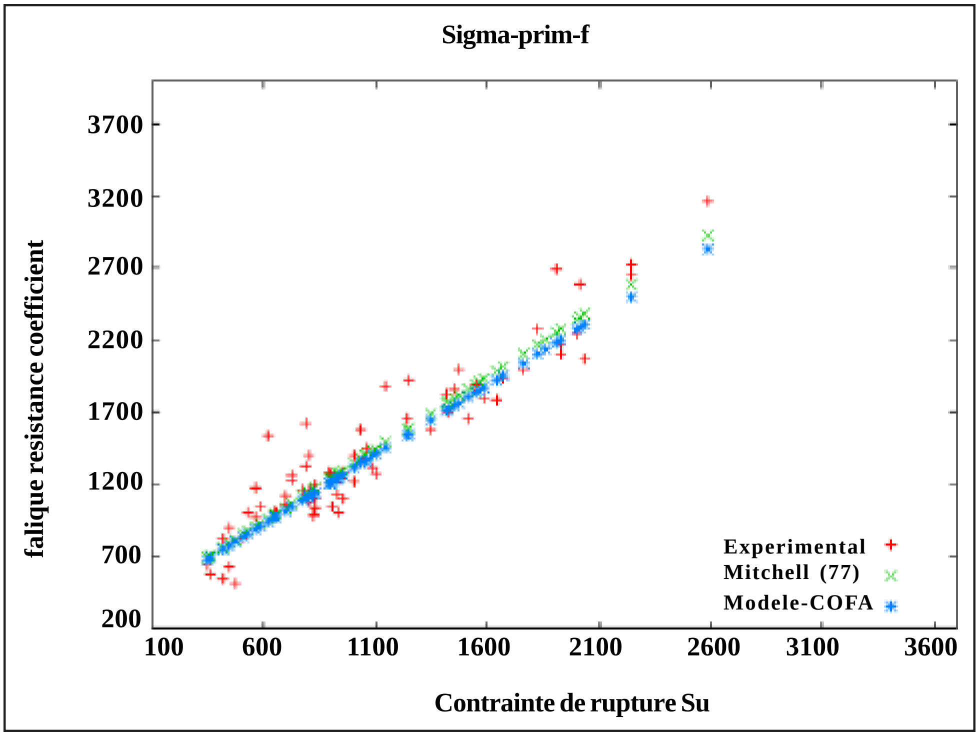

Figure 1. Comparison $\sigma_f'$ obtained with best models based on :

(a) $\sigma_u$ and (b) BHN

## 4. Proposed Model: COFA (Coefficients of Fatigue)

### 4.1 COFA Model

In this paper, we propose a model that estimates reasonably the two coefficients. It is built based on the two best models we have identified: Mitchell model [22] to estimate $\sigma_f'$ and Modified Universal Slope [29, 30] to estimate $\varepsilon_f'$. We have improved these models by adjusting the values of the coefficients of equations (5) and (6) [20].



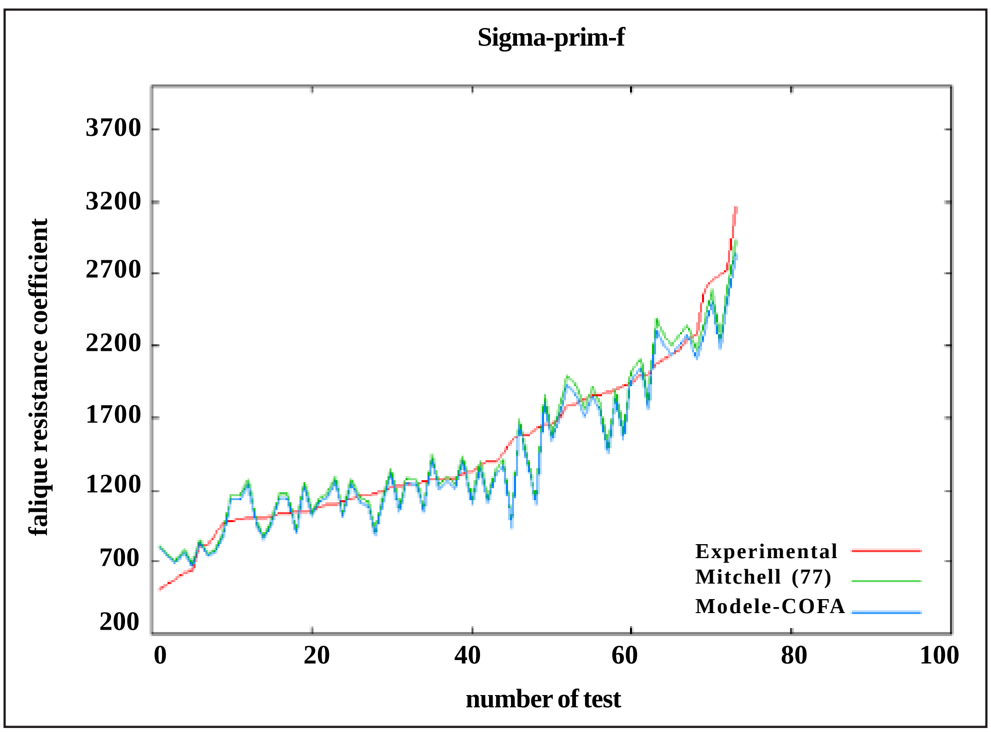




Figure 2. Comparison between experimental values of $\sigma_f'$ and those obtained with COFA and Mitchell

$$\sigma_f' = \sigma_u + 345 \tag{5}$$

$$\varepsilon_f' = 0.0196\,\varepsilon_f'\;0.0155\left(\frac{\sigma_u}{E}\right)^{-0.53} \tag{6}$$

Hence, the equations of COFA model become as follows:

$$\sigma_f' = 0.0965\,\sigma_u + 343 \tag{7}$$

$$\varepsilon_f' = 0.0130\,\varepsilon_f'^{\;0.0155}\left(\frac{\sigma_u}{E}\right)^{-0.53} \tag{8}$$

To validate this model, we carried out several random tests selected from the basis of 82 experimental results [20].

### 4.2 Tests and Validation

To validate COFA model, we performed several tests by sampling randomly from the 82 experimental results [20]. In Figure 2, we represent a comparison of the results obtained by three methods: Mitchell model [22], COFA model andexperimental values [20]. To estimate $\sigma_f'$, COFA model ($\sigma_f' = 0.965\sigma_u + 343$) has slightly improved the results: error ($\sigma_f'$) = 0.107 against 0.109 to Mitchell [22]. Figure 2 shows the results on $\varepsilon_f'$ coefficient determined by Manson's Universal Slopes model [26, 27], COFA model and experimental values [20]. For the estimation of $\varepsilon_f'$, COFA model improve here significantly the results: error ($\varepsilon_f'$) = 0.5856 against 0.655 obtained by Manson's Universal Slopes [26, 27].

Note that the estimation of $\varepsilon_f'$ remains difficult for $\varepsilon_f' \geq 0.4$ (Figure 3). This deficiency can be reduced if the model takes into account the Brinell hardness (BHN) [3, 5, 12].To this purpose, we introduce into COFA model a regularization parameter R which depends on BHN and which is computed as follows: R = m ∗ (BHN)$^n$.

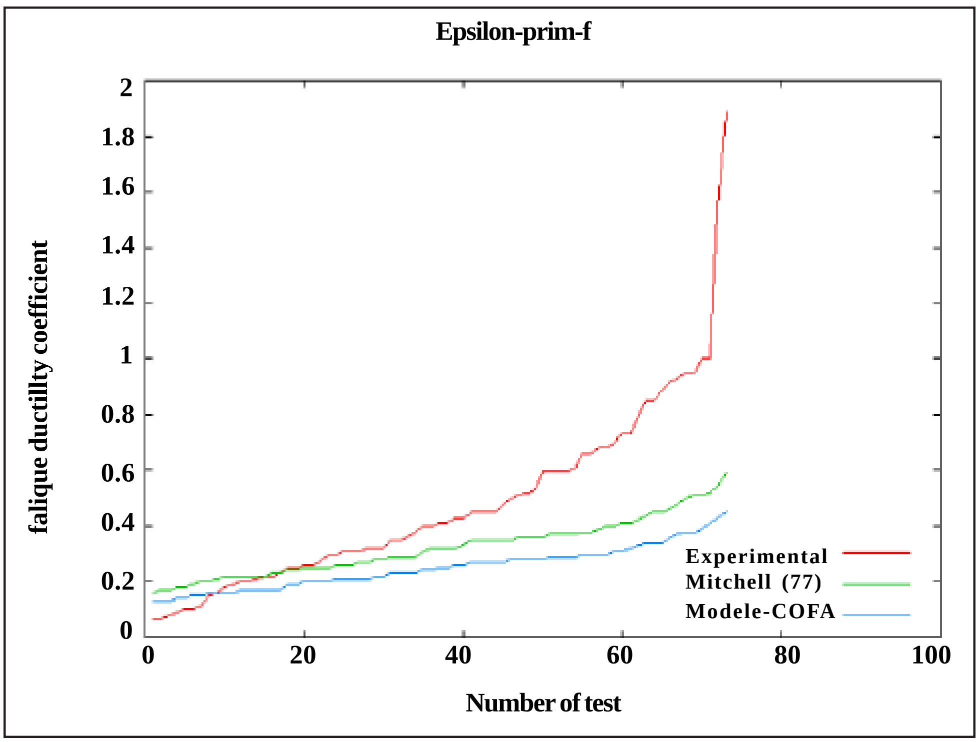



Figure 3. Comparison between experimental values of $\sigma_f'$ and those obtained with COFA and MU-Slope

The formula for modelling the fatigue ductility coefficient becomes:

$$\varepsilon_f' = 0.0130\,\varepsilon_f'^{\;0.155}\left(\frac{\sigma_u}{E}\right)^{-0.53} + 0.1(BHN)^{-0.09}$$

### 4.3 Final Model

We note that the curve of $\varepsilon_f'$ coefficients estimated by COFA model follows the shape of the curve of $\varepsilon_f'$ coefficients determined experimentally (Figure 4).

R parameter has allowed us to minimize the error in estimating the coefficient $\varepsilon_f'$ (error ($\varepsilon_f'$) = 0.5647). This has significantly improved the estimated coefficient $\varepsilon_f'$ with respect to the estimation given by the models discussed above. The COFA model equations are finally:

$$\sigma_f' = 0.0965\,\sigma_u + 343$$

$$\varepsilon_f' = 0.0130\,\varepsilon_f^{\;0.155}\left(\frac{\sigma_u}{E}\right)^{-0.53} + 0.1 \times BHN^{-0.09}$$

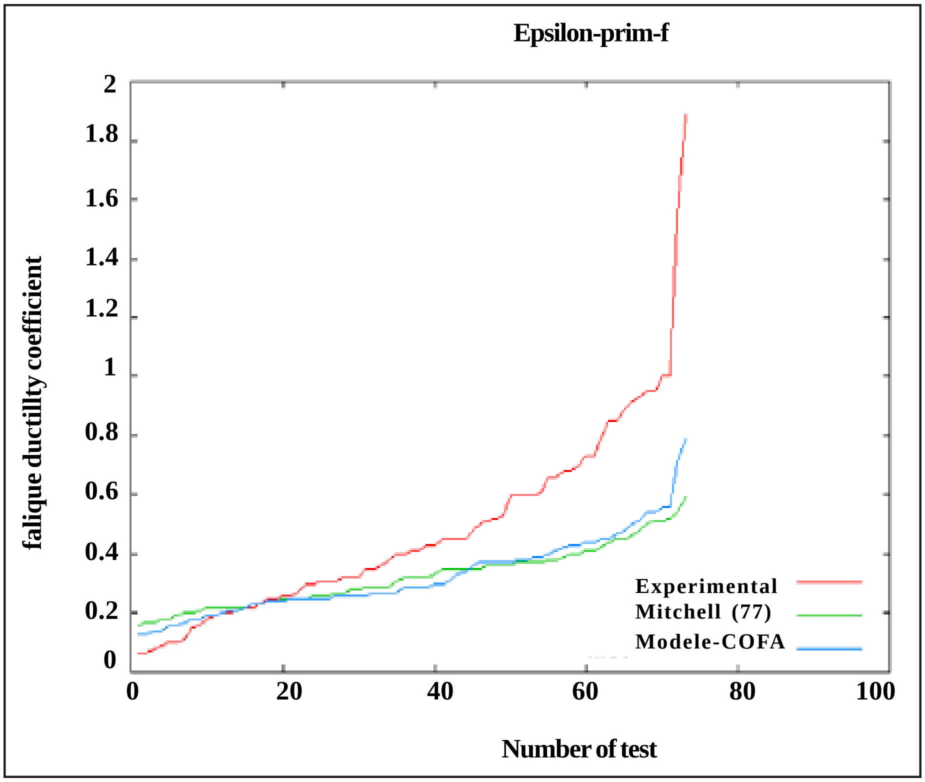



Figure 4. The influence of regularization parameter R on $\varepsilon_f'$

**5. Conclusions and Perspectives**

In this paper, we have proposed a model to estimate the fatigue coefficients of a material. It is based on the traction parameters known to be measurable by a simple and cheap mechanical test. Initially, we have done a comparative study of models available in the literature. Then, we pointed out their shortcomings and suggested improvements that led to COFA model. Based on tests we have done, we can conclude that the formula used to compute $\varepsilon_f'$ and $\sigma_f'$ with COFA give the best results.

Our next work consists in the validation of COFA model by laboratory mechanical tests. We plan in the next step to estimate the exponents of strength and ductility in fatigue. These exponents allow the prediction of the lifetime of the materials, while avoiding expensive experiments. Thereafter, our aim is to make our model more robust.

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