Study of Methods for Constructing Families of Odd-periodic Perfect Complementary Sequence Pairs

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ABSTRACT: *Some characteristics of the families of odd-periodic perfect complementary sequence pairs (OPCSPF) are discussed. The methods for constructing an OPCSPF are given and proved. The families of periodic perfect complementary sequence pairs can be constructed correspondingly by using the equivalent relationships, which further expanded the existence space of the complementary sequence.*

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1. Introduction

Signals with good correlation properties have been widely used in modern communications, radar, sonar, navigation, location, guidance, space monitoring and control, signal processing, source coding and other fields[1]. Periodic binary complementary sequence sets are signals of this kind, which have such properties: the sum of aperiodic autocorrelation of sequences is zero everywhere except "0"shift [2], in practice odd-periodic correlation function is of same importance as periodic correlation function, on certain conditions, periodic binary complementary sequences sets and odd-periodic binary sequences sets can transform into each other [3-4]. Ever since 1999 when the theory of sequences pairs was raised [5], many scholars have researched on it, and it has developed from sequences pairs to almost sequences pairs, families of sequences pairs and other fields, and it has achieved certain fruits [6-7] in characteristics of the spectrum, the necessary condition for existence, methods for construction and so on.

In this paper, we advance a new kind of binary sequences pairs sets-families of odd-periodic perfect complementary sequences pairs (OPCSPF), two kinds of methods for constructing OPCSPF are given, which can be used to construct OPCSPF from low level to high level.

2. Basic Concept

In order to facilitate the discussion of the issue, let sequences a and a' be 2 sequences of length N, be labeled as $a=(a(0), a(1),..., a(N-1))$, $a'=(a'(0), a'(1),..., a'(N-1))$

Definition 1[3] : For sequence $a(n)$, each element of which is $+1$ or -1. We use the symbol of periodic cyclic alternate reverse sequence to write out the odd-periodic sequence $\hat{a}(n)$ of odd sequence $a(n)$: $\hat{a}(n)=(-1)^{[n/N]}a([n]_N)$, $[x]$ is the biggest integer that is smaller than x, Foundation item: $[n]_N=(n) \bmod N$.

Definition 2: Binary sequences pairs $(a(n), a'(n))$[5] of length N is called odd-periodic perfect binary sequences pairs, if its odd-periodic autocorrelation function, OPCF, $\psi_{(a,a')}(\tau)$ satisfies

$$\psi_{(a,a')}(\tau)=\sum_{j=0}^{N-1} a(j)\,\hat{a}'(j+\tau)=\begin{cases} E & \tau=0 \\ E & \tau\neq 0 \end{cases} \qquad (1)$$

Definition 3: Let $\{(a_i(n), a_i'(n))\} \mid 0 \le i \le P-1\}$ be a family of binary sequences pairs of period N, it is called a family of odd-periodic perfect complementary sequences pairs, labeled as $OPCSPF_P^N(a_i, a_i')$, if and only if it satisfies the expression

$$\sum_{i=0}^{P-1}\psi_{(a,a')}(\tau)=\begin{cases}E & \tau=0\\ E & \tau\neq 0\end{cases}$$

Definition 4: For the set of families of binary sequence pairs of length N $\{(a_i(n), a_i'(n)), (b_i(n), b_i'(n)), \ldots, (f_i(n), f_i'(n)), 0 \le i \le p-1\}$. Their family of alternate sequences pairs $\{s_i(n), s_i'(n), 0 \le i \le p-1\}$ is defined as:

$\{s_i(n), s_i'(n)\} = (a_i(n), a_i'(n)) \otimes (b_i(n), b_i'(n)) \otimes \ldots \otimes (f_i(n),$

$f_i'(n), 0 \le i \le P-1 = (a(0), a'(0))\,(b(0), b'(0)) \ldots (f(0), f'(0)) \ldots (a(1), a'(1))\, b(1), b'(1)) \ldots (f(1), f'(1)) \ldots (a(N-1), a'(N-1))\,(b(N-1), b'(N-1)) \ldots (f(N-1), f'(N-1))$

3. Transfomation Properties Of Families Of Odd-Periodic Perfect Complementary Sequeces Pairs

Let $\{(a_i(n), a_i'(n)) \mid 0 \le i \le P-1\}$ be a family of odd-periodic perfect complementary sequences pairs of period N, and $\{(b_i(n), b_i'(n)) \mid 0 \le i \le P-1\}$ be it's the family of its transformation sequences pairs, then the families of sequences pairs we get by 3 transformations bellow are still odd-periodic perfect sequences pairs.

Property 1 (Negative element transform) $b_i(n) = -a_i(n)$, $b_i(n) = -a_i'(n)$.

Property 2 (Linear Phase Transformation)

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$$b_i(n) = (-1)^n\, a_i(n),\ b_i'(n) = (-1)^n\, a_i'(n)$$

Property 3 (Reverse transform) if

$$b_i(n) = a_i(N-1-n)^n,\ b_i'(n) = a_i'(N-1-n)$$

Where $0 \le n \le N-1, 0 \le i \le P-1$

According to definition 2, 3, it's easy to prove those properties, so we omit it.

4. The Construction Of Odd-Periodic Perfect Complementary Sequences Pairs

We can get more and longer new families of odd-periodic perfect complementary sequences pairs by applying the follow theory to the families of odd-periodic perfect complementary sequences pairs we have already known.

Theorem 1: Let $a_i = (a_i(0), a_i(1), \ldots, a_i(N-1))$, $a_i' = (a_i'(0), a_i'(1), \ldots, a_i'(N-1))$ are sequences of period N, $0 \le i \le P-1$. The number of the two sequences is "+1" or "–1". If $a_i(x) = \sum_{j=0}^{2N-1} a_i(j)\, x^j$, $\hat{a}_i'(x) = \sum_{j=0}^{2N-1} (-1)\, x^{[j/N]} a_i[j]_N)\, x^j$, then, the families of complementary sequence pairs $\{a_i, a_i'\}$ is $OPCSPF_P^{2N}$ where $a_i = a_{i \bmod N}$, if and only if ,

$$\sum_{j=0}^{P-1} a_i(x)\,\hat{a}_i'(x^{-1}) = \begin{cases} PN-2\sum_{j=0}^{P-1} d_i & 0 \le j \le N-1\\ 2\sum_{j=0}^{P-1} d_i - PN & N \le j \le 2N-1\end{cases}$$

Proof.

When $0 \le j \le N-1$,

$$\sum_{j=0}^{P-1} a_i(x)\,\hat{a}_i'(x^{-1})$$

$$=\sum_{i=0}^{P-1}\sum_{j=0}^{N-1}\sum_{k=0}^{N-1} a_i(x)\,\hat{a}_i'(k)\, x^{j-k}$$

$$=\sum_{i=0}^{P-1}\sum_{j=0}^{N-1} a_i(j)\,\hat{a}_i'(j) + \sum_{i=0}^{P-1}\ \sum_{j,k=0, j\neq k}^{N-1} a_i(j)\,\hat{a}_i'(k)\, x^{j-k}$$

$$= PN-2\sum_{j=0}^{P-1} d_i + \sum_{\mu=1}^{N-1} x^{\mu} \sum_{i=0}^{P-1}\sum_{j=0}^{N-1} a_i(k+\mu)\,\hat{a}_i'(k)$$

$$= PN-2\sum_{j=0}^{P-1} d_i + \sum_{\mu=1}^{N-1} x^{\mu} \sum_{i=0}^{P-1} \psi_{(a_i, a_i')}(N-\mu)$$

$\sum_{i=0}^{P-1}\psi_{(a_i, a_i')}(N-\mu) = 0$, if and only if

$$\sum_{j=0}^{P-1} a_i(x)\,\hat{a}_i'(x^{-1}) = PN-2\sum_{j=0}^{P-1} d_i\,.$$

For $N \le j \le 2N-1$, we can easy proof

$$\sum_{i=0}^{P-1}\psi_{(a_i, a_i')}(N-\mu) = 0$$

When

$$\sum_{j=0}^{P-1} a_i(x)\,\hat{a}_i'(x^{-1}) = 2\sum_{j=0}^{P-1} d_i - PN$$

So, $\{(a_i, a_{i'})\}$ is a $OPCSPF_P^{2N}$, if and only if

$$\sum_{j=0}^{P-1} a_i(x)\,\hat{a}_i'(x^{-1}) = \begin{cases} PN-2\sum_{i=0}^{P-1} d_i & 0 \le j \le N-1\\ 2\sum_{i=0}^{P-1} d_i - PN & N \le j \le 2N-1\end{cases}$$

Theorem 2: if there exists a set of $OPCSPF_{P_1}^{N_1}$, $OPCSPF_{P_2}^{N_2}$,...,$OPCSPF_{P_b}^{N_b}$ and $a \times b$ columns orthogonal binary matrix, then there exists $OPCSPF_{aP}^{N}$ $N = N_1 + N_2 + \ldots + N_b$, $P = L.C.M\,(p_1, p_2, \ldots, p_b)$.

Proof.

Let $[h(x, y)]$ is a $a \times b$ columns orthogonal binary matrix, and $\sum_{w=1}^{a} h(w, r)\, h(w, q) = 0, 1 \le r \neq q \le b$.

Let $\{(a_i^{(r)}, a_i'^{(r)})\}$, $0 \le i \le P_r - 1\}$ is a $OPCSPF_{P_r}^{N_r}$, $a_i^{(r)}(x)$ and $\hat{a}_i'^{(r)}(x)$ denote respectively the related polynomials of

$a_i^{(r)}$ and $\hat{a}_i'^{(r)}$, $d_i^{(r)}$ - the Hamming distance between the former sequence and the odd-periodic sequence of the latter sequence in every sequences pair when sequence $a_i'^{(r)}$ doesn't shift, where $i = 0, 1, ..., P-1$.

From Theorem 1, we can get

$$\sum_{j=0}^{P-1} a_i^{(r)}(x)\,\hat{a}_i'^{(r)}(x^{-1}) = \begin{cases} P_r N_r - 2\sum_{i=0}^{P-1} d_i^{(r)} & 0 \le j \le N_r - 1 \\ 2\sum_{i=0}^{P-1} d_i^{(r)} - P_r N_r & N_r \le j \le 2N_r - 1 \end{cases}$$

For $i \ge P_r$, let $a_i^{(r)} = a_{i \bmod P_r}^{(r)}$, c_{wi} and $\hat{e}_{wi}$ denote respectively the binary sequence of $N = N_1 + N_2 + ... + N_b$,

$$c_{wi} = h(w,1)a_i^{(1)} \oplus h(w,2)a_i^{(2)} \oplus ... \oplus h(w,b)a_i^{(b)}$$

$$\hat{e}_{wi} = h(w,1)\hat{a}_i^{(1)} \oplus h(w,2)\hat{a}_i^{(2)} \oplus ... \oplus h(w,b)\hat{a}_i^{(b)}$$

where $1 \le w \le a$, $0 \le i \le P-1$.

Let

$$C_{wi}(x) = \sum_{r=1}^{b} h(w,r)\, a_i^{(r)}(x)\, x^{N(r)}$$

$$\hat{E}_{wi}(x) = \sum_{r=1}^{b} h(w,r)\, \hat{a}_i'^{(r)}(x)\, x^{N(r)}.$$

Where

$N(1) = 0, N(2) = 2N_1, ... N(r) = 2N_1 + 2N_2 + ... 2N_{r-1}, r \ge 2.$

When $0 \le j \le N_r - 1$,

$$\sum_{w=1}^{a}\sum_{j=0}^{p-1}(C_{wi}(x)\,\hat{E}_{wi}(x^{-1})) + 2d(C_{wi}, \hat{e}_{wi})$$

$$= \sum_{w=1}^{a}\sum_{i=0}^{p-1}((\sum_{w=1}^{b} h(w,r)a_i^{(r)}(x)\,x^{N(r)})(\sum_{r=1}^{b} h(w,r).$$

$$\hat{a}_i'^{(r)}(x^{-1})\,x^{-N(r)}) + 2\sum_{r=1}^{b} d_i^{(r)}$$

$$= \sum_{i=0}^{p-1}((\sum_{r,p=1}^{b} a_i^{(r)}(x)\hat{a}_i'^{(p)}(x^{-1})\,x^{N(r)-N(p)}.$$

$$\sum_{w=1}^{a} h(w,r)\,h(w,q) + 2a\sum_{r=1}^{b} d_i^{(r)}$$

For $[h(x,y)]$ is a columns orthogonal binary matrix, so the equality can transform as

$$a\sum_{r=1}^{b}(\sum_{i=0}^{P-1} a_i^{(r)}(x)\,\hat{a}_i'^{(r)}(x^{-1}) + 2d_i^{(r)} = a\sum_{r=1}^{b} PN_r = aPN$$

When $N_r \le j \le 2N_r - 1$,

$$\sum_{w=1}^{a}\sum_{j=0}^{p-1}(C_{wi}(x)\,\hat{E}_{wi}(x^{-1})) - 2d(C_{wi}, \hat{e}_{wi})$$

$$= \sum_{w=1}^{a}\sum_{i=0}^{p-1}((\sum_{r=1}^{b} h(w,r)a_i^{(r)}(x)\,x^{N(r)})(\sum_{r=1}^{b} h(w,r).$$

$$\hat{a}_i'^{(r)}(x^{-1})\,x^{-N(r)}) - 2\sum_{r=1}^{b} d_i^{(r)}$$

$$= \sum_{i=0}^{p-1}((\sum_{r,p=1}^{b} a_i^{(r)}(x)\hat{a}_i'^{(p)}(x^{-1})\,x^{N(r)-N(p)}.$$

$$\sum_{w=1}^{a} h(w,r)\,h(w,q) - 2a\sum_{r=1}^{b} d_i^{(r)}$$

$$a\sum_{r=1}^{b}(\sum_{i=0}^{P-1} a_i^{(r)}(x)\,\hat{a}_i'^{(r)}(x^{-1}) + 2d_i^{(r)} = (-1)\,a\sum_{r=1}^{b} PN_r$$

$$= (-1)\,aPN.$$

So $\{(c_{wi}, e_{wi}), 1\}$, $1 \le w \le a$, $0 \le i \le P-1$ is a $OPCSPF_{aP}^{N}$.

5. Conclusion

This paper is based on odd-periodic perfect sequences, by bringing the concept of "*families of pairs*", and constructing families of odd-periodic perfect complementary sequences pairs (OPCSPF), these are new binary sequences pairs sets. OPCSPF can get much more and complicated sequencebased on several old methods for studying perfect signals.

References

[1] Yang Yixian. (1996). Theory and design of perfect signal, Beijing: Posts & Telecom Press, p. 15-45.

[2] Bomer, L., Antweiler, M. (1990). Periodic complementary binary sequences. *IEEE Trans on Inform Theory*, 36 (6)1487-1497.

[3] Luke, H. D. (1997). Binary odd-periodic complementary sequences. *IEEE Trans On Inform Theory*, 43 (1) 365-367.

[4] Wen Hong, Hu Fei, Fu Chusheng, Jin Fan. (2006). Odd-periodic complementary sequence sets. *Chinese Journal of Radio Science*. 26 (1) 70-73.

[5] Zhao Xiaoqun, He Wencai, Wang Zhongwen, so on. (1999). Study on Perfect Binary Arrays Pairs. *Acta Electronica Sinica*, 27 (1) 34-35.

[6] Xu Chengqian, Jin Huilong. (2003). Families of Almost Perfect Periodic Complementary Binary Sequences Pairs. *Systems Engineering and Electronics*. 25 (9) 1086-1089.

[7] Jin Hui-long, Xu Cheng-qian. (2010). The Study of Methods for Constructing a Family of Pseudorandom Binary Sequence Pairs Based on the Cyclotomic Class. *Acta Electronica Sinica*, 38 (7) 1608-1611.