The Computer Design of Practical Filter

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ABSTRACT: *This paper discussed how to use computer to directly transfer the simulative low pass prototype design to all kinds of needed principles and design methods. This method's advantage is that it is very direct, and universal. So, this method is simple, convenient, practical and fast.*

Categories and Subject Descriptors:
J.6 [**COMPUTER-AIDED ENGINEERING**]: Computer-aided design (CAD)

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Computer-aided Design, Design Models, Filter Design

1. Introduction

The traditional simulative filter mostly is designed according to formula computing and diagram [1] [2]. It is inconvenient, and what's more, it is also not convenient for the parameter's optimized analysis. Therefore this article proposed a transmission function $H(s)$, which was obtained from a known simulative low pass prototypical system function to the low pass, high pass band pass and band stop function under the matrix transformation, and then obtain the figure's low pass, high pass, band pass and band stop filter's function $H(z)$ under polynomial shift. As a result, there will be many different type of construction. The whole process can be accomplished on computer fast and efficiently for one time.

2. The Design of Simulative Filter

Pass filter is known:

$$H(s) = \frac{a_0 s^n + a_1 s^{n-1} + \ldots + a_n}{b_0 s^n + b_1 s^{n-1} + \ldots + b_n} = \frac{\sum_{i=0}^{n} a_i s^{n-i}}{\sum_{i=0}^{n} b_i s^{n-i}} \qquad (1)$$

If the $H(s)$ has been directly transferred to the needed simulative low pass, high pass, band pass and band stop filter, then the W_c in the $H_L(s) = H(s)_{s1 \to \frac{s}{w_l}}$ of low pass filtering is the needed low pass filter's cut-off frequency. The transmission function $H_{BP}(s)$ of band pass filter [3] [4] is:

$$H_{BP}(s) = H(s)_{s1 \to \frac{s^2 + w_0^2}{BS}} \qquad (2)$$

The W_0 is the needed band pass filter's centre frequency, B is the band pass's band width. Transmission function $H_{BS}(s)$ of band stop filter is:

$$H_{BP}(s) = H(s)_{s1 \to \frac{BS}{s^2 + w_0^2}} \qquad (3)$$

W_0 is the needed band stop filter's centre frequency, B is the stop band width. The transmission function $H_H(S)$ of high pass filter is:

$$H_H(S) = H(S)_{s1 \to \frac{w_c}{s}} \qquad (4)$$

W_c is the needed high pass filter's cut-off frequency. It can be found when examine the formula (1), (2), (3), (4): if order the w_0 and w_c to be equal to 0 and B separately, as a result, the formula (2) and (3) are equal to (1) and

(4). Hence, it is easy to get the transmission functions of these four filters when understand the transformation from low pass to band pass and band stop.

2.1 The transformation of band pass
From formula (2):

$$H_{BP} = H(s)_{s1 \to \frac{s^2 + w_0^2}{BS}} = \frac{a_0 \left[\frac{s^2 + w_0^2}{BS}\right]^n + a_1 \left[\frac{s^2 + w_0^2}{BS}\right]^{n-1} + \ldots + a_n}{b_0 \left[\frac{s^2 + w_0^2}{BS}\right]^n + b_1 \left[\frac{s^2 + w_0^2}{BS}\right]^{n-1} + \ldots + b_n}$$

(5)

$$= \frac{a_n(BS)^n + a_{n-1}(s^2 + w_0^2)^n (BS)^{n-1} + \ldots + a_k(s^2 + w_0^2)^{n-k} s^k B^k + \ldots + a_0(s^2 + w_0^2)^n}{b_n(BS)^n + b_{n-1}(s^2 + w_0^2)^n (BS)^{n-1} + \ldots + b_k(s^2 + w_0^2)^{n-k} s^k B^k + \ldots + b_0(s^2 + w_0^2)^n}$$

It is easy to find out that to calculate it by hand is very inconvenient. Therefore, write the numerator and the denominator of the $H_{BP} = H(s)$ is to be matrix form, hence, to obtain the $H_{BP}(s) = \frac{E(S)}{F(S)}$

$$E(S) = [a_n B^n \ a_{n-1} B^{n-1} \ldots a_1 B \ a_0] \begin{bmatrix} s^n \\ (s^2 + w_0^2) s^{n-1} \\ \vdots \\ (s^2 + w_0^2)^k s^{n-k} \\ \vdots \\ (s^2 + w_0^2)^n \end{bmatrix}$$

$$= \underbrace{[a_n \ a_{n-1} \ldots a_1 \ a_0]}_{\vec{a}} \underbrace{\begin{bmatrix} B^n & 0 & 0 & \ldots & 0 \\ 0 & B^{n-1} & 0 & \ldots & 0 \\ 0 & 0 & 0 & B^{n-2} & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \ldots & B^0 \end{bmatrix}}_{\vec{B}} \underbrace{\begin{bmatrix} s^n \\ (s^2 + w_0^2) s^{n-1} \\ \vdots \\ (s^2 + w_0^2)^{n-1} s \\ \vdots \\ (s^2 + w_0^2)^n \end{bmatrix}}_{\overrightarrow{s(w_0)}}$$

$$= \vec{a} \vec{B} \overrightarrow{S(w_0)}$$

(6)

In the same way to get the denominator polynome's matrix form

$$F(S) = \vec{b} \vec{B} \overrightarrow{S(w_0)}$$

(7)

Here the $\vec{b} = [b_n \ b_{n-1} \ldots b_1 \ b_0]$, therefore the matrix [5] form of

$$H_{BP}(s) \text{ is: } H_{BP} = \frac{\vec{a} \vec{B} \overrightarrow{S(w_0)}}{\vec{b} \vec{B} \overrightarrow{S(w_0)}}$$

(8)

If the matrix $\overrightarrow{S(w_0)}$ can be worked out, then the rest problems also can be solved out. From the analysis above, it can be found that the general term in the matrix $\overrightarrow{S(w_0)}$ is $(s^2 + w_0^2)^k s^{n-k}$, and its expansion is the polynome of S to the power of $2n$, so $\overrightarrow{s(w_0)}$ can be written as:

$$\overrightarrow{s(w_0)} = \begin{bmatrix} w_{00} & w_{01} & \ldots & w_{02n} \\ w_{10} & w_{11} & \ldots & w_{12n} \\ \vdots & \vdots & \ldots & \vdots \\ w_{n0} & w_{n1} & \ldots & w_{n2n} \end{bmatrix} \begin{bmatrix} s^{2n} \\ s^{2n-1} \\ \vdots \\ 1 \end{bmatrix} = \vec{w} \vec{s}$$

(9)

Next is the discussion of the calculation for matrix $\vec{w}$. Since $\vec{w}$ is the coefficient matrix of $\overrightarrow{s(w_0)}$, to choose the number K term and the number $K + 1$ term in $\overrightarrow{s(w_0)}$ to expand:

$$(s^2 + w_0^2)^k s^{n-k} = s^{n-k-1}(c_k^0 s^{2k+1} + \ldots + c_k^p s^{2k-2p+1} w_0^{2p} + \ldots + w_0^{2k} s)$$

$$(s^2 + w_0^2)^{k+1} s^{n-k-1} =$$

$$s^{n-k-1}(c_{k+1}^0 s^{2k+2} + \ldots + c_{k+1}^p s^{2k-2p+2} w_0^{2p} + \ldots + w_0^{2k+2})$$

Pay attention to all powers' coefficients of S, and then get:

$$\begin{cases} w_{1,j} = 0 & (j \neq n, j = 0, 1, \ldots, 2n) \\ w_{1n} = 1 \\ w_{kj} = w_0^2 \ w_{k-1, j-1} + w_{k-1, j-1} & (2 \leq k \leq n+1, j \leq 2n+1) \end{cases}$$

(10)

Therefore, from formula (10), all the elements of $\vec{w}$ can be worked out. If the low pass filter's transmission function $H_L(s)$ is needed from low pass prototype $H(s)$, then from formula (1), and based on the formula (8), to let the $\vec{a}, \vec{B}, \vec{b}, \vec{s}, \vec{w}$ invariable, choose the matrix elements $B = w_c, w_0 = 0$, and put them back to the formula, and then the result is coming.

2.2 The transformation from simulative low pass to simulative band-stop
From formula (3):

$$H_{BS} = H(s)|_{s1 \to \frac{BS}{s^2 + w_0^2}} = \frac{a_0 B^n s^n + \ldots + a_n (s^2 + w_0^2)^n}{b_0 B^n s^n + \ldots + b_1 B^{n-1}(s^2 + w_0^2) s^{n-1}}$$

(11)

To compare formula (11) and (5): there is no difference except he marshalling sequence of $\{a_n\}, \{b_n\}$ in the multinomial coefficient's numerator and the denominator of $H_{Bs}(s)$ is opposite. Hence, the $H_{Bs}(s)$ can be calculated by the same matrix expression. That is:

$$H_{BS}(s) = \frac{\vec{a} \vec{B} \overrightarrow{s(w_0)}}{\vec{b} \vec{B} \overrightarrow{s(w_0)}} = \frac{\vec{a} \vec{B} \vec{w} \vec{s}}{\vec{b} \vec{B} \vec{w} \vec{s}}$$

(12)

Here, $\vec{a} = [a_0 \ a_1 \ldots a_{n-1} \ a_n], \vec{b} = [b_0 \ b_1 \ldots b_{n-1} \ b_n]$. And other calculating methods of matrix are as the same as band pass transformation. If the high pass filter's transmission function $H_H(s)$ is needed from low pass prototype $H(s)$, then from formula (4), it is known that based on the formula (11), to order the $\vec{a}, \vec{B}, \vec{b}, \vec{s}, \vec{w}$ invariable, and then to put the matrix elements $B = w_c, w_0 = 0$ back to the formula., and it is accomplished. It is obvious from the above analysis:

① The four transformed filters can be expressed by the

same matrix form totally, which is that the transmission function= $\dfrac{\vec{a}\,\vec{B}\,\vec{w}\,\vec{s}}{\vec{b}\,\vec{B}\,\vec{w}\,\vec{s}}$. If the supreme power and the coefficient in the polynome of low pass prototype $H(s)$'s numerator and the denominator is known, and the transformed parameter B and w_0 are also known, then the matrix of $\vec{a}$, $\vec{B}, \vec{b}, \vec{s}, \vec{w}$ can be formed naturally.

② As a result that the matrix $\vec{w}$ is made up of the elements that calculated from a unified and closed relation under the simple arithmetic, hence, the whole procedure [5] [6] can be prepared by computer in one time.

③ if to program the numerator and the denominator of the transformed transmission function to be a subroutine, and to call it out twice separately, then the needed transmission function of filter can be worked out. This process has been fulfilled on microcomputer.

3. The Design of Simulative Filter

The above analyzed the design of simulative filter [7], and the general form of all sorts of simulative filter is $H(s)$, if to do a bilinear transformation on the simulative low pass, high pass, band pass, and band stop, then the needed digit low pass, high pass, band pass, and band stop filter can be obtained. To avoid the difficulty of calculation, here, the polynomial shift is applied. Suppose that the transmission function of filter in simulative region is showing in formula (1), then put $s = C\,\dfrac{z+u}{z-u}$ in the formula, the design from simulative to digital (also is bilinear transformation) has been fulfilled. That is

$$H(z) = H(s)_{s1 \to C\frac{z+u}{z-u}} = \frac{(z-u)^n \sum\limits_{i=0}^{n} c^{n-i} a_i \left[z + u \big/ z - u \right]^{n-i}}{(z-u)^n \sum\limits_{i=0}^{n} c^{n-i} a_i \left[z + u \big/ z - u \right]^{n-i}}$$

$$= \frac{D(z)}{E(z)} = \frac{\sum\limits_{i=0}^{n} d_i\, z^{n-i}}{\sum\limits_{i=0}^{n} e_i\, z^{n-i}}$$

Therefore, next discussion is about

$$H(z) = \sum_{i=0}^{n} a_i\, s^{n-i} \qquad (13)$$

$s = c\,(z+u)/(z-u)$ under $H^*(z) = (z-u) \sum\limits_{i=0}^{n} a_i c^{n-i} \left[z + u \big/ z - u \right]^{n-i}$

expansion.

The $H^*(z)$ can be expanded as following:

Translate $H(s)$ for one unit, and therefore get $H'(s) = \sum\limits_{i=0}^{n} c_i\, s^{n-i}$ Order $H''(s) = \sum\limits_{i=0}^{n} b_i\, z^{n-i}$, and translate $H''(z)$ for c units, and get the expansion form of $H^*(z)$, $H^*(z) = \sum\limits_{i=0}^{n} d_i\, z^{n-i}$, and here $b_i = (2c)^i\, c_{n-i}\,(i=0, 1, 2, ...,n)$ And

the proof is as follows:

To order $x = z - c$, so $s = (x + 2c)/x$, and put it in formula (13), thus, theresult is:

$$f'(x) = H^*\left(\frac{z+u}{z-u}\right) = x^n \sum_{i=0}^{n} \left(1 + \frac{2c}{x}\right)^{n-i} = x^n \sum_{i=0}^{n} c_{n-i} \left(\frac{2c}{x}\right)^i$$

Here, $c_i\,(i = 0, 1, 2,..., n)$ is the result of $G(s)$' translation for -1 unit. To order $b_i = (2c)^i c_{n-i}$, then $f'(x) = \sum\limits_{i=0}^{n} b_i\, z^{n-i}$ because $x = z - c$ so $H^*(z) = f'(z - c) = \sum\limits_{i=0}^{n} b_i\,(z - c)^{n-i} = \sum\limits_{i=0}^{n} d_i\, z^{n-i}$ and here d_i is the result of $f'(x)$'s translation for c units. Next, to illustrate the design process by giving example:

$H(s) = \dfrac{1}{s^3 + 2s^2 + 2s + 1}$, now transform it to the digital low pass filter. Solution: the denominator $F(s) = s^3 + 2s^2 + 2s + 1$ can be known from $H(s)$, Translate $F(s)$ for -1 unit., and therefore, $c_0 = 1\ c_1 = 5\ c_2 = 9\ c_3 = 6\ (c_i(i = 0, 1, 2, 3,))\ b_i = 2^i$ So, $b_0 = c_3 = 6\ b_1 = 2c_2 = 18\ b_2 = 4c_1 = 20\ b_3 = 8c_0 = 8$ And then get the center polynome $F'(x)$, $F'(x) = 6x^3 + 18x^2 + 20x + 8$ translate the $F'(x)$ for -1 unit, and as a result, get the digital filter $H(Z)$'s denominator polynome $F(Z)$. Hence, $F(z) = 6z^3 + 36z^2 + 74z + 52$ the numerator $E(s) = 1$ can be known from $H(s)$, and in the same way, the $H(Z)$'s numerator polynome $E(Z)$ can be worked out as: $E(z) = z^3 + 3z^2 + 3z + 1$. The transformed digital low pass filter's transmission function is :

$$H(z) = \frac{z^3 + 3z^2 + 3z + 1}{6z^3 + 36z^2 + 74z + 52}$$

As a result, there are 3 features of this design [8] [9]: It is practical for hand computation. All the procedure can be undertaken only with add operation, and it only occupies a little internal memory of computers. Hence, it is at a high speed. The programming is standard.

4. Conclusion

For the sake of conveniently design the simulative and digital filter rapidly and precisely, the whole process is written as procedure for computer to fulfill it.

(1) At the procedure entrance, let $P = 0, 1$; $P_1 = 0, 1, 2, 3$; $P_2 = 0, 1, 2, 3$; $P_3 = 0, 1, 2$; $P_4 = 0, 1$, and therefore, the prototype simulative low pass can be transformed to the data and its frequency characteristic diagram of the required simulative or digital low pass, high pass, band pass, and band stop filter's transmission functions.

(2) If vary the input data in the procedure (the coefficient of $H(s)$'s numerator and denominator and B, w_0), then, there will be different simulative and digital filter.

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