# **Backstepping Control of an Induction Motor**

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**ABSTRACT:** This paper presents the backstepping nonlinear control of an induction motor. This control is based on the stability of the system from the Lyapunov theory with taking account the nonlinearity of our system. To validate the effectiveness and robustness of the proposed solution some simulation results are provided.

Keywords: Backstepping, Nonlinear Control, Induction Motor, Indirect Vector Control

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### 1. Introduction

Induction motor is the most used in industrial applications compared to other electric machines, due to its excellent reliability, great robustness and reduced maintenance... [1]. Moreover, the induction motor is complex because its dynamic model is nonlinear [2-5]. In industrial application, the design of control law is essentially based on classical regulator for their simple structures; however this type of controllers ignores the nonlinearity of the system. Several types of nonlinear control have been introduced in the last two decades and have been applied to the induction motor as the backstepping method. This type of control is a systematic and recursive design of controlling nonlinear system.

The backstepping technique is applied to the induction motor to design a speed controller. The present paper is organized as follows; in section II, the model of induction motor as defined. Section III is devoted to developing the control law by the backstepping technique. Improving performance and robustness of the proposed control is established in section [3-4].

### 2. Induction Motor Modeling

The model of the induction motor on the axis "d" can be described in a reference connected to the rotating field by the following equation [3-6]:

$$\frac{d}{dt}i_{ds} = \alpha_1 + \delta V_{ds}$$

$$\frac{d}{dt}i_{qs} = \alpha_2 + \delta V_{qs}$$
(1)

$$\frac{d}{dt}\phi_{dr} = M\beta_r i_{ds} - \beta_r \phi_{dr}$$
$$\frac{d}{dt}\omega = \frac{n_p^2}{jL_r}\phi_{dr} i_{qs} - \frac{n_p}{j}T_l - \frac{f}{j}\omega$$

With  $i_s$ ,  $\phi_r$ , are stator currents, rotor flux, the index s and r representing stator and rotor,  $\omega$  is the rotor speed and  $\sigma$  is the mutual and leakage inductance.

Where:

$$\alpha_{1} = -\gamma i_{ds} + \omega_{r} i_{qs} + k\beta_{r} \phi_{dr} + M\beta_{r} \frac{i_{qs}^{2}}{\phi_{dr}}$$
$$\alpha_{2} = -\omega_{r} i_{ds} - M\beta_{r} \frac{i_{ds} i_{qs}}{\phi_{dr}} - \gamma i_{qs} - k\omega_{r} \phi_{dr}$$

The constants are defined as follows:

$$\gamma = \delta R_{sr}, \delta = \frac{1}{\sigma L_r}, \quad R_{sr} = R_s + \frac{M^2}{L_r^2} R_r, \quad \upsilon = \frac{M}{L_r}$$
$$\beta_r = \frac{1}{T_r}, \quad k = \delta \upsilon, \quad \sigma = 1 - \frac{M}{L_r L_s}$$

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#### 3. Backstepping Control

The backstepping control law can be obtained in several steps. Each step will provide a reference for the next step. Stability and performance of our system will be studied using Lyapunov theory [4-6]. In the first step, we consider the trajectories of speed and flux as a reference, and we define the tracking errors as follows:

$$e_{\omega} = \omega_{ref} - \omega$$

$$e_{\phi} = \phi_{ref} - \phi_{dr}$$
(2)

By deriving equation (2) we obtain:

$$\dot{e}_{\omega} = \dot{\omega}_{ref} - \dot{\omega}$$

$$\dot{e}_{\phi} = \dot{\phi}_{ref} - \dot{\phi}_{dr}$$
(3)

When replacing  $\dot{\omega}$ ,  $\dot{\phi}_{dr}$  with these expressions from the system of equations (1), equations (3) become:

$$\dot{e}_{\omega} = \omega_{ref} - \frac{n_p^2 M}{jL_r} \phi_{dr} i_{qs} + \frac{f}{j} \omega + \frac{n_p}{j} T_l$$

$$\dot{e}_{\omega} = \dot{\phi}_{ref} - M \beta_r i_{ds} + \beta_r \phi_{ref}$$
(4)

The Lyapunov function associated with the error and flux velocity, to achieve the objective of pursuit is chosen as follows:

$$f_1 = \frac{1}{2} \left( e_{\omega}^2 + e_{\phi}^2 \right) \tag{5}$$

The derivate of equations (5) is written as follows:

$$\dot{f}_1 = -k_\omega e_\omega^2 - k_\phi e_\phi^2 \tag{6}$$

With  $k_{\omega}$  and  $k_{\phi}$  are positive constant, chosen so as to guarantee the exponential convergence errors of flux and speed.

To satisfy equation (6), we must choose the dynamic errors as the following form:

$$\dot{e}_{\omega} = -k_{\omega}e_{\omega}$$

$$\dot{e}_{\phi} = -k_{\phi}e_{\phi}$$
(7)

Considering that  $i_{qs}$  and  $i_{ds}$  as virtual control inputs, then equations (3) and (7) can generate the stabilizing functions based on the stability condition of Lyapunov theory to achieve the objective of pursuit, which can be written as follows:

$$(i_{qs})_{ref} = \frac{1}{M\beta_r} (\dot{\phi}_{ref} + \beta_r \phi_{dr} + k_{\phi} e_{d\phi})$$

$$(i_{qs})_{ref} = \frac{jL_r}{n_p^2 M \phi_{dr}} \left( \dot{\omega}_{ref} + \frac{f}{j} \omega + \frac{n_p}{j} T_l + k_{\omega} e_{\omega} \right)$$
(8)

The system of equations (4) highlights the desired behavior of flux and the stator currents to ensure the pursuit of speed and rotor flux.

To achieve these desired behaviors, we will define in the second step the error between stator currents, direct flux and their references as follows:

$$e_{iq} = (i_{qs})_{ref} - i_{qs}$$

$$e_{id} = (i_{ds})_{ref} - i_{ds}$$
(9)

By replacing equation (8) into equation (9) we obtain:

$$e_{iq} = \frac{jL_r}{n_p^2 M \phi_{dr}} \left( \dot{\omega}_{ref} + \frac{f}{j} \omega + \frac{n_p}{j} T_l + k_\omega e_\omega \right) - i_{qs}$$

$$e_{id} = \frac{1}{M\beta_r} \left( \dot{\phi}_{ref} + \beta_r \phi_{dr} + k_\phi e_\phi \right) - i_{ds}$$
(10)

Then we can write equation (3) as follows:

$$\dot{e}_{\omega} = -k_{\omega}e_{\omega} + \frac{n_{p}^{2}M\phi_{dr}}{jL_{r}}e_{iq}$$

$$\dot{e}_{z} = -k_{z}e_{z} + M\beta \ e_{zz}$$
(11)

The dynamics errors of  $e_{ia}$  and  $e_{id}$  are respectively given by:

$$\dot{e}_{iq} = \frac{jL_r}{n_p^2 M \phi_{dr}} \left( \ddot{\omega}_{ref} + \frac{f}{j} \dot{\omega} - k_\omega^2 e_\omega \right) + k_\omega e_{iq} - (\alpha_1 + \delta V_{qs})$$

$$\dot{e}_{id} = \frac{1}{M\beta_r} \left( \dot{\phi}_{ref} - M \beta_r^2 i_{ds} + \beta_r \phi_{dr} - k_\phi e_\phi^2 \right) + k_\phi e_{id} - (\alpha_1 + \delta V_{ds})$$
(12)

To have an exponential decrease error of  $e_{iq}$  and  $e_{id}$  we must impose that:

$$\dot{e}_{iq} = -k_{iq}e_{iq}$$

$$\dot{e}_{id} = -k_{id}e_{id}$$
(13)

Where  $k_{ia}$  and  $k_{id}$  are positives parameters.

Final step in the design of the control law is to determine the expressions of the stator voltages  $V_{ds}$  and  $V_{qs}$  from a suitable choice of the new Lyapunov function associated with flux errors, speed and errors of currents which is given by the following expression:

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Figure 1. Block diagram of Backstepping control

Designation	Notations	Rating values
Stator resistance	R <sub>s</sub>	$4.580\Omega$
Rotor resistance	R <sub>r</sub>	$4.468\Omega$
Stator self-inductance	$L_s$	253mH
Rotor self-inductance	L <sub>r</sub>	253mH
Mutual inductance	М	242.3mH
Moment of inertia	j	$0.023 kgm^2$
Friction coefficient	f	0.0026Nm
Number of poles	n <sub>p</sub>	2
Rated power	$P_n$	1.5 <i>Kw</i>
Rated voltage	V <sub>sn</sub>	220V

Table 1. Parameters Of Induction Motor

$$f_2 = \frac{1}{2} \left( e_{\omega}^2 + e_{\phi}^2 + e_{iq}^2 + e_{id}^2 \right)$$
(14)

By replacing equations (7) and (13) in the derivative of the Lyapunov function  $f_2$  we have:

$$\dot{f}_{2} = -k_{\omega}e_{\omega}^{2} - k_{\phi}e_{\phi}^{2} - k_{iq}e_{iq}^{2} - k_{id}e_{id}^{2} + e_{iq\phi}\left(k_{iq}e_{iq} + (\dot{i}_{qs})_{ref} - (\alpha_{1} + \delta V_{qs})\right) + k_{id}\left(k_{id}e_{id} + (\dot{i}_{qs})_{ref} - (\alpha_{2} + \delta V_{ds})\right)$$
(15)



Figure 2. Simulated results to a step speed 1500tr/mn



Figure 3. Simulated results for tracking speed

To ensure that the derivative of the Lyapunov function  $f_2 \mbox{ must}$  be negative:

$$k_{iq} e_{iq} + (\dot{i}_{qs})_{ref} - (\alpha_1 + \delta V_{qs}) = 0$$

$$k_{id} e_{id} + (\dot{i}_{qs})_{ref} - (\alpha_2 + \delta V_{ds}) = 0$$
(16)

Using the equation system (16), the expression of the control law can be writing as follows:

$$V_{qs} = (k_{iq} e_{iq} + (\dot{i}_{qs})_{ref} - \alpha_1)/\mu$$
(17)





Figure 4. Torque response (second test)



Figure 5. Direct and quadratic flux response

# 4. Simulation Results

To verify the proposed solution of the nonlinear control using backstepping control of induction motor, a discrete model in Matlab-Simulink is built with a 10ms sample time. The parameters of induction motor are shown in table 1.

Figure 1 shows the architecture of the vector control algorithm incorporating backstepping technique to design the control law.

Figure 2 shows the first test of the machine to a step speed 1500r/mn under a load torque of 10Nm. In 0.21s the motor reach the steady state. Figure 3 shows the second test concerning the tracking speed. The electromagnetic torque developed by the motor in the second test is showing in figure 4.

Figure 5 shows the dynamic responses of the rotor flux for the tracking speed.

# 5. Conclusion

In this paper, the nonlinear control of induction motor has been presented and verified with simulation results by using Matlab-Simulink environment. The proposed concept is based on the backstepping technique. The different results illustrate the good performance on the tracking speed and flux regulations which certify the robustness of the vector control using this type of control.

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