Duopoly Game Models of Water Supply in the Presence of Market Power

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ABSTRACT: *The purpose of this paper is to investigate the allocation of output and the water supply for two firms to produce differentiated product in two oligopoly game models established under the settings of complete information and incomplete information respectively. By solving a linear demand system model, this paper obtains the corresponding allocation results of output and the water supply for two firms under the condition that the market power exists and the information on the marginal costs is complete. Furthermore, the relation between the allocation result of output and the equilibrium price of water markets in the framework of complete information is analyzed. The study result shows that when the equilibrium price of water markets raises, the total equilibrium output will decrease, while the change rule of the individual output is complicated. Under the condition that water consumption parameters in model are different, and the other parameters in model satisfy suitable conditions, the market shares of output due to the change of the equilibrium price of water markets transforms from the firm who requires more water quantity to the firm who requires less water quantity. Finally, this paper compares the allocation results obtained under the setting of incomplete information on marginal cost with that obtained under the setting of complete information on marginal cost.*

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1. Introduction

Water resource trading is a water market policy which can reduce unnecessary and exorbitant waste of water resource by using economic means and it has features of both water quantity assurance and cost-effective in the control of water consumption [1-2]. It is a more effective means by using the invisible hand of market to control water resource utilization. To some extent, although economists mainly concerned with fair and efficient problems, they have committed themselves to realizing water resource trading in the process of the international political agenda [3-4]. Now in the process of the implementation of water market policy, economists must deliberate on the effective allocation of water supply in the real world. Therefore, water resource trading attracts the universal concern of all countries in the world. In order to implement the system of the tradable water supply, we must first solve a key problem on the allocations of water supply from both a theoretical and a practical level [5].

According to the existing literatures, we find that many authors made a lot of efforts to study the allocation of emission right and water supply by using homogeneous oligopoly game model. Heller [6] started to pay great attention to the initial emission permits allocation problem and carried on the discussion in this aspect. By using a homogenous oligopoly game model and auction theory, Sunnevåg [7] studied the allocation of permits under two auction mechanisms. Recently, by using homogeneous oligopoly game theory, Foellmi and Meister [8] established a model of water supply and product market competition. On the basis of the results of the model, they analyzed and compared the relevant welfare gains and showed that production efficiency and retail prices may differ depending on the initial cost differential, the application of regulations

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and the distribution of bargaining power. Using a theoretical model, they showed that at higher initial production cost differentials, welfare is higher under competitive conditions, even in a lower-bound benchmark case without any regulation. Considering market power, Ansin and Houba [9] established a water market model as multi-market Cournot competition with a river structure. In their model, suppliers are connected through water balances, which impose resource constraints, and they are connected to heterogeneous water users via a water delivery infrastructure. Furthermore, they gave conditions for the existence of water market equilibrium and assessed the effects of market power on water extraction, delivery, and water prices.

Inspired and motivated by the above results in this research field, in this paper, we establish two duopoly game models of product quantity and the water supply for two firms to produce differentiated product under the settings of complete information and incomplete information, respectively and obtain the corresponding allocation results of product quantity and the water supply under the condition that the market power exists, and next, we compare the allocation results obtained under the setting of incomplete information on marginal cost with that obtained under the setting of complete information on marginal cost.

2. The basic model

In this paper, we consider the allocation model of the water supply of two firms producing two differentiated good. Let $p_1 = A - Bq_1 - Dq_2$ and $p_2 = A - Bq_2 - Dq_1$ be the linear inverse demand curves facing firm 1 and firm 2 respectively, where q_i denotes the output of firm i $(i = 1, 2)$; p_1 and p_2 denote the price of two different products produced by firm 1 and firm 2 respectively. Parameter A measures the market size or the reservation price, which is assumed to be equal across varieties for the sake of simplicity. As for parameters B and D, we assume that $0 \leq D \leq B$. Notice that parameter D captures the degree of substitutability between the two different goods produced by two firms. In the limit case $D = 0$, goods are independent and each firm becomes a monopolist. In the opposite limit case $D = B$, the goods produced by two firms are perfect substitutes and the model collapses into the homogenous oligopoly model. Thus, the higher is parameter D, the lower is the degree of differentiation.

In the process of production, we suppose that each unit product requires water quantity at the proportional rate of δ_i. However, each firm can substitute away from permits either by engaging in water consumption abatement or reducing production; thus, the ultimate water consumption of firm i is given as follows:

$$Q_i = \delta_i q_i - d_i q_i$$

where d_i is the abatement level of firm i. Consequently, decisions in the product and water markets are linked. Without loss of generality, we assume that the cost of abatement is assumed to be quadratic in both output and abatement per unit of output:

$$k_i(d_i, q_i) = \alpha d_i q_i + \beta (d_i q_i)^2 \tag{1}$$

where $i = 1, 2$ and the nonnegative parameters α and β denote technological parameters. Therefore, by the above formula with the assumption on α and β, we know that

$$\partial k_i / \partial q_i \geq 0, \partial k_i / \partial d_i \geq 0 \ (i = 1, 2)$$

that is, the cost of abatement is non-decreasing function on α and β.

3. Duopoly Game and Water Supply Allocation

Now we consider the above duopoly game model with the production of heterogeneous products. Tradable water markets are considered, as an input with a fixed water supply $\overline{Q}$, which is exogenously determined by the authorities. Let (Q_1, Q_2) be the water supply of two firms such that $\overline{Q} = Q_1 + Q_2$. If both firms are assumed to be price takers in the water markets, then each firm i's profit maximization problem becomes:

$$\underset{q_i, d_i}{Max}\ \Pi_i = p_i q_i - c_i q_i - k_i(d_i, q_i) - \eta Q_i \tag{2}$$

where $i = 1, 2$, $Q_i = \delta_i q_i - d_i q_i$, c_i is the marginal cost of firm i and η is the equilibrium price of water markets. Substituting (1) into (2) and using the first order condition for profit maximization lead to the following results:

$$\begin{aligned} \partial \Pi_i / \partial q_i &= -\alpha q_i - 2\beta d_i q_i^2 + \eta q_i \leq 0 \\ d_i \times \partial \Pi_i / \partial d_i & \\ &= d_i(-\alpha q_i - 2\beta\alpha d_i q_i^2 + \eta q_i) \\ &= 0 \end{aligned} \tag{3}$$

Usually, we assume $d_i > 0$ $(i = 1, 2)$, thus from (3), we obtain $d_i = (\eta - \alpha) / 2\beta q_i$, $i = 1, 2$.

At the interior Cournot equilibrium, the first order condition of profit maximization for firm i is given as follows:

$$\partial \Pi_i / \partial d_i = p_i - c_i + \partial p_i / \partial q_i - \eta(\delta_i - d_i) = 0$$

Therefore, we have

$$q_i = (A - c_i - Dq_j - \eta\delta_i) / 2B$$

where $i \neq j$, i, $j = 1, 2$. In addition, we assume that:

i. $A - c_i - \eta\delta_i > 0$, $i = 1, 2$;

ii. $0 < \dfrac{D}{2B} < min\ \{ \dfrac{A - c_1 - \eta\delta_1}{A - c_2 - \eta\delta_2}, \dfrac{A - c_2 - \eta\delta_2}{A - c_1 - \eta\delta_1} \}$;

Solving the system of linear equations composed by two firms' reaction functions leads to the following Nash equilibrium product quantity of two firms and the total production quantity of two firms:

$$q_1^{\,*} = \frac{2B\,(A - c_1 - \eta\delta_1) - D\,(A - c_2 - \eta\delta_2)}{4B^2 - D^2},$$

$$q_2^{\,*} = \frac{2B\,(A - c_2 - \eta\delta_2) - D\,(A - c_1 - \eta\delta_1)}{4B^2 - D^2},$$

$$q_1^{\,*} + q_2^{\,*} = \frac{(2B - D)\,[2A - c_1 - c_2 - \eta\,(\delta_1 + \delta_2)]}{4B^2 - D^2}.$$

By assumptions I and II, it is easy to check that the above Nash equilibrium product quantity of two firms are interior solutions, that is, $q_1^{\,*} > 0, q_2^{\,*} > 0$. In fact, we prove the fact under two different conditions that $A - c_1 - \eta\delta_1 \geq A - c_2 - \eta\delta_2$ and $A - c_1 - \eta\delta_1 < A - c_2 - \eta\delta_2$.

Case 1. If $A - c_1 - \eta\delta_1 \geq A - c_2 - \eta\delta_2$, then we have $2B > D$ by assumption II. Therefore, by assumptions I and II, we know that $q_1^{\,*} > 0, q_2^{\,*} > 0$;

Case 2. if $A - c_1 - \eta\delta_1 < A - c_2 - \eta\delta_2$, then we can easily prove that $q_1^{\,*} > 0, q_2^{\,*} > 0$. By the above formulas, we have

$$\frac{d\,(q_1^{\,*} + q_2^{\,*})}{d\eta} = -\frac{\delta_1 + \delta_2}{2B + D} < 0,$$

$$q_1^{\,*} - q_2^{\,*} = (2B + D)[(c_2 + \eta\delta_2) - (c_1 + \eta\delta_1)].$$

Obviously, the sign of $q_1^{\,*} - q_2^{\,*}$ depends on the sign of $(c_2 + \eta\delta_2) - (c_1 + \eta\delta_1)$ and we have:

$$q_1^{\,*} \geq q_2^{\,*} \Leftrightarrow (c_2 + \eta\delta_2) \geq (c_1 + \eta\delta_1);$$
$$q_1^{\,*} < q_2^{\,*} \Leftrightarrow (c_2 + \eta\delta_2) < (c_1 + \eta\delta_1).$$

To examine the effects of the equilibrium price of water markets on the Nash equilibrium quantity of two firms, we differentiate $q_1^{\,*}$ and $q_2^{\,*}$ with respect to η as follows:

$$\frac{dq_1^{\,*}}{d\eta} = \frac{D\delta_2 - 2B\delta_1}{4B^2 - D^2}, \qquad (4)$$

$$\frac{dq_2^{\,*}}{d\eta} = \frac{D\delta_1 - 2B\delta_2}{4B^2 - D^2}. \qquad (5)$$

By (4), (5) and assumption II, we know that $dq_i^{\,*}/\,d\eta < 0$ ($i = 1, 2$), which implies that an increase in η decreases the Nash equilibrium product quantity of two firms under the condition that the assumption III is satisfied. Furthermore, by (4) and (5), we obtain the following two results:

1. $\delta_1 = \delta_2 \Leftrightarrow dq_1^{\,*}/\,d\eta = dq_2^{\,*}/\,d\eta < 0$, which implies that an increase in η decreases the same Nash equilibrium product quantity of two firms if and only if $\delta_1 = \delta_2$;

2. $\delta_1 > \delta_2 \Leftrightarrow dq_1^{\,*}/\,d\eta < dq_2^{\,*}/\,d\eta < 0$, which implies that under the condition that $\delta_1 > \delta_2$, we see a redistribution of equilibrium product quantity, where the firm who less uses water actually increases its product quantity somewhat on behalf of the firm who less uses water. The inverse is also true;

3. $\delta_1 < \delta_2 \Leftrightarrow dq_2^{\,*}/\,d\eta < dq_1^{\,*}/\,d\eta < 0$, which implies that under the condition that $\delta_1 < \delta_2$, we also see a redistribution of equilibrium product quantity, where the firm who less uses water actually increases its product quantity somewhat on behalf of the firm who less uses water. The inverse is also true.

Now, we give the following assumptions on δ_1 and δ_2 as follows: $\delta_1 > \delta_2$ and

$$\frac{\delta_2}{\delta_1} < \frac{D}{2B} < min\,\left\{\frac{A - c_1 - \eta\delta_1}{A - c_2 - \eta\delta_2}, \frac{A - c_2 - \eta\delta_2}{A - c_1 - \eta\delta_1}\right\}.$$

Then we obtain the following proposition:

Proposition 1. Under assumption I and the above two assumptions, the linear duopoly game model admits a unique interior Nash equilibrium product quantity of two firms. The effect of η on the Nash equilibrium product quantity of the firm who overuses water is negative; while the effect of η on the Nash equilibrium product quantity of the firm who less uses water is non-negative.

Proof. The conclusion that the linear duopoly game model admits a unique interior Nash equilibrium product quantity of two firms is obviously under assumption I and the above two assumptions. By use of the assumption $\delta_1 > \delta_2$ and

$$\frac{\delta_2}{\delta_1} < \frac{D}{2B} < \min\,\left\{\frac{A - c_1 - \eta\delta_1}{A - c_2 - \eta\delta_2}, \frac{A - c_2 - \eta\delta_2}{A - c_1 - \eta\delta_1}\right\},$$

we can obtain the following:

$$\delta_2 /\, \delta_1 < D\,/\,2B < 1 < \delta_1 /\, \delta_2. \qquad (6)$$

Hence by (4), (5) and (6), we can prove that the following results hold:

$$\frac{dq_1^{\,*}}{d\eta} = \frac{D\delta_2 - 2B\delta_1}{4B^2 - D^2} < 0,\; \frac{dq_2^{\,*}}{d\eta} = \frac{D\delta_2 - 2B\delta_2}{4B^2 - D^2} > 0.$$

Proposition 1 states that under some special restrictions, as water market equilibrium price increases, the Nash equilibrium quantity of the firm who overuses water will decrease, whereas the Nash equilibrium quantity of the firm who less uses water will increase.

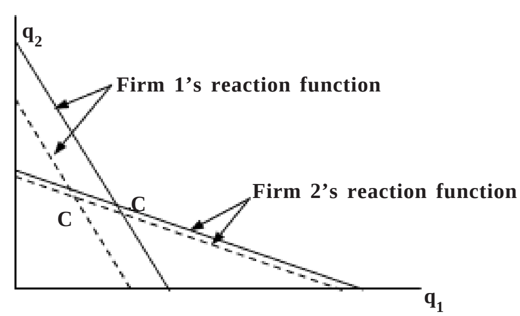

Figure 1[1]. The distribution of product quantity between two firms under the conditions of Proposition 1

[1] Figure 1 is similar to the second part of Figure 1 of K. J. Sunnevåg (2003). But Sunnevåg (2003) discussed how the equilibrium quantities of two firms vary with the equilibrium price of the trade market of emission right.

Now, we can allocate the water supply $\overline{Q}$ on the basis of the above analysis. By the Nash equilibrium product quantity of two firms, we obtain the allocation of water supply for two firms:

$$Q_1^* = \delta_1 \frac{2B(A - c_1 - \eta\delta_1) - D(A - c_2 - \eta\delta_2)}{4B^2 - D^2} - d_1 q_1^*$$

$$= \delta_1 \frac{2B(A - c_1 - \eta\delta_1) - D(A - c_2 - \eta\delta_2)}{4B^2 - D^2} - \frac{\eta - \alpha}{2\beta},$$

$$Q_1^* = \overline{Q} - Q_1^*.$$

Next, we give some numerical examples on the water supply allocation. In these examples, the available water supply quota is $Q = 995$, where each unit product quantity allows the firm to consume one unit of water supply. In addition, we assume $A = 1450$, $B = 0.27$, $D = 0.25$, $c_1 = c_2 = 189$. Now, let the technological parameters be given as follows: $\alpha = 102$, $\beta = 0.42$. Then the resulting product quantity and water quantity allocation for varying parameter values is presented in Table 1-3 below.

Case 1. $\delta_1 = 0.6$, $\delta_2 = 0.4$

Parameters	Firm 1		Firm 2	
	q_1^*	Q_1	q_2^*	Q_2
$\eta = 426$	1189.69	328.10	1473.49	666.90
$\eta = 325$	1278.44	501.59	1502.58	493.41
$\eta = 283$	1319.50	576.22	1514.67	418.78

Table 1. Allocation of product quantity and water supply quota ($\delta_1 = 0.6$, $\delta_2 = 0.4$)

Case 2. $\delta_1 = \delta_2 = 0.5$

Parameters	Firm 1		Firm 2	
	q_1^*	Q_1	q_2^*	Q_2
$\eta = 426$	1326.58	277.58	1326.58	717.42
$\eta = 325$	1390.51	429.78	1390.51	565.22
$\eta = 283$	1417.09	493.07	1417.09	501.93

Table 2. Allocation of product quantity and water supply quota ($\delta_1 = \delta_2 = 0.5$)

Case 3. $\delta_1 = 0.8$, $\delta_2 = 0.2$

Parameters	Firm 1		Firm 2	
	q_1^*	Q_1	q_2^*	Q_2
$\eta = 426$	885.90	323.01	1767.27	671.99
$\eta = 325$	1054.30	577.97	1726.71	417.04
$\eta = 283$	1124.33	683.99	1709.85	311.01

Table 3. Allocation of product quantity and water supply quota ($\delta_1 = 0.8$, $\delta_2 = 0.2$)

In view of Table 3, we can see that the technical data in Table 3 satisfy the condition of Proposition 1, therefore, by Proposition 1, we know that the equilibrium product quantity of Firm 1 decreases with the increase of the equilibrium price of water markets; while the equilibrium product quantity of Firm 2 increases with the increase of the equilibrium price of water markets.

4. Duopoly game model under the setting of incomplete information

In the above section, we establish a duopoly game model of water supply in the presence of market power and obtain the allocation results of output and the water supply for two firms to produce differentiated product under the setting of complete information. The study results verify a fact that when the equilibrium price of water markets raises, the total equilibrium output will decrease, while the change rule of the individual output is complicated. In particular, if the water consumption coefficient in the game model is same, the firms' equilibrium output will decline equivalently. But, assuming that the water consumption parameters in model are different and the other parameters in game model satisfy suitable conditions, we find that the market shares of output induced by the change of the equilibrium price of water markets transforms from the firm who requires more water quantity to the firm who requires less water quantity. Furthermore, with the raising of equilibrium price of water markets, the individual output may increase on the condition that the related conditions hold.

It is needed to point out that in Section 3, the allocation results of output and the water supply for two firms are obtained on the condition that the marginal cost information of two firms is common knowledge; that is, one firm knows the exact marginal cost of the opposite firm, which strictly restricts the applicable area of the game model established in Section 3 under the setting of complete information. As a matter of fact, in the process of utilizing water resource, there exist various information asymmetries; for example, one firm may not know the information on the actual water supply, the marginal benefit and the marginal cost of the other firms. In order to secure these data, one firm has invested substantive manpower, material resources and funds, but the effect is not satisfactory. Therefore, it is necessary to establish a generalized duopoly game model of water supply in the presence of market power and analyze the allocation of output and the water supply for two firms to produce differentiated product under the setting of incomplete information. Furthermore, we compare the allocation results of output and the water supply for two firms with that obtained under the setting of complete information. For the sake of convenience, we suppose that there are two firms 1 and 2 producing two differentiated good, one of which may not know the information on the marginal cost of another firm. Without loss of generality, we may assume that Firm 1 possesses incomplete information on the marginal cost, which implies that the marginal cost c_1 of Firm 1 is common knowledge and the marginal cost of Firm 2 is likely to be c_2^H or c_2^L ($c_2^L < c_2^H$).

Firm 2 knows that his own marginal cost is c_2^H or c_2^L, but Firm 1 only knows that the probability of $c_2 = c_2^L$ is $1-\theta$, then Firm 1 knows that the probability of $c_2 = c_2^H$ is θ, where θ is also common knowledge. We emphasis that the other parameters in this section are the same as in Sections 2 and 3.

Let $q_2^{**}(c_2^H)$ and $q_2^{**}(c_2^L)$ denote the product quantities of Firm 2 with respect to c_2^H and c_2^L. The fact that Firm 1 has only one marginal cost leads to the product quantity q_1^{**} of Firm 1 is single. If the marginal cost of Firm 2 is c_2^H, then Firm 2 will choose $q_2^{**}(c_2^H)$ to maximize the following objective function:

$$\underset{q_2, d_2}{Max}\ \Pi_2 = p_2 q_2 - c_2^H q_2 - k_2(d_2, q_2) - \eta Q_2. \qquad (7)$$

If the marginal cost of Firm 2 is c_2^L, then Firm 2 will choose $q_2^{**}(c_2^L)$ to maximize the following objective function:

$$\underset{q_2, d_2}{Max}\ \Pi_2 = p_2 q_2 - c_2^L q_2 - k_2(d_2, q_2) - \eta Q_2. \qquad (8)$$

In (7) and (8), $p_2 = A - Bq_2 - Dq_1^{**}$. Since Firm 1 only knows that the probability of $c_2 = c_2^L$ is $1-\theta$, and hence, the probability of the probability of $c_2 = c_2^H$ is θ, it follows that Firm 1 will choose q_1^{**} to maximize the following expect payoff:

$$\underset{q_1, d_1}{Max}\ \Pi_1 = \theta[p_1^H q_1 - c_1 q_1 - k_1(d_1, q_1) - \eta Q_1] + (1-\theta)[p_1^L q_1 - c_1 q_1 - k_1(d_1, q_1) - \eta Q_1],$$

where $p_1^H = A - Bq_1 - Dq_2^{**}(c_2^H)$ and $p_1^L = A - Bq_1 - Dq_2^{**}(c_2^L)$. By using the first order conditions of the above three optimization problems, we can obtain the following:

$$d_i = \frac{\eta - \alpha}{2\beta q_i},\ i = 1, 2.$$

At the interior Cournot equilibrium, the first order conditions with respect to product quantity for the above three optimization problems are given as follows:

$$q_2^{**}(c_2^H) = \frac{A - c_2^H - \eta\delta_2 - Dq_1^{**}}{2B}, \qquad (9)$$

$$q_2^{**}(c_2^L) = \frac{A - c_2^L - \eta\delta_2 - Dq_1^{**}}{2B}, \qquad (10)$$

$$q_1^{**} = \frac{A - [\theta Dq_2^{**}(c_2^H) + (1-\theta)Dq_2^{**}(c_2^L)] - \eta\delta_1 - c_1}{2B} \qquad (11)$$

In addition, we assume that:

$$A - c_1 - \eta\delta_1 > 0, \qquad (12)$$

$$A - c_2^H - \eta\delta_2 > 0, \qquad (13)$$

$$0 < \frac{D}{2B} < min\left\{\frac{A - c_1 - \eta\delta_1}{A - [\theta c_2^H + (1-\theta)c_2^L] - \eta\delta_2}, \frac{A - c_2^H - \eta\delta_1}{A - c_1 - \eta\delta_1}, \frac{A - c_2^L - \eta\delta_2}{A - c_1 - \eta\delta_1}, \frac{A - c_2 - \eta\delta_2}{A - c_2^L - \eta\delta_2}\right\} \qquad (14)$$

$$4B^2(A - c_2^L - \eta\delta_2) > D^2\theta(c_2^H - c_2^L) + 2BD(A - c_1 - \eta\delta_1) \qquad (15)$$

Solving the above systems of linear equations composed by two firms' reaction functions leads to the following Nash equilibrium product quantities of two firms and the total production quantity of two firms:

$$q_1^{**} = \frac{2B(A - c_1 - \eta\delta_1) - D\{A - [\theta c_2^H + (1-\theta)c_2^L] - \eta\delta_2\}}{4B^2 - D^2},$$

$$q_2^{**}(c_2^H) = \frac{4B^2(A - c_2^H - \eta\delta_2) - D^2(1-\theta)(c_2^H - c_2^L) - 2BD(A - c_1 - \eta\delta_1)}{8B^3 - 2BD^2},$$

$$q_2^{**}(c_2^L) = \frac{4B^2(A - c_2^L - \eta\delta_2) - D^2\theta(c_2^H - c_2^L) - 2BD(A - c_1 - \eta\delta_1)}{8B^3 - 2BD^2}.$$

By (12)-(14), we can easily prove that the above Nash equilibrium product quantity of two firms are interior solutions, that is, $q_1^{**} > 0, q_2^{**}(c_2^H) > 0$. By (15), we know that $q_2^{**}(c_2^L) > 0$. In order to examine the effects of the equilibrium price of water markets on the Nash equilibrium quantity of two firms, we differentiate q_1^{**}, $q_2^{**}(c_2^H)$ and $q_2^{**}(c_2^L)$ with respective to η as follows:

$$\frac{dq_1^{**}}{d\eta} = \frac{D\delta_2 - 2B\delta_1}{4B^2 - D^2}, \qquad (16)$$

$$\frac{dq_2^{**}(c_2^H)}{d\eta} = \frac{dq_2^{**}(c_2^L)}{d\eta} = \frac{D\delta_1 - 2B\delta_2}{4B^2 - D^2}, \qquad (17)$$

By (14), we have $dq_1^{**}/d\eta < 0$ and $dq_2^{**}(c_2^H)/d\eta = dq_2^{**}(c_2^L)/d\eta < 0$, which implies that in the setting of complete information, an increase in η decreases the Nash equilibrium product quantities of two firms under (14) holds. Differentiating $q_1^{**} + q_2^{**}(c_2^H) + q_2^{**}(c_2^L)$ with respect to η leads to the following:

$$\frac{d(q_1^{**} + q_2^{**}(c_2^H) + q_2^{**}(c_2^L))}{d\eta} = \frac{\delta_1 + \delta_2}{2B + D} < 0,$$

which implies that when the equilibrium price of water markets raises, the sum of the equilibrium output will decrease.

Now, we compare the allocation results of output and the water supply for two firms with that obtained under the setting of complete information. Let $c_2 = c_2^H$. Straightforward calculation shows that the following holds:

$$\frac{A - c_1 - \eta\delta_1}{A - [\theta c_2^H + (1-\theta)c_2^L] - \eta\delta_2} \leq \frac{A - c_1 - \eta\delta_1}{A - c_2^H - \eta\delta_2}. \qquad (18)$$

So, utilizing (14) and (18) leads to the following:

$$0 < \frac{D}{2B} < min \{ \frac{A - c_1 - \eta\delta_1}{A - c_2^H - \eta\delta_2}, \frac{A - c_2^H - \eta\delta_2}{A - c_1 - \eta\delta_1} \}. \qquad (19)$$

Therefore, by (12), (13) and (19), in the framework of complete information, we can see that assumptions I and II in Section 3 under the condition that $c_2 = c_2^H$ hold. Thus, by (12)-(14), we have

$$q_2^{**}(c_2^H) = \frac{4B^2 (A - c_2^H - \eta\delta_2) - D^2 (1-\theta)(c_2^H - c_2^L) - 2BD (A - c_1 - \eta\delta_1)}{8B^3 - 2BD^2}$$

$$\geq \frac{2B (A - c_2^H - \eta\delta_2) - D(A - c_1 - \eta\delta_1)}{4B^2 - D^2} = q_2^*(c_2^H),$$

which implies that if the marginal cost of Firm 2 is c_2^H, then the corresponding Nash equilibrium product quantity of Firm 2 under the setting of incomplete information is greater than the corresponding Nash equilibrium product quantity of Firm 2 under the setting of complete information. At the same time, the fact that the corresponding water supply of Firm 2 under the setting of incomplete information is greater than the corresponding water supply of Firm 2 under the setting of complete information is guaranteed by the above formula and the followings:

$$Q_2^{**} = \delta_2 q_2^{**}(c_2^H) - \frac{\eta - \alpha}{2\beta},$$

$$Q_2^{*} = \delta_2 q_2^{*}(c_2^H) - \frac{\eta - \alpha}{2\beta}.$$

Let $c_2 = c_2^L$. Then by (13) and (14), we have

$$A - c_2^L - \eta\delta_2 > 0, \qquad (20)$$

$$0 < \frac{D}{2B} < min \{ \frac{A - c_1 - \eta\delta_1}{A - c_2^L - \eta\delta_2}, \frac{A - c_2^L - \eta\delta_2}{A - c_1 - \eta\delta_1} \}. \qquad (21)$$

Therefore, by (12), (20) and (21), in the framework of complete information, we can see that assumptions I and II in Section 3 under the condition that $c_2 = c_2^L$ hold. Thus, by (12)-(15), we have

$$q_2^{**}(c_2^L) = \frac{4B^2 (A - c_2^L - \eta\delta_2) - D^2 \theta (c_2^H - c_2^L) - 2BD (A - c_1 - \eta\delta_1)}{8B^3 - 2BD^2}$$

$$\leq \frac{2B (A - c_2^L - \eta\delta_2) - D(A - c_1 - \eta\delta_1)}{4B^2 - D^2} = q_2^*(c_2^L),$$

which implies that if the marginal cost of Firm 2 is c_2^L, then the corresponding Nash equilibrium product quantity of Firm 2 under the setting of incomplete information is lower than the corresponding Nash equilibrium product quantity of Firm 2 under the setting of complete information. At the same time, the fact that the corresponding water supply of Firm 2 under the setting of incomplete information is lower than the corresponding water supply of Firm 2 under the setting of complete information is guaranteed by the above formula and the followings:

$$Q_2^{**} = \delta_2 q_2^{**}(c_2^L) - \frac{\eta - \alpha}{2\beta},$$

$$Q_2^{*} = \delta_2 q_2^{*}(c_2^L) - \frac{\eta - \alpha}{2\beta}.$$

For Firm 1, we have the conclusion different from Firm 2. In fact, let $c_2 = c_2^H$. Then by (12), (13) and (19), in the framework of complete information, we can see that assumptions I and II in Section 3 under the condition that $c_2 = c_2^H$ hold. Thus, by (12)-(14), we have

$$q_1^{**} = \frac{2B (A - c_1 - \eta\delta_1) - D\{A - [\theta c_2^H + (1-\theta)c_2^L] - \eta\delta_2\}}{4B^2 - D^2}$$

$$\leq \frac{2B (A - c_1 - \eta\delta_1) - D(A - c_2^H - \eta\delta_2)}{4B^2 - D^2} = q_1^*(c_2^H),$$

which implies that if the marginal cost of Firm 2 is c_2^H, then the Nash equilibrium product quantity of Firm 1 under the setting of incomplete information is lower than the corresponding Nash equilibrium product quantity of Firm 1 under the setting of complete information. At the same time, the corresponding water supply of Firm 1 under the setting of incomplete information is lower than the corresponding water supply of Firm 1 under the setting of complete information. By (12)-(15), we have

$$q_1^{**} = \frac{2B (A - c_1 - \eta\delta_1) - D\{A - [\theta c_2^H + (1-\theta)c_2^L] - \eta\delta_2\}}{4B^2 - D^2}$$

$$\geq \frac{2B (A - c_1 - \eta\delta_1) - D(A - c_2^L - \eta\delta_2)}{4B^2 - D^2} = q_1^*(c_2^L),$$

which implies that if the marginal cost of Firm 2 is c_2^L, then the Nash equilibrium product quantity of Firm 1 under the setting of incomplete information is greater than the corresponding Nash equilibrium product quantity of Firm 1 under the setting of complete information. At the same time, the corresponding water supply of Firm 1 under the setting of incomplete information is greater than the corresponding water supply of Firm 1 under the setting of complete information.

5. Conclusion

In this paper, the allocation problems of product quantity and the water supply for two firms to produce differentiated product in two oligopoly game models established under the settings of complete information and incomplete information are studied, respectively. On the basis of a linear demand system, we obtain the allocation results of product quantity and the water supply under the condition that the market power exists and the information on the marginal costs is complete. The study result shows that as the equilibrium price of water markets increases, the total Nash equilibrium product quantity will decrease, whereas the change rule of the individual product quantity is complicated. If the water consumption coefficient in the game model is same, the firms' Nash equilibrium product quantity will decline equivalently. Assuming that the water consumption coefficient in model is not same, we find that the market shares of product quantity due to the change of the equilibrium price of water markets transforms from the firm who need more water quantity to less one. With the raising of equilibrium price of water markets, the individual product quantity may increase on the condition that the related conditions hold. Furthermore, we give three numerical examples about the distribution of product quantity and water supply quota to verify the above facts. Finally, we establish a generalized Duopoly game model under the setting of incomplete information on marginal cost and compare the corresponding allocation results with that obtained under the setting of complete information on marginal cost.

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