# Adaptive Full Order Observer for Sensorless Induction Motor Control

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**ABSTRACT:** This paper proposes a method for designing a sensor less induction motor drive based on a robust rotor flux observer. The industrial world demands reliable and inexpensive systems. For this reason, the number of sensors is reduced. Accordingly, this study will focus on the design of a robust rotor flux observer using both the Lyapunov theory and the linear matrix inequality (LMIs). Thus, the development of an estimator of the rotor speed. The proposed solution always results in a stable and robust controller against variations of resistances. The solution is evaluated by simulation.

## Keywords: Induction motor, FOC, LMI, Robust Observer

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## 1. Introduction

Thanks to a century of the existing development in the technology of rotating electrical machines, the electric motors have been widely used as a source of electromotive force. More than half of the total electric energy produced in industrialized countries is indeed consumed by electric motors [1]. Asynchronous machines or induction machines are currently the most prevailing (widespread) machines in industrial applications. They represent at least 80% of electrical industrial drive [2]. The choice of the induction motor is unbeatable in many industrial applications with constant or adjustable speed due to its reliability, its power density, robustness, low manufacturing cost and exploitation.

The vector control schema is a method which reduces to a linear control structure by the flux orientation. It was proposed by Blaschkeen 1972. The capability of such modern microcontrollers made it possible to implement sophisticated controls like Vector Control. This command, which allows decoupling the control variables, remains the most widely used for the high dynamic performance it offers for a wide range of applications. In order to improve the dynamic performance of an adjustable speed induction motor, we thought it interesting to use a robust rotor flux observer as well as an estimator for a rotor speed based on the control variables.

The present article is organized as follows: in section II, we present the model of an induction motor. Section III is dedicated to present the optimized field-oriented control. An adaptive flux observer and an online tracking mechanical speed mechanism are established in Section IV. The validation of the proposed solution as well as the discussion of the results are carried out in Section IV. We end our article with a conclusion.

#### 2. Induction Motor Model

In this section, we present a dynamic model of induction motor in relation to its control. The model must be able to accurately represent the different dynamics. The induction machine is, by nature, a three- phase model but under certain simplifying assumptions, we can move to an equivalent twophase representation, thus reducing model complexity. This model is established in terms of differential equations and of using the Park transformation (or is based primarily on the Park transformation).

Let  $\lambda_r$ ,  $v_s$  and  $i_s$  be the rotor flux, the stator current and the stator voltage, respectively. With  $x = [i_s^T \lambda_r^T]^T$ , The dynamic of an induction motor is defined by the following state equation (1) in a rectangular coordinate fixed to the stator.

$$\dot{x} = (\mathbf{A} + \omega \mathbf{A}_{\omega}) x + B v_s$$

$$i_s = C x$$
(1)

Where

$$A = \begin{bmatrix} -\left(Rr \frac{1-\sigma}{\sigma Lr} + \frac{Rs}{\sigma Ls}\right)I\left(\frac{R_r}{\varepsilon Lr}\right)I \\ \left(\frac{MR_r}{Lr}\right)I & -\left(\frac{R_r}{Lr}\right)I \end{bmatrix}$$
$$A_{\omega} = \begin{bmatrix} 0_{2x2} & -(1/\varepsilon)J \\ 0_{2x2} & J \end{bmatrix}$$
$$B = \begin{bmatrix} \frac{1}{\sigma Ls}I & 0_{2x2} \end{bmatrix}^T$$
$$C = \begin{bmatrix} I & 0_{2x2} \end{bmatrix};$$
$$C_1 = \begin{bmatrix} 0_{2x2} & I \end{bmatrix};$$

Rs, Rr: Resistance of Stator and Rotor phase winding

Ls, Lr: Stator and Rotor inductance

 $\omega$ : Mutual inductance

 $\sigma = 1 - (M^2 / L_s L_r)$ : Angular rotor speed  $\varepsilon = \sigma LsLr/M$ : Total leakage factor

*I* and *J* are the unit matrix and the skew symmetric matrix of  $2 \times 2$ , respectively. The electromagnetic torque in the stator reference frame is expressed by the following equation (2).

$$Ce = p \left( 3M/2L_r \right) \left( \lambda_{r\alpha} i_{s\beta} - \lambda_{r\beta} i_{s\alpha} \right)$$
<sup>(2)</sup>

#### 3. Field - Oriented Control

In this section, basic principles of field-oriented control is explained for induction motor control

#### 3.1 Rotor flux-oriented control

In the literature, there are two recommended techniques for high performance induction machine control. The first one is called indirect field-oriented control (IFO) and the second one is the direct field-orientation control (DFO) [3]. In order to optimize the performance of the induction motor and reduce the sensitivity of the stability of the controller to the variation of rotor resistance, we will implement the DFO technique. The field-oriented control has been widely studied in the literature [4–6]. Among others we adopt the direct rotor flux-oriented control (DRFOC); It is achieved by aligning the d-axis of the synchronous reference frame with the rotor flux vector, and imposing a fast dynamic loop current [7]. Indeed, this solution offers many advantages in

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monitoring and analysis of (or analyzing) the induction machine. The expression of electromagnetic torque and the rotor flux are simplified to the following two equations (3).

$$\begin{cases} Ce = p \frac{3M}{2L_r} \lambda_{rd} i_{lq} \\ \lambda_{rd} = Mi_{Ld} - \frac{L_r}{R_r} \frac{d\lambda_{rd}}{dt} \end{cases}$$
(3)

So, the Alignment of the "d" axis of the synchronous reference frame with the rotor flux vector results in:

$$\begin{cases} \lambda_r = \lambda_{rd} \\ \omega_s = \frac{MR_r}{L_r \lambda_{rd}} i_{Lq} + p\omega \end{cases}$$
(4)

Equations (4) define the relationship of mechanical speed and the angular velocity of rotating reference frame d, q

#### 3.2 Power losses minimization

The classical rotor field-oriented control imposes a rated value to the magnitude of the rotor flux. This control technique causes unnecessary consumption of electrical energy when the motor is under-loaded. Almost half of them operate at less than 40% of their rated loads [8]. In the paper [7] an optimal control law of rotor flux based on the research of optimal flux magnitude is defined by the following equation (5).

$$\lambda_{rd} = M_{4} \sqrt{\frac{R_q}{R_d}} \sqrt{kCe}$$
<sup>(5)</sup>

where

$$R_{d} = Rs + (Rs + 1) (\omega_{s}Ls)^{2}/Rc$$

$$R_{q} = Rs + (RsRc + 1) ((\omega_{s}Ls\sigma)^{2}/Rc) + Rr (M/Lr)^{2}$$

$$k = (2L_{r})/(3pM^{2})$$

#### 4. FLUX Observer Design

In this section, we focus on the problem of the synthesis of observers for induction motor. In general, an observer is a dynamic sub-system that allows estimating the state of a real system based on its measurable inputs and outputs. Thus, this kind of dynamic systems is used in the field of asynchronous machines control. Indeed, most of the control laws for this type of machines is based on immeasurable states like rotor flux [6], [7]. In addition, there is also an important industrial demand for machine control without speed sensor [9–11].

It is well known that the change in operation parameters in induction machines causes problems for the flux estimation. Furthermore, the behavior of the control system is directly related to the flux estimation errors.

State-space expression (or State estimation strategy ) of the proposed observer in Figure.1 can be expressed in the following differential equation (6) where H stands for the observer gain. The symbol ^ denotes an estimated value of a nominal parameter or a state variable.

$$\begin{cases} \hat{\dot{x}} = (\hat{A} + \hat{\omega}_r A_{\omega}) \hat{\dot{x}} + \hat{B} v_s + H(\hat{is} - is) \\ \hat{is} - C \hat{\dot{x}} \end{cases}$$
(6)

Where

$$\hat{A} = A + \Delta A, \ \hat{B} = B + \Delta B, \ \hat{C} = C + \Delta C \text{ and } \ \hat{\omega}_r = \hat{\omega}_r + \Delta \omega_r$$

The state estimation error  $e = x - \hat{x}$  is defined as the difference between the estimated and the real states.

The dynamics of estimation error estimation is simulated by the following equation (7)

$$\dot{e} = (A + HC + \omega_r A_{\omega}) e + \Delta \omega_r A_{\omega} \hat{x}$$
<sup>(7)</sup>

to ensure the asymptotic stability of the state estimation error, we consider the following quadratic Lyapunov function

$$V(e, \Delta \omega_r) = e^T P e + (2\Delta \omega_r^2 / \mu)$$
(8)

Where *P* is a positive-definite symmetric matrix and  $\mu$  is *a* positive design constant.

The time derivative of V becomes

$$\dot{V}(e,\Delta\omega_r) = \dot{e}^T P e + e^T P \dot{e} + \frac{2}{\mu} \left(\Delta\omega_r\right) \left(\frac{d\Delta\omega_r}{dt}\right)$$
(9)

After some calculation, the equation (9) becomes

$$\dot{V}(e,\Delta\omega_{r}) = e^{T} \left( (A + HC)^{T} P + P (A + HC) e + e^{T} (\omega_{r} (PA_{\omega} + A_{\omega}^{T} P)) e + \Delta\omega_{r} (\hat{x}A_{\omega}^{T} P e + e^{T} PA_{\omega} \hat{x}) + \frac{2}{\mu} (\Delta\omega_{r}) \left( \frac{d\Delta\omega_{r}}{dt} \right)$$

$$(10)$$

The state error (e) is quadratically stable if the equation (9) is defined negatively. So

$$\dot{V}(e,\Delta\omega_{\rm r}) < 0 \tag{11}$$

If the first condition of Lyapunov is

$$\Delta \omega_r (\hat{x} A_\omega^T P e + e^T P A_\omega \hat{x}) + \frac{2}{\mu} (\Delta \omega_r) \left( \frac{d\Delta \omega_r}{dt} \right) = 0$$
(12)

The second Lyapunov condition is:

$$(A + HC + \omega_r A_{\omega})^T P + P (A + HC + \omega_r A_{\omega}) < 0$$
(13)

Since  $\dot{\omega}_r \approx 0$  we can rewrite the equation (12) in the following form

$$\frac{2}{\mu} \left( \frac{d\omega_r}{dt} \right) = -(\hat{x} A_{\omega}^T P e + e^T P A_{\omega} \hat{x})$$
(14)

Hence

$$\frac{d\omega_r}{dt} = -\mu \, e^T P A_\omega \hat{x}$$

The adaptive law for rotor speed estimation is given by

$$\hat{\omega}_{r} = K_{pv} (e^{T} P A_{\omega} \hat{x}) + K_{iv} \int (e^{T} P A_{\omega} \hat{x}) dt$$
(15)

Where  $K_{pv}$ ,  $K_{iv}$ , are positive design constants

# 5. Gain-Scheduled Calculation

To optimize the dynamic convergence of the proposed observer to the real system, we use the technique poles placement in a complex sub-region  $\mathcal{D}$ .  $\mathcal{D}$  is the intersection between a disk centered at center (0,0) with radius "*r*" and the left half plane limited by a vertical line with the abscissa "-*h*", where h is a positive constant (Figure 2). These conditions are expressed in terms of bilinear matrix inequality (BMI) defined by the following equation (16)

$$\begin{cases} \begin{bmatrix} -rP & A_{e\omega}^{T}P + C^{T}H^{T}P \\ PA_{e\omega}PHC & -rP \end{bmatrix} < 0 \\ PA_{e\omega} + A_{e\omega}P + PHC + C^{T}H^{T}P + 2hP < 0 \end{cases}$$
(16)

with  $P = P^T > 0$  and  $A_{e\omega} = A + \omega_r A_{\omega}$ 

Let R = PH, the bilinear matrix inequality (16) becomes a LMI

$$\begin{cases} \begin{bmatrix} -rP & A_{e\omega}^{T}P + C^{T}R^{T} \\ PA_{e\omega}^{T}RC & -rP \end{bmatrix} < 0 \\ PA_{e\omega} + A_{e\omega}^{T}P + RC + C^{T}R^{T} + 2hP < 0 \end{cases}$$
(17)

The matrix  $A_{e\omega}$  which depends affine on the mechanical rotor speed  $\omega_r \in [\omega_{r1} \ \omega_{r2}]$ ; where  $A_{e\omega}$  represents the parameter values of the matrix  $A_{e\omega}$  at the vertices  $\omega_{ri}$  of the parameter polytope. Accordingly, [12] it is then possible to design the observer gain H if and only if there exists a real positive matrix  $P = P^T$ ,  $R_1$  and  $R_2$  correspond simultaneously in the four following LMIs:

$$\begin{cases} \begin{bmatrix} -rP & A_{e\omega i}^{T}P + C^{T}R_{i}^{T} \\ PA_{e\omega i} + R_{i}C & -rP \end{bmatrix} < 0 \\ PA_{e\omega i} + A_{e\omega i}P + R_{i}C + C^{T}R_{i}^{T} + 2hP < 0 \end{cases}$$
(18)

 $R_1$ ,  $R_2$  and P can be calculated numerically under LMI constraints of (18) by using the cross decomposition technique. The observer sub-gain  $H_i$  is synthetized by the following equation

$$H_i = P^{-1}R_i \quad i \in \{1, 2\}$$
(19)

The observer gain H for a given  $\omega_r \in [\omega_{r_1} \ \omega_{r_2}]$  can be interpolated between the two sub-gains  $H_1$  and  $H_2$ . Let



Figure 2. Constraints state observer of induction machine

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 $H(\omega_{r}) = \frac{H_{1}(\omega_{r2} - \omega_{r}) + H_{2}(\omega_{r} - \omega_{r1})}{(\omega_{r2} - \omega_{r1})}$ (20)



Figure 2. Constraints of pole placement

# 6. Simulation Results

To verify the proposed adaptive and robust observer, simulations are performed. Figure 3 presents the block diagram of the sensor less vector control scheme without the speed sensor. Three-phase induction motor parameters are given in Table 1. The simulation algorithm of the full drive system was designed by using the Simulink. The sampling step is  $10 \,\mu$ s in material platforms Dspic/microchip. Figure.4 shows the system response to a step speed (1000tr/mn) under a resistive torque of 15Nm. The variation of the electromagnetic is illustrated in Figure.5; the motor reaches its steady state after 0.3s. At t =1s load torque is changing from 15Nm to 40Nm. The speed distribution is rejected.



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Figure 4. Simulated speed using the proposed speed adaptive scheme



# 7. Conclusions

Figure 5. Torque response for step varying of load torque

In this study, an adaptive observer and speed estimator has three-phase induction motor drives. In the steady state, the



Figure 6. The curve of real and estimated rotor flux

Symbol		Quantity	UM
Rs	Stator resistance	0.477	[Ω]
Rr	Rotor resistance	0.893	[Ω]
Rc	Core-loss resistance	92	[Ω]
Ls	Stator inductance	0.0955	[H]
Lr	Rotor inductance	0.893	[H]
М	Mutual inductance	0.1040	[H]
J	Moment of inertia	0.22	[Kgm <sup>2</sup> ]
Vn	Rated voltage	380	[V]
In	Rated current	10.4	[A]
Pn	Rated power	3.4	[KW]
Р	Number of pole pair	2	-

Table 1. Parameters of the Induction Motor

controller changes the components of the stator current vector in search of a minimum power loss. The simulation results show that rotor flux and speed estimation can be achieved properly by the proposed solution. Therefore, the proposed algorithm can be used for high performance industrial applications of tree-phase induction motor. The experiment setup is needed to the validity of the proposed scheme.

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