Construction of Physical Rebound Model Between Ping-Pong and Table /Racket Based on Critical Friction Angle

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ABSTRACT: *Based on the analysis of force, impulse and impulse moment in the rebound phenomenon between ping-pong and the table/racket, this paper introduced the concept of critical friction angle and provided a criterion to determine whether it was rolling or sliding between ping-pong and table/ racket so that the physical rebound model was acquired. Besides, a kind of rebound model based on multiple linear regression(MLR) was acquired by the knowledge of algorithm. Finally, this paper proved the effectiveness of the two rebound models by parameter estimate, experiment verification and error analysis.*

Keywords: Ping-pong robot, rebound model, Critical friction angle, Multiple linear regression, Parameter estimation.

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1. Introduction

Because of its rapid and accurate feature, this sport has raised more requirements for the ping-pong robot, which attracts great attention of many researchers both at home and abroad. This paper gave a summary of the research status on the ping-pong robots both at home and abroad from the perspective of visual system, control system and performing structure[1].

In playing ping-pong by robots, it is required to manage the tracking and trajectory prediction of fast-moving ping-pong balls, as well as control the robots to hit the ball[2]. In other words, as to robots, the task of playing ping-ping with human partners can be divided into several subtasks. Firstly, they should acquire the flying trajectory of ping-pong by visual sensor(intelligent high-speed camera in general), carry out image processing at once and then measure the location, velocity, angular velocity and other flight conditions of ping-pong. Secondly, they should make a trajectory prediction in the next period by analyzing the acquired flight conditions and flying trajectory of ping-pong, namely the location, velocity and angular velocity. Thirdly, there

should be a motion planning for these robots so that they can be controlled to hit the ball in a certain location and in a certain posture and finally reach the goal of hitting balls.

It is shown in the process of controlling ping-pong robots that the rebound model between ping-pong and racket/table plays an important role in the accurate prediction of the flying trajectory and in the design of the motion control planning. When ping-pong collides with the table/racket, it will produce some rotation under the effect of friction. Thus there is supposed to be a shift in the flying trajectory after rebound under Magnus Effect. Therefore, the rebound model between ping-pong and racket/table is attracting more attention of more researchers. Zhang, et al.[3] assumed that there was a linear relation between the velocity in two directions parallel to the contact plane. In the normal line direction on the contact plane, the velocity after collision was acquired by elastic restitution coefficient. There would certainly be an energy loss in the rebound process between ping-pong and table. The energy loss was expressed by the rate of velocity loss in the horizontal direction and the restitution coefficient in the vertical direction[4]. The two models above failed to take the rotation of ping-pong into consideration, which failed to be applied widely thus. Andersson [5] revealed that the rebound result was decided by the velocity of ball, angular velocity, and the property of ping-pong and table. In the normal line direction on the contact plane, the status after rebound was decided by restitution coefficient. In the horizontal direction, the rebound velocity of ball was dependent on the rotation velocity of ball, friction of table and the rebound coefficient in the vertical direction. However, he simply provided the qualitative theory in his paper without the analytic expression of rebound model. There were also two papers[6-7] providing a criterion to determine whether it was rolling or sliding between ping-pong and hitting surface by the analysis of the contact types between ping-pong and hitting surface. Then, the analytic expression of rebound model was acquired, but too many assumptions within these two papers failed the wide application of this model.

2. Problem Analysis

The difficulty in analyzing the collision process between ping-pong and some plane lies in three aspects. Firstly, the process of energy transformation is quite complex. The ping-pong itself and the contact plane is likely to be elastic. When the ping-pong collides with the hitting surface, it is hard to determine whether and how many the kinetic energies of ping-pong will be transformed into elastic potential energy. Secondly, it is hard to determine the relative motion between ping-pong and hitting surface is rolling or sliding or the combination. Thirdly, it is hard to determine whether there is any friction between ping-pong and hitting surface and how much friction there is. The uncertainty of these problems leaves some difficulties on construction of this model.

Physicians have made some scientific achievements in deeper studies on this. Garwin introduced the restitution coefficient parallel to the contact plane, and put forward a method to detect whether the mutual reaction between ping-pong and plane was rolling or sliding or the combination[8] and detect whether the kinetic energy was reserved as elastic potential energy. Without considering the ping-pong compression, Brody analyzed the effect of plane friction and acquired that it was rolling friction between ping-pong and plane, which was related to the translation and rolling velocity before rebound. Cross made an analysis of the effect of ping-pong with different properties and fixed plane, and made deeper studies on the mutual reaction and restitution coefficient in the vertical direction. It is shown in all literature above that friction and resilience play a decisive role in the rebound process. In the rebound process of table, the friction plays a major role, and its amount depends on the fact whether the relative motion between ping-pong and table is rolling, sliding or the combination. However, in the rebound process of ping-pong and racket, resilience plays a leading role. Physical scientists also made several studies and acquired a lot of qualitative results. For example, in the two papers[6-7], they analyzed the collision process between ping-pong and racket based on rotation theorem. Moreover, they introduced the definition of hitting friction angle and made a qualitative discussion over the influence of hitting friction angle on the passing velocity, rotation effect and hitting action.

This paper based on the papers[6-7] introduced the concept of critical friction angle, provided the criterion to determine whether it was rolling or sliding between ping-pong and table/racket, and it further acquired the physical rebound model between the two. Beside, based on the knowledge of algorithm, a rebound model was acquired by MLR. Finally, it validated the effectiveness of the two rebound models based on parameter estimation, experiment verification and error analysis.

3. Critical Friction Angle

3.1 Definition of Critical Friction Angle

When the flying ping-pong hit toward a certain hitting surface, let á denote the angle between velocity of oncoming ball and normal line on the hitting surface. When á is no less than ö, the relative motion between ping-pong and hitting surface is sliding. Otherwise, it is a rolling friction, and this angle ö is the so-called critical friction angle. The critical friction angle is related to the material of contacting plane and property of ping-pong, and is in a close relation with friction coefficient and rebound coefficient of contacting plane in particular. The derivation process for its analytic expression is shown as follows.

3.2 Derivation Pprocess of Critical Friction Angle

This section takes the example of rebound phenomenon between ping-pong and table(figure1), and shows the solution of critical friction angel. Firstly, it builds a world coordinate system(WCS) which takes the longer edge of table as X axis, the shorter edge as Y axis, and the upward direction perpendicular to the plane of table as the positive direction of Z axis. Secondly, let V_i denote the incident velocity of ping-pong and ω_i denote the angular velocity acquired from the flight trajectory, let V_e denote the passing velocity and ω_e denote the angular velocity of ping-pong after collision. Velocity and angular velocity was expressed by x, y and z respectively in the measure of X, Y and Z axis. For example, the velocity of oncoming ball is V_{iz} in the normal line of ping-pong table, namely the measure of Z axis. The variables of incidence and emergence are expressed in subscript i and e respectively.

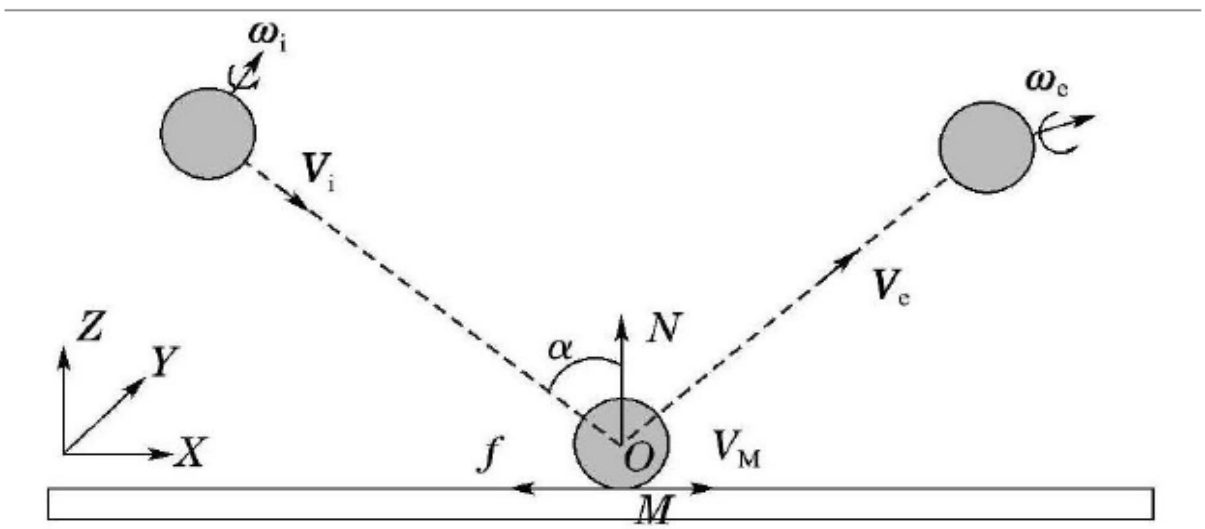

Figure1. Rebound phenomenon between ping-pong and table

In the collision between ping-pong and table, ping-pong is constantly impacting the table, and the velocity of Z axis is gradually reduced. When reaching the minimum M, the velocity measure of ping-pong in Z axis is equal to zero, namely $V_{iz} = 0$ When neglecting the rotation of ping-pong, it can be considered as the mass point. When taking account of its rotation , the rebound process between ping-pong and table is analyzed based on the force analysis of the minimum. Let r denote the radius, and there is no deformation of ping-pong in the rebound process between ping-pong and table, the vector from the center O to contact point M is expressed as $\overline{OM} = (0,0, -r)^T$, and the velocity at the point M is:

$$V_M = V + \overline{OM} \times \omega = \begin{bmatrix} v_x - r\omega_y \\ v_y - r\omega_x \\ 0 \end{bmatrix}$$

Where in $\omega = \lfloor \omega_x \, \omega_y \, \omega_z \rfloor$ was the angular velocity of ping-pong.

The velocity at point M parallel to table is shown in figure 2. It is shown from figure 2 that the velocity at point M is:

$$v_{xy} = \sqrt{(v_x - r\omega_y)^2 + (v_y + r\omega_x)^2}$$

And

$$\sin\theta = \frac{v_x - r\omega_y}{\sqrt{(v_x - r\omega_y)^2 + (v_y + r\omega_x)^2}} = \frac{v_x - r\omega_y}{\| v_{xy} \|}$$

$$\cos\theta = \frac{v_y - r\omega_x}{\sqrt{(v_x - r\omega_y)^2 + (v_y + r\omega_x)^2}} = \frac{v_y + r\omega_x}{\| v_{xy} \|}$$

It is shown in papers[6-7] that before and after rebound between ping-pong and table, when point M enjoys a consistent velocity and direction parallel to the measure of table, a relative sliding is induced between ping-pong and table. Otherwise, a relative rolling is induced. Next, the boundary conditions are acquired by analyzing the rebound model when there is a relative sliding between ping-pong and table, and then the analytic expression of critical friction angle is further obtained.

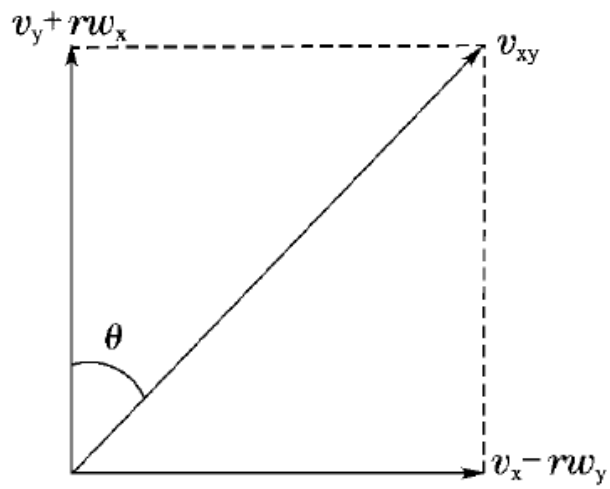

Figure 2. Velocity in the horizontal direction on table at the point M

3.3 Rebound Model in the Relative Sliding Between Ping-Pong and Table

When there is a relative sliding between ping-pong and table, make a force analysis of point M. Table enjoys a vertical and upward support N and friction f in the horizontal direction. Let μ denote the sliding friction coefficient between the two, and then f meets the need of equation as follows:

$$f = \mu N,$$

The friction f and the velocity measure in the tangent line direction on hitting surface are in the opposite direction . Let m denote the mass of ping-pong, P denote its impulse, and $I = \frac{2}{3} mr^2$ denote its rotational inertia. The rebound process between ping-pong and table is shown in figure 1. From the impulse theorem and the impulse moment theorem, we have:

$$mV_e - mV_i = P \qquad (1)$$

$$I\omega_e - I\omega_i = OM \times P \qquad (2)$$

The motion of ping-pong in the normal line direction on hitting surface is decoupled with its motion parallel to the hitting surface. Next, this paper makes a discussion over the motion in these two directions respectively. Ping-pong simply receives a support in the normal line direction on hitting surface, thus the change in the motion velocity in the normal line direction on hitting surface is only related to N. Let e denote the elastic restitution coefficient of table, and then we have:

$V_{ez} = -eV_{iz}$

Based on impulse theorem, then:

$$mV_{ez} - mV_{iz} = P_z = \int_0^S N dt = -(1+e)mV_{iz}$$

Wherein S refers to the action cycle in the collision between ping-pong and table. Let P_{xy} denote the impulse suffering the ping-pong parallel to the hitting surface. According to the equation $f = \mu N$ and the impulse theorem, the P_{xy} meets the need of following equation:

$$mV_{exy} - mV_{ixy} = P_{xy} = -\int_0^S f dt = -\mu \int_0^S N dt = -\mu P_z = (1+e)m\mu V_{iz}$$

P_{xy} and the velocity v_{xy} at the point M in the tangent line direction on table enjoy a same direction, while the friction f and v_{xy} are in an opposed direction. This is why there is a Minus after the second sign of equality. It is shown in figure2 that the impulse in the direction of X and Y axis are respectively:

$$P_x = P_{xy} \sin\theta = -\mu |P_z| \sin\theta$$

$$P_y = P_{xy} \cos\theta = -\mu |P_z| \cos\theta$$

Then,

$$P = \begin{bmatrix} P_x \\ P_y \\ P_z \end{bmatrix} = \begin{bmatrix} -\mu |P_z| \sin\theta \\ -\mu |P_z| \cos\theta \\ P_z \end{bmatrix}$$

Then substitute this into equation (1) and (2), and then have them simplified,

$$V_e = V_i + \frac{P}{m} = \begin{bmatrix} V_{ix} - (1+e)|V_{iz}|\mu \dfrac{V_{iz} - r\omega_{iy}}{\| v_{ixy} \|} \\ V_{iy} - (1+e)|V_{iz}|\mu \dfrac{V_{iy} - r\omega_{ix}}{\| v_{ixy} \|} \\ -eV_{iz} \end{bmatrix}$$

$$\omega_e = \omega_i + \frac{\overline{OM} \times P}{I} = \begin{bmatrix} \omega_{ix} - \dfrac{3(1+e)}{2r}|V_{iz}|\mu \dfrac{V_{ix} - r\omega_{iy}}{\| v_{ixy} \|} \\ \omega_{iy} - \dfrac{3(1+e)}{2r}|V_{iz}|\mu \dfrac{V_{iy} - r\omega_{ix}}{\| v_{ixy} \|} \\ \omega_{iz} \end{bmatrix}$$

$$\begin{cases} V_e = V_i + \dfrac{P}{m} = \begin{bmatrix} V_{ix} - (1+e)\mu ctg\alpha (V_{ix} - r\omega_{iy}) \\ V_{iy} - (1+e)\mu ctg\alpha (V_{iy} - r\omega_{ix}) \\ -eV_{iz} \end{bmatrix} \\ \omega_e = \omega_i + \dfrac{\overrightarrow{OM} \times P}{I} = \begin{bmatrix} \omega_{ix} - \dfrac{3(1+e)}{2r}\mu ctg\alpha (V_{iy} - r\omega_{ix}) \\ \omega_{iy} - \dfrac{3(1+e)}{2r}\mu ctg\alpha (V_{ix} - r\omega_{iy}) \\ \omega_{iz} \end{bmatrix} \end{cases} \quad (3)$$

Let α denote the angle between the velocity of oncoming ball and normal line on the hitting surface, namely let α denote the incidence angle, and then we have $ctg\ \alpha = \frac{|V_{iz}|}{\|v_{ixy}\|}$. Therefore, when considering the angular velocity, the rebound model between ping-pong and table can be expressed as follows:

3.4 Determination of Critical Friction Angle

It is shown in the rebound model acquired from the relative sliding between ping-pong and table in section 3.1 that the velocity at the point M after rebound between ping-pong and table can be acquired from the following equation:

$$V_{eM} = \begin{bmatrix} v_{ex} - r\omega_{ey} \\ v_{ey} - r\omega_{ex} \\ 0 \end{bmatrix} = \begin{bmatrix} v_{ix} - r\omega_{iy} - \frac{5(1+e)}{2r}\ ctg\ \alpha\mu\,(V_{ix} - r\omega_{iy}) \\ v_{iy} - r\omega_{ix} - \frac{5(1+e)}{2r}\ ctg\ \alpha\mu\,(V_{iy} - r\omega_{ix}) \\ 0 \end{bmatrix} =$$

$$\left(1 - \frac{5(1+e)}{2}\mu ctg\ \alpha\right)\begin{bmatrix} V_{ix} - r\omega_{iy} \\ V_{iy} - r\omega_{ix} \\ 0 \end{bmatrix} = \left(1 - \frac{5(1+e)}{2}\mu ctg\ \alpha\right)V_{iM} \qquad (4)$$

Because V_{eM} and V_{iM} are in a same direction, we can get $1 - \frac{5(1+e)}{2}\mu ctg\ \alpha \geq 0$, that is $tg\alpha \geq \frac{5(1+e)}{2}\mu$, $\alpha \geq \arctan\left(\frac{5(1+e)}{2}\mu\right)$.

The boundary conditions for a relative sliding between ping-pong and table are $\alpha \geq \arctan\left(\frac{5(1+e)}{2}\mu\right)$. In other words, the critical friction angle $\varphi = \arctan\left(\frac{5(1+e)}{2}\mu\right)$. We can reach a similar conclusion by analyzing the rebound process between ping-pong and racket.

3.5 Rebound Model in Relative Rrolling Between Ping-Pong and Table

The derivation process of critical friction angle is stated as above. When the angle between the velocity of oncoming ball and normal line on hitting surface is greater than or equal to the critical friction angle, there is a sliding friction between ping-pong and table, and the rebound model is shown in equation (3).

When the incidence angle of velocity of oncoming ball is less than the critical friction angle, there is a relative rolling between the two, and a smaller rolling friction which can be neglected. Then we have $P_{xy} = P_x = P_y = 0$. From impulse theorem and impulse moment theorem, there is a fixed velocity and angular velocity parallel to the hitting surface. In the normal line direction on hitting surface, there is a reduced velocity in the rate of restitution coefficient but a fixed angular velocity. When there is a relative rolling between spinning ball and table, the rebound model is as follows:

$$\begin{cases} V_e = \begin{bmatrix} V_{ix} \\ V_{iy} \\ -eV_{iy} \end{bmatrix} \\ \omega_e = \omega_i. \end{cases} \qquad (5)$$

4. Rebound Model Between Ping - Pong And Table Acquired From The Knowledge of Algorithm

For stress of ping-pong in flight process is decoupled in the tangent line direction and normal line direction on hitting surface, the rebound model is separately analyzed in two directions.

Based on the analysis of force, impulse and impulse moment in the rebound between ping-pong and table, there is a reduced speed in the rate of restitution coefficient in the normal line direction on hitting surface but a fixed angular velocity. In a word, the velocity and angular velocity before and after rebound meets the following equation:

$$\begin{cases} V_{ez} = -eV_{iz} \\ \omega_{ez} = \omega_{iz}. \end{cases} \tag{6}$$

In the tangent line direction on hitting surface, it is shown in the analysis process of section 3 that the velocity in direction of X axis after rebound is related to the velocity in direction of X axis and angular velocity in direction of Y axis before rebound, and the angular velocity is related to that in the direction of X axis and velocity in the direction of Y axis before rebound. Correspondingly, the velocity in the direction of Y axis after rebound is related to the angular velocity in the direction of X axis and velocity in the direction of Y axis before rebound, and the angular velocity is related to that in the direction of Y axis and velocity in the direction of Y axis before rebound. This paper supposes this to be a linear relation and fits the relation between velocity and angular velocity before and after rebound by MLR.

$$\begin{cases} V_{ex} = k_{vx1}V_{ix} + k_{vx2}\omega_{iy} + b_{vx} \\ V_{ey} = k_{vy1}V_{ix} + k_{vy2}\omega_{ix} + b_{vy} \\ V_{ex} = k_{vx1}V_{iy} + k_{vx2}\omega_{ix} + b_{vx} \\ V_{ey} = k_{vy1}V_{ix} + k_{vy2}\omega_{iy} + b_{vy} \end{cases} \tag{7}$$

Based on the detection data from several groups, the value of all parameters is acquired by MLR.

5. Rebound Model Between Ping - Pong And Racket

The difference of racket and table from the ping-pong sport lies in the following aspects: racket is moving but the table is static. Therefore, the absolute motion of ping-pong the relative motion between ping-pong and racket + motion of racket. Thus the rebound model between ping-pong and racket is introduced.

The relation between velocity in WCS and velocity in racket coordinate system is expressed in the equation(8):

$$V = \dot{T} \cdot {}^{e}Q + T \cdot {}^{e}V. \tag{8}$$

Wherein $\boldsymbol{V}$ refers to the velocity in the WCS, $\boldsymbol{T}$ refers to the transformation matrix from WCS to racket coordinate system. $\dot{\boldsymbol{T}}$ refers to the derivative of $\boldsymbol{T}$, ${}^{e}Q$ refers to the homogeneous coordinate in the racket coordinate system, and ${}^{e}V$ refers to the velocity expressed in racket coordinate system.

Suppose that the position of racket at hitting moment stays static, it is all zero to solve the derivative of time by some elements of rotation matrix in $\boldsymbol{T}$ matrix. However, it is not zero to solve the derivative of time by the translation vector $\boldsymbol{T}$ in matrix. Let the derivative of time by translation vector in $\boldsymbol{T}$ matrix be, $\left(\dot{q}_x, \dot{q}_y, \dot{q}_z\right)^T$, and the velocity of racket at the hitting moment be, $V_{ph} = (v_{phx}, v_{phy}, v_{phz})^T$, then:

$$\dot{T} \cdot {}^{e}Q = \begin{bmatrix} 0 & 0 & 0 & \dot{q}_x \\ 0 & 0 & 0 & \dot{q}_y \\ 0 & 0 & 0 & \dot{q}_z \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} {}^{e}q_x \\ {}^{e}q_y \\ {}^{e}q_z \\ 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & v_{phx} \\ 0 & 0 & 0 & v_{phy} \\ 0 & 0 & 0 & v_{phz} \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} {}^{e}q_x \\ {}^{e}q_y \\ {}^{e}q_z \\ 1 \end{bmatrix} = \begin{bmatrix} v_{phx} \\ v_{phy} \\ v_{phz} \\ 0 \end{bmatrix} = V_{ph} \tag{9}$$

Substitute equation(9) into equation (8), then

$$V = V_{ph} + T\,{}^{e}V \qquad (10)$$

Substitute the velocity of oncoming ball V_i, and angular velocity ω_i acquired from flight trajectory, the emergence velocity V_e and angular velocity ω_e of ping-pong after collision into equation (10) respectively. Besides, transform the equations into forms as follows:

$${}^{e}V_i = T^{-1}(V_i - V_{ph}) \qquad (11)$$

$${}^{e}V_e = T^{-1}(V_e - V_{ph}) \qquad (12)$$

As to angular velocity, for the position of racket at the hitting moment stays static, the relation between angular velocity expressed in WCS and that expressed in racket coordinate system meets the following equation:

$$\omega = T\,{}^{e}\omega, \text{ that is } {}^{e}\omega = T^{-1}\omega$$

Substitute ${}^{e}V_i$, ${}^{e}V_e$ and the former equation into equation(1) and (5) or (6) and (7), the rebound model of racket can be acquired.

6. Parameter Estimation

Let $(1 + e)\mu = \eta$, then the equation (3) can be simplified into the linear system about parameter η and e.

$$V_e - \begin{bmatrix} V_{ix} \\ V_{iy} \\ 0 \end{bmatrix} = \eta \begin{bmatrix} ctg\,\alpha\,(V_{ix} - r\omega_{iy}) \\ ctg\,\alpha\,(V_{iy} - r\omega_{ix}) \\ 0 \end{bmatrix} - e \begin{bmatrix} 0 \\ 0 \\ V_{iz} \end{bmatrix}$$

The motion trajectory for ping-pong in 12 groups is acquired based on experimental platform. The velocity and angular velocity before and after collision between ping-pong and table are acquired by method in literature[9]. Please refer to table 1.

No.	$V_i/(m\cdot s^{-1})$	$\omega_i/(rad\cdot s^{-1})$	$V_e/(m\cdot s^{-1})$	$\omega_e/(rad\cdot s^{-1})$
1	(−0.92,2.89,−2.46)T	(−35.44,2.89,−7.29)T	(0.54,−1.97,−2.22)T	(11.35,0.85,−0.0)T
2	(−0.20,−3.03,−2.57)T	(−45.9,−43.7,−41.1)T	(−0.15,−2.08,2.28)T	(12.4,13.5,−9.98)T
3	(0.45,−3.21,−2.60)T	(30.2,−23.4,−15.86)T	(0.26,−2.14,2.31)T	(17.4,7.45,−6.44)T
4	(0.72,−2.96,−2.61)T	(47.2,2.32,12.83)T	(0.41,−1.92,2.31)T	(20.3,8.03,−5.70)T
5	(0.41,−3.14,−2.64)T	(−12.5,−77.0,−85.9)T	(0.24,−2.14,2.32)T	(14.2,−11.7,12.6)T
6	(0.29,−2.70,−2.50)T	(−33.2,−38.8,−43.3)T	(0.16,−1.82,2.25)T	(14.0,5.38,1.30)T
7	(−0.27,−2.71,−2.59)T	(−37.3,−55.6,−52.3)T	(−0.19,−1.82,2.3)T	(16.5,15.1,−6.47)T
8	(−0.41,−3.04,−2.86)T	(−47.1,−23.2,−17.2)T	(−0.29,−2.09,2.45)T	(14.3,24.6,−18.7)T
9	(−0.46,−264,−2.98)T	(−54.1,−59.2,−49.5)T	(−0.32,−1.78,−2.51)T	(16.5,16.7,−8.02)T
10	(−0.24,−3.07,−2.62)T	(−45.0,−26.5,−20.1)T	(−0.18,−2.11,−2.31)T	(12.9,20.9,−16.8)T
11	(−052,−3.18,−2.68)T	(−27.2,−51.5,−43.1)T	(−0.35,−2.17,−2.35)T	(16.1,20.6,−16.5)T
12	(0.35,−3.17,−2.64)T	(−53.6,−21.5,−19.1)T	(−0.25,−2.2,−2.32)T	(11.5,20.2,−18.0)T

Table1. Measured Value of Velocity and Angular Velocity

That is

$$\begin{bmatrix} V_{ex}-V_{ix} \\ V_{ey}-V_{iy} \\ V_{ez} \\ \omega_{ex}-\omega_{ix} \\ \omega_{ey}-\omega_{iy} \\ \omega_{ez}-\omega_{iz} \end{bmatrix} = \eta \begin{bmatrix} -ctg\alpha\left(V_{ix}-r\omega_{iy}\right) \\ -ctg\alpha\left(V_{iy}-r\omega_{ix}\right) \\ 0 \\ \dfrac{3ctg\alpha\left(V_{iy}-r\omega_{ix}\right)}{2r} \\ \dfrac{3ctg\alpha\left(V_{ix}-r\omega_{iy}\right)}{2r} \\ 0 \end{bmatrix} + e \begin{bmatrix} 0 \\ 0 \\ -V_{iz} \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{13}$$

Given the velocity and angular velocity before and after the collision between ping-pong and table, there are only the parameter η and e unknown in equation (13). Then the estimation of parameter η and e is acquired by the least square parameter identification, and the estimation value of η is obtained by the formula of $\mu = \eta/(1+e)$, and the critical friction angle by the equation of $\varphi = \arctan\left(\frac{5(1+e)}{2}\mu\right)$. The results are shown as follows:

$e = 0.8788$, $\mu = 0.1049$, $\varphi = 26.2318$

7. Experment Verification

This paper names the model made up of equation (3) and(5) as the physical model, and the model made up of equation(6) and (7) as the learning model. Besides, this paper substitutes the estimation value of parameter μ and e acquired from the least square parameter identification, velocity and angular velocity before collision between ping-pong and table into the two models in this paper and the model in literature[3,6,7] respectively. Next, it makes a comparison among the velocity and angular velocity after rebound from calculation and the measured value. The estimated error among the velocity and angular velocity after rebound from calculation for four rebound models and the measured data are shown in table 2 together with the comparison of error in these models.

Variable	Physical model in this paper	learning model in this paper	Model in literature[6-7]	Model in literature[3]
$V_e/(m\cdot s^{-1})$	Mean error (0.0420，0.5344，0.0023)T	(0, 0, 0.0023)T	(0.0951, 0.3805, 0.0023)T	(0, 0, 0.0023)T
	Maximal error (0.2524，1.0700，0.0582)T	(0.0073, 0.0081, 0.0582)T	(0.2524, 0.5490, 0.0582)T	(0.0119, 0.0866, 0.0582)T
	Mean variance (0.0374，0.0631，0.0022)T	(0, 0, 0.0022)T	(0.0204, 0.0195, 0.0022)T	(0, 0.0012, 0.0022)T
$\omega_e/(rad\cdot s^{-1})$	Mean error (9.19，40.33，24.11)T	(0, 0, 24.11)T	(-2.36, 36.35, 24.11)T	—
	Maximal error (27.46，67.62，98.59)T	(2.31, 2.43, 98.59)T	(27.46, 67.62, 98.59)T	—
	Mean variance (282.90，558.39，976.80)T	(2.49, 3.87, 976.80)T	(1734.3, 821.8, 976.8)T	—

Table 2. Mean Error, Maximal Value and Variance of Estimated Results and Measured Value of Four Rebound Models

In conclusion, two rebound models in this paper achieves good estimation results, and proves the effectiveness. By making a comparison of two models in this paper with two models in literature[6,7] in their defects and merits, we can get the following results:

1) These four rebound models are same in the direction of normal line on table. They enjoy a perfect estimation result for velocity, but a greater error in estimation for angular velocity. However, the angular velocity in the direction of normal line leaves little effect on the accuracy of trajectory prediction, which thus fails to affect the effectiveness of this model.

2) In the direction parallel to table, learning model in this paper achieves the optimal estimation result. The estimation error for angular velocity is obviously less than the physical model in this paper and models in the literature[6,7] in particular. The model in literature [3] enjoys a better estimation result for velocity. However, it fails to be applied widely without considering the angular velocity. The estimation result for parameters in learning model is dependent on the selected measured data. In practical application, it can help to further reduce the estimation error for the model by constantly updating the parameters in model from online learning.

3)The merits of the physical model in this paper compared with the rest three models lie in following aspects: there is more definite physical significance in the derivation process, and the determination criteria for motion types between ping-pong and hitting surface is related to the incidence information. However, the determination criteria in literature[6,7] is related to the output information. The determination criteria in physical model from this paper can be acquired from the measured data before rebound, which is more real-time and accurate. When there is a relative rolling between ping-pong and table, the physical model in this paper enjoys a much more precise result compared with that in literature[6,7], which especially enjoys less estimation error for angular velocity.

8. Conclusion

Based on analysis of momentum and angular momentum in the rebound between ping-pong and table/racket, this paper introduced the concept of critical friction angle. Besides, it provided the criterion to determine whether it was sliding or rolling between ping-pong and table-racket. When the flying ping-pong hit toward the certain hitting surface, the angle between the velocity of oncoming ball and the normal line direction in the hitting surface is greater than or equal to the critical friction angle, there is a sliding motion between ping-pong and hitting surface. Otherwise, when the incidence angle is less than critical friction angle, there is a rolling motion between the two and a rebound model is further acquired between ping-pong and table based on that. According to the analysis of physical process, the mutual relation between all variables is acquired and a kind of linear rebound model is acquired by learning and linear simulation of multiple variables. The simulation experiment proved the effectiveness of this model.

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