The Study on PenduBot Control Linear Quadratic Regulator and Particle Swarm Optimization

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Journal of Digital
Information Management

ABSTRACT: *In this paper, a new algorithm to optimize the state feedback linearization matrix of two-link PenduBot control system. Mature method of comparison, the best of the simulation results of the particle swarm optimization algorithm can make the object more stable. Obviously, the particle swarm algorithm is introduced to the angular displacement and speed of smaller each link, less overshoot and a relatively short period of time to adjust. The algorithm through the control control ability proves two-link PenduBot. However, it needs modification parameter and run the program several times, so the need of further research for further improvement.*

Categories and Subject Descriptors: **I.2.9 [Robotics] G.1.6 [Optimization]**

General Terms:
Swarm optimization, Linear Quadratic Regulator

Keywords: PenduBot, PSO, LQR, State-Feedback

Received: 1 August 2012, Revised 2 October 2012, Accepted 9 October 2012

1. Introduction

1.1 Introduction of PenduBot

PenduBot is the abbreviation of pendulum and robot. The research scope includes single pendulum and robot. PenduBot also called inverted pendulum arm drive or rotary inverted pendulum. Inverted pendulum is the world recognized problem and valuable a typical physical model, automatic control theory research. For equipment, inverted pendulum of simple structure, small volume, and the price is cheap. As a kind of control system, it is a very complicated system, fast-responding more variables, tight coupling, the typical nonlinear, high order and unstable [1]. From the 1960 s, has been the object of inverted pendulum, the effectiveness of the new method to control the ability of typical nonlinear and natural unstable objects. In the study of this field can solve many problems in the control theory and practice, such as the rockets attitude and natural unstable system the stabilization problem of the control problem. As a robot, it can simulate the human to perform some task, especially the danger. PenduBot is a typical owe actuated robot representation [2], and on the basis of the robot to attract expert in the field. So, PenduBot is a research focus and robot control field, caused the people's extensive concern, domestic and foreign scholars.

This paper introduces a kind of new experiment device-two-link PenduBot (as shown in figure 1 below). In tradition, the bottom of the link is directly driven by motor called slewing. But in this thesis, the two link is determined for 1 and link 2, as shown in figure 1 below. In fact, the definition is simpler and easier to use [3].

The Pendubot is a double plane robot with a single the actuator shoulder of the first link, the combination of both two links are unsaturated and allow swing freely [1]. It is in combination with other mechanical systems, such as down the pendulum on a cart [2], Acrobot [3] and [4], it is to use control and robot education and for research as a typical owe drive mechanical system example less executive branch of freedom than [5]. Many researchers study pendulum type level 2 inverted pendulums is swinging of control system, for example, [1]-[4], [6]. Level 2 inverted pendulums is swinging the control problem. For Pendubot is a sad Pendubot the integrity of the unstable balance (two links are in upright position), then balance it integrity vertical [1]. To solve these problems, [1] use part of the feedback linear technology to level 2 inverted pendulum is swinging of control, implementation of the linear zed balance of integrity then use linear quadratic regulator balance control. However, no stability analysis is the premise. In recent years, considerable progress has been made the research method, the passive control or energy the work is part of the subsidy support of scientific research (C) grant of have no. Some groundbreaking secondary inverted pendulum is swinging works of control related Pendubot is [8] and [9], the control aim is to drive total mechanical energy of potential energy, man of integrity balance and stability of the Angle and horns the first link speed. This paper is to examine the energy control system [8] and [9]. The paper compared, we obtained the following results:

1) We provide a greater control parameters area Achieve control of purpose. Specifically, we put forward a sufficient and necessary condition to avoid singular point in the control law. We obtain the necessary and sufficient conditions of the control parameters so, the upper and lower balance (link 1 and 2 in integrity and downward position, respectively) was the only unexpected closed-loop balance point.

2) We prove up and down on the saddle point (hyperbola and unstable). In fact, we provide a basis through the use of watts standards of proof show that Jacobin matrix is the point value two and two characteristics in the value of the left wing and right wing open the plane, respectively. Therefore, we prove Pendubot eventually will be entered into the attraction of the basin any stable controller for the entire original. In addition to a set of conditions of measurement zero, these conditions improve provide the control parameters is satisfied.

1.2 Introduction of Algorithms

Linear Quadratic Regulator (LQR) and Particle Swarm Optimization (PSO) were used in the thesis. The following is the introduction of the two algorithms.

LQR is one of mature state space methods that developed early in the modern control theory [4]. So we need not to introduce it in details.

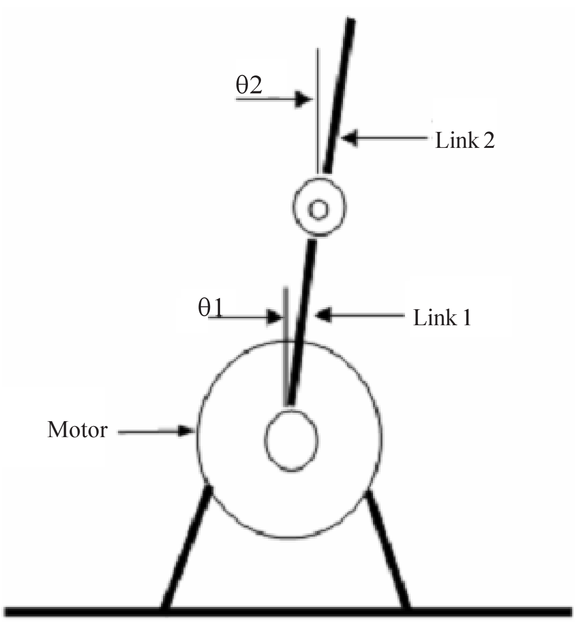

Figure 1. Schematic of two-link PenduBot

The particle swarm optimization algorithm is a kind of bionic computing algorithm, developed in 1995 by Kennedy and Eberhart. The algorithm from the movement of birds suddenly research. Similar genetic algorithm, the algorithm is a kind of based on iterative optimization method. It can be applied to many fields because of its simple definition and easy to realize and strong convergence and robustness. This is the study focused on the academic interest [5].

And so on, and has been widely applied to control PenduBot as a mature state space method. However, the particle swarm optimization algorithm is a kind of new bionic computing algorithm, and a few examples to control PenduBot. Considering the characteristics of good, I think this is a good path will go into this field of particle swarm optimization algorithm.

In the paper, I finished the programming based on state feedback theory according to the algorithm. The simulation results are compared, and between the two algorithms, we also research the two-link PenduBot control. Goal is to acquire control ability of the particle swarm optimization algorithm of strong nonlinear and instability in nature, and applying the algorithm to control PenduBot.

2. System Modeling

Two-link PenduBot consisted of two links connected by one pivot with low friction (shown in Figure.1). The bottom of Link 1 was directly driven by a DC motor. The links of the PenduBot can rotate in the vertical plane.

The angles of the two links θ_1 and θ_2 were measured by two angle sensors. If the control torque supplied by the DC motor can make θ_1 and θ_2 at zero degree, the two links of PenduBot were upright.

A mathematical model of two-link PenduBot was derived from classical mechanics method. The nonlinear model was shown in (1) in matrix form. Table I defined the parameters illustrated in Figure.1. The numerical values of the parameters were obtained by measuring and testing.

Before simulation, the model's linearization must be done. As important states, θ_1 and θ_2 were the controlled objects θ_1 and θ_2 and were the observed variables. Based on the nonlinear theory, the physical system could be linearized in a small neighborhood of stable points. So as the angles θ_1 and θ_2 were in a small neighborhood of zero degree, that is $\sin\theta_i$ and $\cos\theta_i\,(i=1,2)$ were approximately to 0 and 1 respectively, the nonlinear model can be linear zed [6] [7]. The preliminary results were as follows in (2).

System linearization was a common practice and a useful method in automatic control engineering. Linear systems had strong tools, such as LQR etc. For linear systems, the state model takes the special form:

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

where

$$\begin{bmatrix} J_0+J_1+M_1L_1^2+M_2L_3^2 & M_2L_3L_2+\cos(\theta_2-\theta_1) \\ M_2L_3L_2+\cos(\theta_1-\theta_2) & J_2+M_2L_2^2 \end{bmatrix}\begin{bmatrix}\ddot{\theta}_1\\ \ddot{\theta}_2\end{bmatrix}+$$

$$\begin{bmatrix} -(M_1L_1+M_2L_3)g\sin\theta_1 \\ -M_2gL_2\sin\theta_2 \end{bmatrix}+$$

$$\begin{bmatrix} \mu_1+\mu_2 & -\mu_1-M_2L_3L_2\sin(\theta_2-\theta_1)\dot{\theta}_2 \\ -\mu_1-M_2L_3L_2\sin(\theta_1-\theta_2)\dot{\theta}_1 & \mu_2 \end{bmatrix}\begin{bmatrix}\dot{\theta}_1\\ \dot{\theta}_2\end{bmatrix}$$

$$=\begin{bmatrix}G_0\\0\end{bmatrix}U \qquad (1)$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & J_0+J_1+M_1L_1^2+M_2L_3^2 & M_2L_3L_2 \\ 0 & 0 & M_2L_3L_2 & J_2+M_2L_2^2 \end{bmatrix}\begin{bmatrix}\dot{\theta}_1\\ \dot{\theta}_2\\ \ddot{\theta}_1\\ \ddot{\theta}_2\end{bmatrix}=$$

$$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ -(M_1L_1+M_2L_3)g & 0 & \mu_1+\mu_2 & -\mu_2 \\ 0 & -M_2L_2g-\mu_2 & & \mu_2 \end{bmatrix}\begin{bmatrix}\theta_1\\ \theta_2\\ \dot{\theta}_1\\ \dot{\theta}_2\end{bmatrix}+\begin{bmatrix}0\\0\\G_0\\0\end{bmatrix}U \qquad (2)$$

$$x=[\theta_1\ \ \theta_2\ \ \dot{\theta}_1\ \ \dot{\theta}_2]$$

$$y=\begin{bmatrix}\theta_1\\ \theta_2\end{bmatrix}$$

Substitute the corresponding numeric values in the table I, the results were:

$$A=\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 63 & -14 & -6.8 & 0.164 \\ -31 & 31 & 3.4 & 0.36 \end{bmatrix}$$

$$B=\begin{bmatrix}0\\0\\3.74\\-1.84\end{bmatrix}$$

$$y=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

The matrix $M_C=[B\quad AB\quad A^2B\quad A^3B]$ was defined as the controllable matrix. According to linear system theory, if MC ranks 4, the expected poles of the close-loop system can be placed everywhere in complex plane. After computing, it was easily known that the system was controllable. So we could place any poles of which the real parts were negative.

3. PSO Optimization Design

3.1 Standard PSO Algorithm

Now, most of the algorithm is based on the position and speed the evolution equations and the internal variables self-adapting inertia expanded and modified parameters. So we choose this development equation of particle swarm optimization algorithm simulation. The particle swarm optimization algorithm is called the standard version of the PSO algorithm, or the standard particle swarm optimization algorithm for short. The evolution equations

	Parameter	Notation	Vvalue and Unit
1	mass of Link 1	M_1	0.195 kg
2	mass of Link 2	M_2	0.152 kg
3	inertia of the rotor	J_0	0.000127 Kg.m^2
4	inertia of Link 1	J_1	0.0039 Kg.m^2
5	inertia of Link 2	J_2	0.00684Kg.m^2
6	the displacement from the joint to the c.m. of Link 1	L_1	0.1m
7	the displacement from the pivot to the c.m. of Link 2	L_2	0.15m
8	the displacement from the joint to the pivot	L_3	0.2m
9	viscous coef. of joint	μ_1	0.05N.m.s
10	viscous coef. of pivot	μ_2	0.0026N.m.s
11	angular displacement of Link 1 relative to vertical position	θ_1	rad
12	angular displacement of Link 2 relative to vertical position	θ_2	rad
13	angular velocity of Link 1	$\dot{\theta}_1$	rad/s
14	angular velocity of Link 2	$\dot{\theta}_2$	rad/s
15	the ratio of deflecting torque and control voltage	G_0	0.029N.m/V
16	control voltage	U	V
17	acceleration of gravity	g	9.8m/s^2

Table 1. Parameters Of Two-Link Pendubot

of particles are shown in (3) where v_{ij}, x_{ij} are the velocity and position components of particle i, dimension j respectively; w is inertial weight, it can regulate v and balance global and local search abilities; c_1 and c_2 are acceleration constants, restricted to the interval $0 \le c_1, c_2 \le 2$; $r_{1j}(t)$; and $r_{2j}(t)$ are random functions, generating random numbers restricted to the interval [0,1]; p_{ij} is optimal position of the particle i, j means dimension; p_{gj} is the global optimal position of dimension j [8].

$$v_i(t+1) = w \bullet v_{ij}(t) + c_1 \bullet r_{1j}(t) \bullet [p_{ij}(t) - x_{ij}(t)] + c_2 \bullet r_{2j}(t) \bullet [p_{gj}(t) - x_{ij}(t)]$$

$$x_{ij}(t+1) = x_{ij}(t) + v_{ij}(t+1) \qquad (3)$$

3.2 Selection of Parameters

1) Selection of w. The bigger w is, the faster the particles fly, and then they will do global detect in a longer step; the reverse is also true. By experiment, Shi and Eberhart found that when w was restricted to the interval [0.9, 1.2], the search performance was ideal. So, we chose $w = 1.15$ in order to do global detection in a longer step [9].

2) Selection of c_1 and c_2. Considering the interval restriction, we chose $c_1 = 1.0$, $c_2 = 1.0$ in order to increase the influence of global optimal position on the optimal process.

3) Selection of $r_{1j}(t)$ and $r_{2j}(t)$. They are random functions as mentioned previously.

4) Selection of the velocity interval. In order to keep the particles in the search space, the range of v_{ij} is frequently limited, but still restricted to the interval [v_{max}, v_{max}]. According to equation $v_{max} = k \bullet x_{max}$ $(0.1 \le k \le 1.0)$, we chose an intermediate value, that is, $k = 0.55$.

5) Selection of x_{max}, x_{max}, The bigger the interval, the bigger possibility of the optimal results we get. By experience, we set $x_{max} = 6000$, $x_{max} = -4500$.

3.3 Selection of Fitness Function

Because PenduBot characteristics, how to solve the problem is the main problem quick search state feedback matrix. At the same time, we must be as small as possible overshoots is impossible. Based on linear system theory, if all of the closed-loop system poles of negative real parts, the system will be more stable. The key to solving fast was to find the biggest negative real part of absolute value. We hope it can improve fast problem to a larger extent. Reduce overshoots is the second consideration. So we add up all the poles of the closed loop system, to as fitness function and [10].

4. Simulation Results

The algorithm process, after the completion of PenduBot input parameters, then add another program to eliminate the steady-state error, we can run the whole program (we take measures as a signal input). In this paper, the iterative Ncmax and population size was selected as the parameters of the particle swarm optimization algorithm

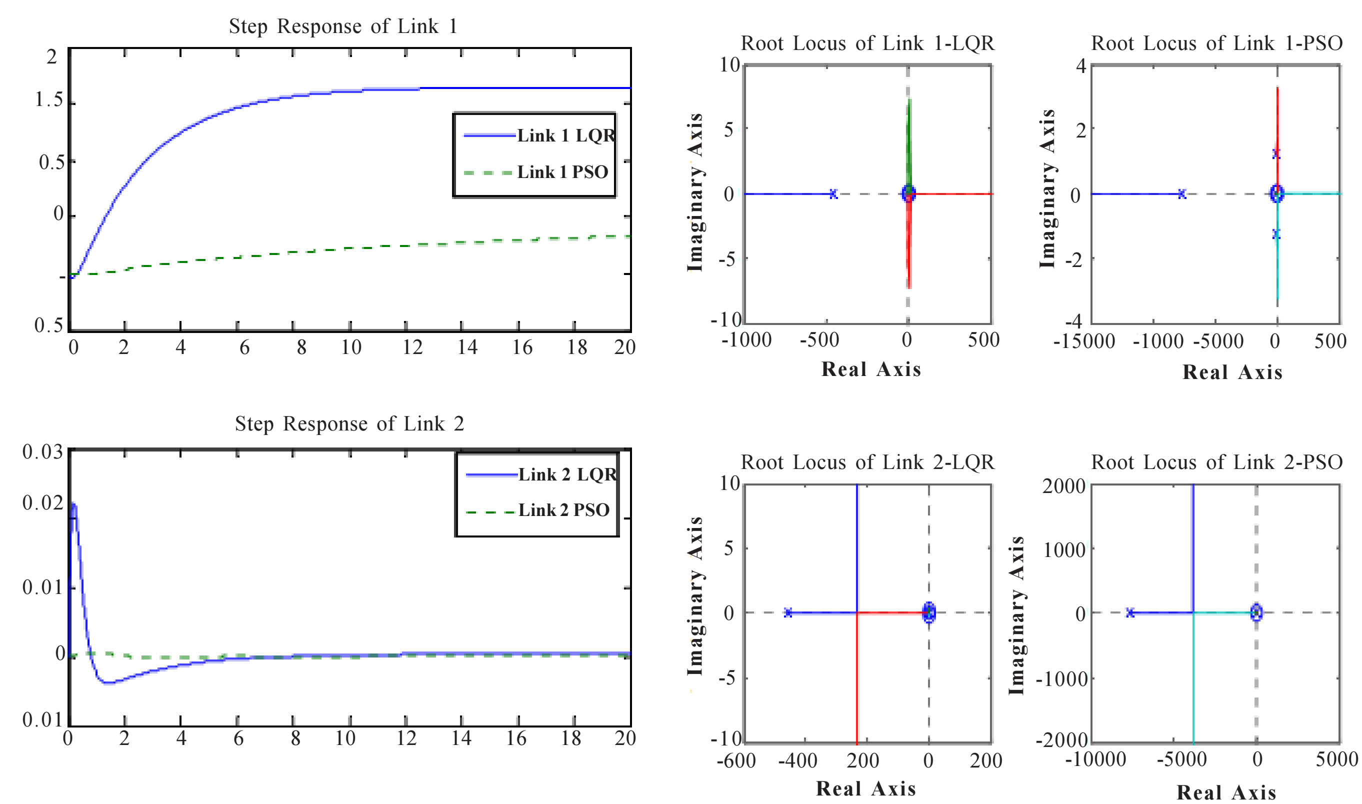

Figure 2. Comparison Results by LQR and PSO: (a) this is the simulation results comparison; (b) this is root locus by LQR and PSO

of fitness function. According to experience, we first set m = 50, adjusted Ncmax from a big number and run ten times. When we set Ncmax to smaller values in numerical order, and also run the modulated program ten times. When Ncmax = 500, we noticed that some simulation results using PSO were better than those using LQR. In order to save time, we decreased the iteration. When Ncmax = 250, and m = 50, the results using PSO were better than using LQR for 8 times. Then we set Ncmax = 250, and adjusted m only. The result was, only when m = 55, the results were better for ten times. The best simulation results were shown in Figure 2 (a). It is clear that using PSO, Link 1 finally stabilized in less than 0.25 degree, and Link 2 finally almost reached stabilization in vertical position. Though Link 1 and Link 2 finally stabilized in certain position in less than ten seconds using LQR, the positions and stabilization processes were no better than those using PSO algorithm. From the root locus in Figure 2 (b), the reason was clear.

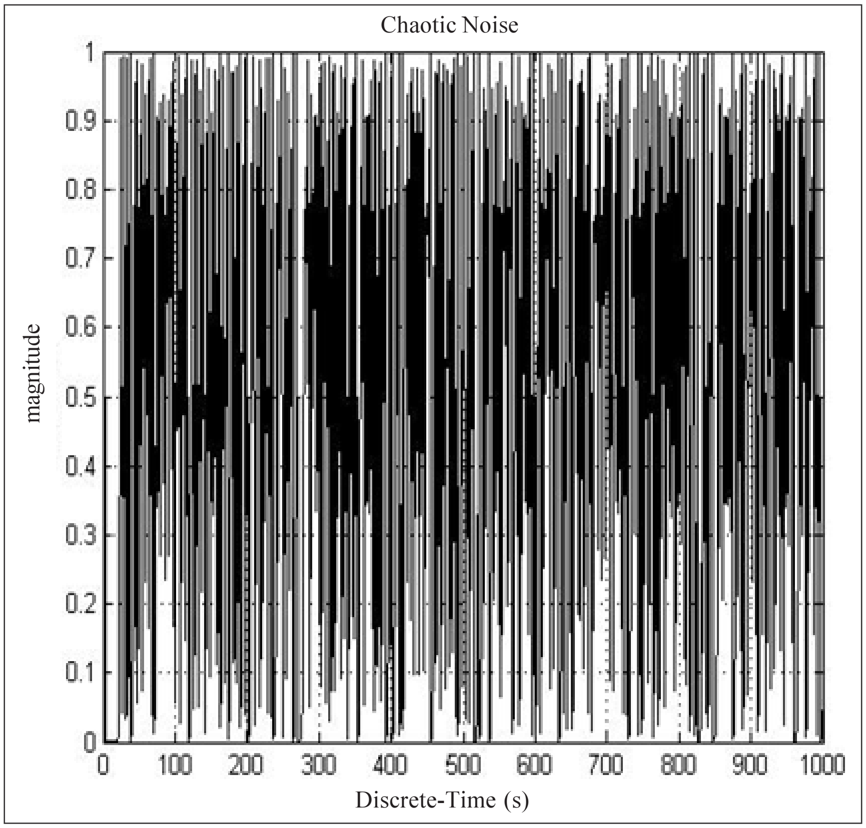

Figure 3. Chaotic noise from the Logistic Map

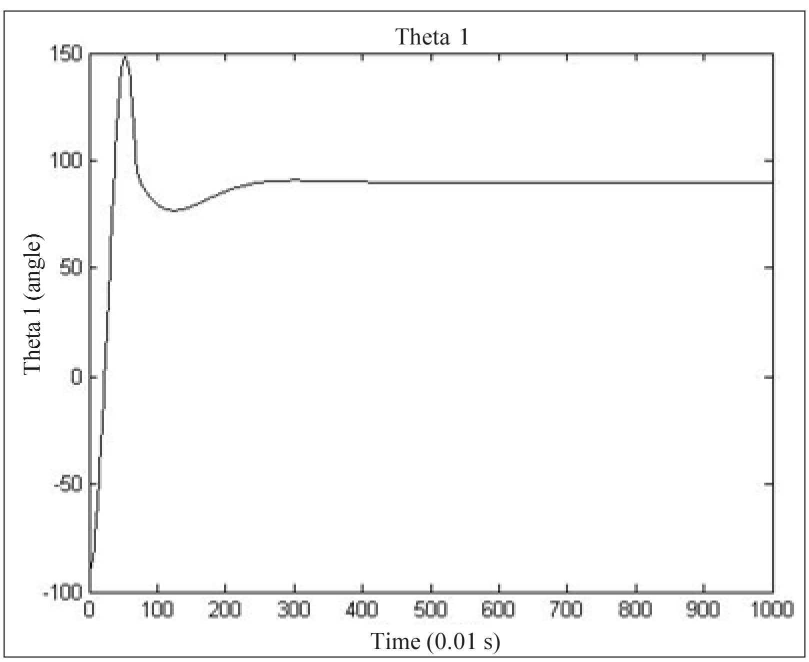

Figure 4. The trajectory of the first joint without input perturbation

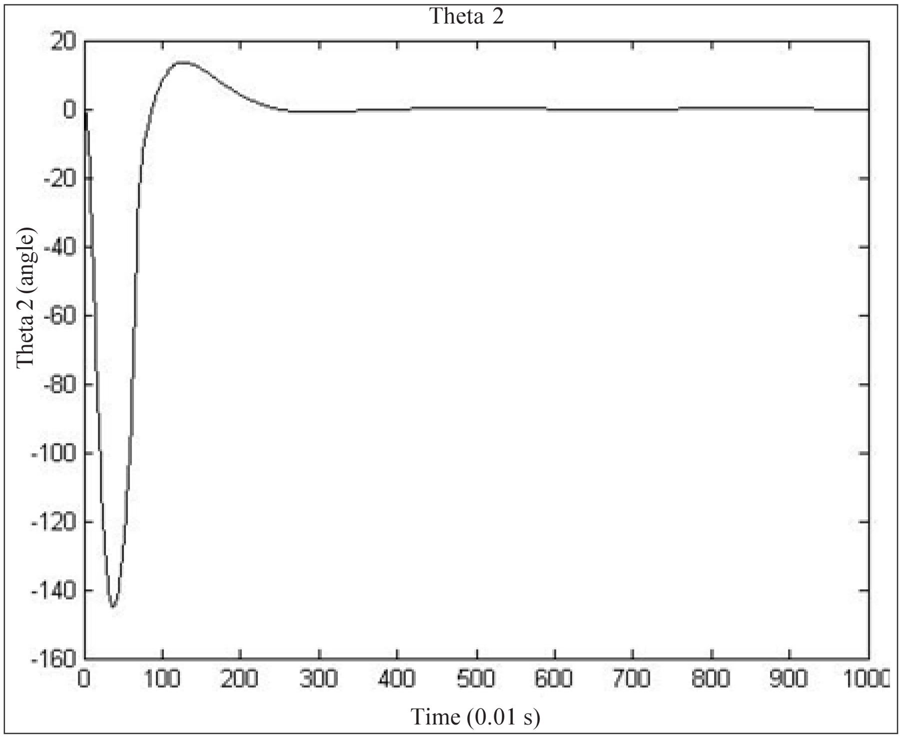

Figure 5. The trajectory of the second joint without perturbation

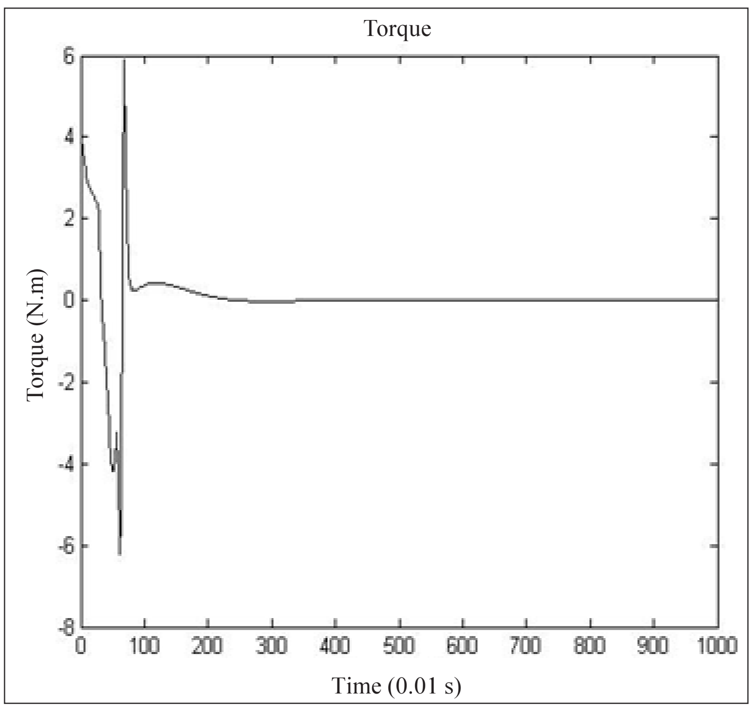

Figure 6. The corresponding torque in case of no input perturbation

5. Conclusions

PenduBot is to place and the abbreviation of robot. The research scope is including single pendulum and robot. PenduBot also called inverted pendulum arm drive or rotating inverted pendulum. Inverted pendulum is the world recognized problem and valuable a typical physical model, automatic control theory research. The inverted pendulum on equipment, simple structure, small volume, and the price is cheap. As a kind of control system, it is a very complicated system, a kind of more variables, tight coupling, the typical nonlinear, high order and unstable [1]. Since the 1960 s, has been the object of inverted pendulum, the effectiveness of the new method of control ability is a typical nonlinear and natural unstable objects. In the study of this field can solve many problems in the control theory and practice, such as the rockets attitude and natural unstable system the stabilization problem of the control problem. As a robot, it can simulate the human to perform some task, especially dangerous. PenduBot is a typical owe actuated robot said [2], and on the basis of the robot to attract expert in the field. So, PenduBot is hot and robot control field research, caused the people’s extensive concern, domestic and foreign scholars.

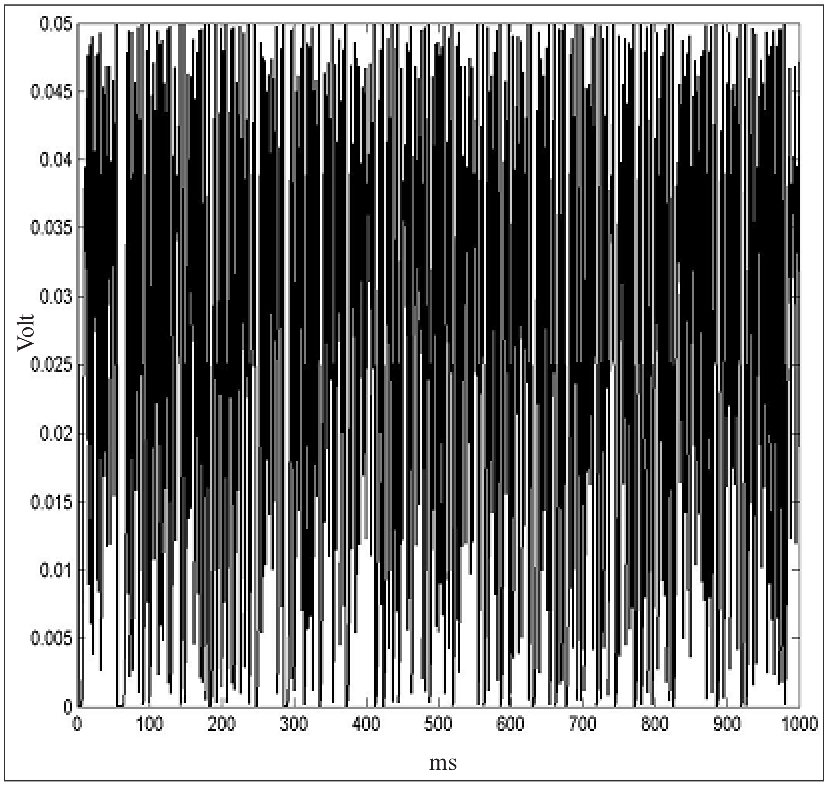

Figure 7. The chaotic perturbation signal with 0.05 V injected at the control input signal

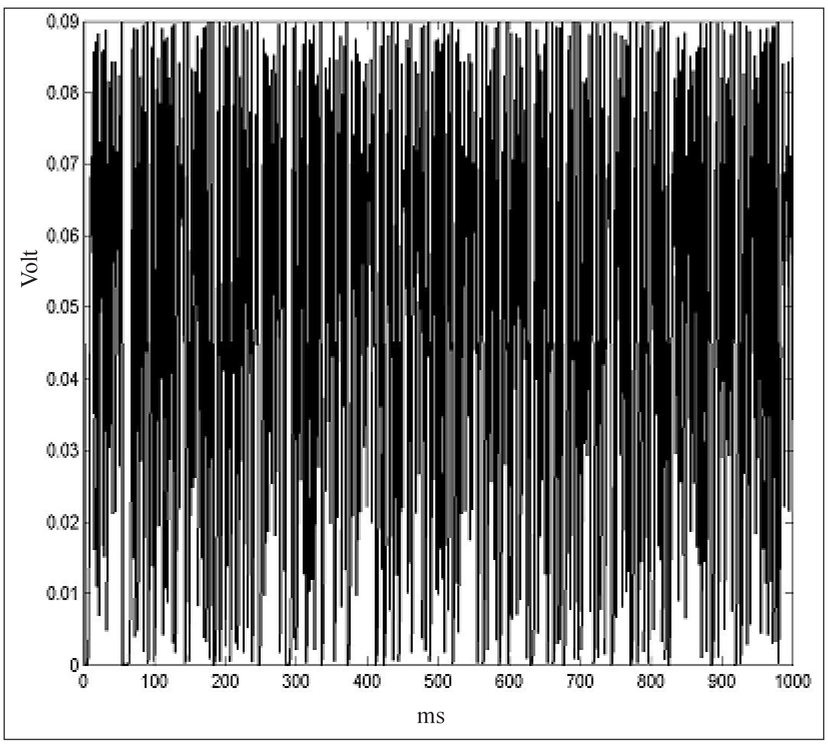

Figure 8. The chaotic perturbation signal with 0.09 V injected at the control input signal

This paper introduces a kind of new experiment device-two-link PenduBot (the following figure 1). In traditional, the bottom of the link is called back directly driven by motor. But in this thesis, two links is determined for 1 and link 2, the following figure 1. In fact, the definition is simpler and easier to use [3].

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Pendubot unstable balance integrity (two links are in the vertical position), then it balance integrity vertical [1]. To solve these problems, [1] use part of feedback linear technology level 2 inverted pendulum swing control, realize the balance of the linear zed complete, using linear quadratic regulator balance control. However, no stability analysis is the premise. In recent years, has made great progress, and research methods, and passive control or energy is part of the job subsidies support of scientific research (C) funded no. Some groundbreaking level 2 inverted pendulum swing in the Pendubot control related work is [8] and [9], the control of the purpose is to promote the total mechanical energy of potential energy, just one's balance and stable Angle and Angle of the first link speed. This paper mainly discusses the energy control system [8] and [9]. The paper compared, the results obtained:

1) we provide a greater control parameters to control area of purpose. Specifically, we proposed a sufficient and necessary condition to avoid singularity in control law. We get the necessary and sufficient conditions of the control parameters; therefore, the upper and lower balance (link 1 and 2 in the integrity and descending position, respectively) is the only accident the closed-loop balance point.

2) Proof we hold a saddle point (hyperbola and unstable). In fact, we provide a foundation, through the use of watts standards of proof shows that, Jacques, than matrix is points two and two features of the values of the left wing and right wing fly, respectively. Therefore, we prove Pendubot will eventually into attractive any stable controller basin in the original. In addition to a set of conditions of the measurement zero, these conditions provide satisfactory control parameters to improve this paper studies how to stabilize two-link PenduBot algorithm. We are here by a mature algorithm simulation and a new algorithm separately. In contrast, our study of the new algorithm can control ability. The particle swarm optimization algorithm is a kind of bionic computing algorithm, formed in 1995, and he is in "Kennedy hart. The algorithm from sports birds suddenly research. Similar genetic algorithm is a kind of based on iterative optimization method. It can be applied to many areas, because of its simple definition and easy to realize and strong convergence and robustness. This is the study of the main academic interest [5].

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In the paper, I finished the programming based on state feedback theory according to the algorithm. The simulation results are compared, both between algorithm, we also double PenduBot control. Goal is to acquire control ability of the particle swarm optimization algorithm of strong nonlinear and unstable properties, and application of the algorithm to control PenduBot.

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