

Low Rank MMSE Channel Estimation in MIMO-OFDM Systems

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ABSTRACT: *In this paper, we propose a low-rank minimum mean-square error (OLR-MMSE) channel estimator for multiple-input multiple-output orthogonal frequency division multiplexing (MIMO-OFDM) systems. We evaluate the performance of the proposed low-rank channel estimator for slowly fading channel environments in terms of the symbol error rate (SER) by computer simulations. It is shown that the proposed channel estimator gives the best tradeoff between performance and complexity.*

Keywords: MIMO-OFDM, Channel Estimation, MMSE, OLR-MMSE

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1. Introduction

OFDM is a multi-carrier modulation technique where data symbols modulate a parallel collection of regularly spaced sub-carriers. It is becoming widely applied in wireless communications systems due to its high rate transmission capability with high bandwidth efficiency and its robustness with regard to multi-path fading and delay [1]. It has been used in digital audio broadcasting (DAB) systems, digital video broadcasting (DVB) systems, digital subscriber line (DSL) standards, and wireless LAN standards such as the American IEEE® Std. 802.11™ (WiFi) and its European equivalent HIPRLAN/2 [2]. It has also been proposed for wireless broadband access standards such as IEEE Std. 802.16™ (WiMAX) and as the core technique for the fourth-generation (4G) wireless mobile communications.

Multiple-input multiple-output (MIMO) antennas and OFDM can be implemented to achieve high spectral efficiency and/or a large coverage area that are critical for future-generation wireless local area networks [3-5]. Realizing these gains requires the channel state information (CSI) at the receiver, which is often obtained through channel estimation. There exist two main types of channel estimation schemes: pilot-assisted schemes [4], [6], [8-11] in which a portion of the bandwidth is allocated to training symbols, and blind approaches [12-14], which can be implemented by exploiting the statistical properties or the deterministic information of the transmitted symbol (e.g., finite alphabet, constant modulus, etc.). For pilot-assisted schemes, CSI can be estimated by exploiting the frequency correlation and/or the time correlation of the pilot and data symbols [3], [4], [8]. The two basic 1D channel estimations for pilot-assisted schemes are block-type pilot channel estimation and comb-type pilot channel estimation [9], [10], [15], in which the pilots are inserted in the frequency direction and in the time direction, respectively. The estimations for the comb-type pilot arrangement includes the least square (LS) estimator with 1D interpolation, the maximum likelihood (ML) estimator, and the parametric channel modeling-based (PCMB) estimator. The estimations for the block-type pilot arrangement can be based on LS, minimum mean-square error (MMSE), and modified MMSE. Other channel estimation strategies were also studied in [16-18]. The MMSE

mean-square error (MMSE), and modified MMSE. Other channel estimation strategies were also studied in [16-18]. The MMSE estimator yields much better performance than LS estimators, especially under the low SNR scenarios. A major drawback of the MMSE estimator is its high computational complexity, especially in matrix inversions. To overcome this drawback, modified MMSE estimators are proposed to reduce complexity [7], [19-21]. Among them, an optimal low-rank MMSE (OLR-MMSE) estimator is proposed in this paper, which is achieved by using the singular value decomposition (SVD) of the channel auto covariance matrix.

This paper is organized as follows: In section 2, we give a description of the MIMO-OFDM system. In section 3, we discuss the LS, MMSE and OLR-MMSE channel estimators for the SISO-OFDM and MIMO-OFDM systems. Next, performance of the proposed estimator is evaluated and compared to LS and MMSE estimators in section 4. Finally, conclusion is given in section 5.

2. Description Of The MIMO-OFDM System

The general transceiver structure of MIMO-OFDM system is presented in Figure 1. It uses N_t transmit antennas and N_r receive antennas with N subcarriers in an OFDM block, a cyclic prefix (CP) is added to avoid inter-symbol interference (ISI). The incoming bits are coded and mapped according to the type of modulation used (binary or quadrature phase shift keying (BPSK/QPSK), 16-quadrature amplitude modulation (QAM), or 64-QAM). The frequency domain transmitted sequence from the n -th ($n = 1, \dots, N_t$) transmit antenna is represented by $X_{n,k}$ where $k = 1, \dots, N$ represents the k -th OFDM sub-carrier. For each modulated signal, an Inverse Fast Fourier Transform (IDFT) of size N is performed. After parallel-to-serial (P/S) conversion, signal is transmitted from the corresponding antenna. The channel between each transmitter/receiver pair is modeled as multipath channel, whose delay characteristic is assumed to be the same for all available channels. The impulse response of the channel is expressed as:

$$g(t) = \sum_{m=1}^M \alpha_m \delta(t - \tau_m T_s) \quad (1)$$

Where α_m is the m -th complex path gain, τ_m is the corresponding path delay, and T_s is the sampling interval.

At the receiver side, first serial-to-parallel (S/P) conversion is performed and the CP is removed. After DFT operation, the sequence received by the m -th ($m = 1, \dots, N_r$) receive antenna is expressed as:

$$Y_{m,k} = \sum_{n=1}^{N_t} H_{m,n,k} X_{n,k} + W_{m,k} \quad (2)$$

where, $H_{m,n,k}$ is the frequency response of the channel between the n -th transmit antenna and the m -th receive antenna for the k -th sub-carrier, $W_{m,k}$ is the frequency response of zero-mean additive white Gaussian noise (AWGN) with variance σ_N^2 .

The system is modeled using the N -point discrete-time Fourier transform (DFTN) as:

$$y = DFT_N \left(IDFT_N(x) \otimes \frac{g}{\sqrt{N}} + w \right) \quad (3)$$

where $x = [x_{n,k}]^T$, ($n = 1, \dots, N_t$), ($k = 1, \dots, N$) denote the input data of IDFT block at the transmitter, $y = [y_{m,k}]^T$, ($m = 1, \dots, N_r$) the output data of DFT block at the receiver, $w = [w_{m,k}]^T$ is the vector of i.i.d complex Gaussian variables, and \otimes denotes cyclic convolution.

The vector $\frac{g}{\sqrt{N}}$ is the observed channel impulse response after sampling the frequency response of $g(t)$, and

$$g_k = \frac{1}{\sqrt{N}} \sum_m \alpha_m e^{-j\frac{\pi}{N}(k+(N-1)\tau_m)} \frac{\sin(\pi\tau_m)}{\sin\left(\frac{\pi}{N}(\tau_m - k)\right)} \quad (4)$$

Let us define the signal transmitted on the k -th sub-carrier from all the N_t transmit antennas as $X_k = [X_{1,k}, X_{2,k}, \dots, X_{N_t,k}]^T$

The received signal as a function of the respective channel state information (CSI) matrix $H_{m,n,k}$ can be expressed as:

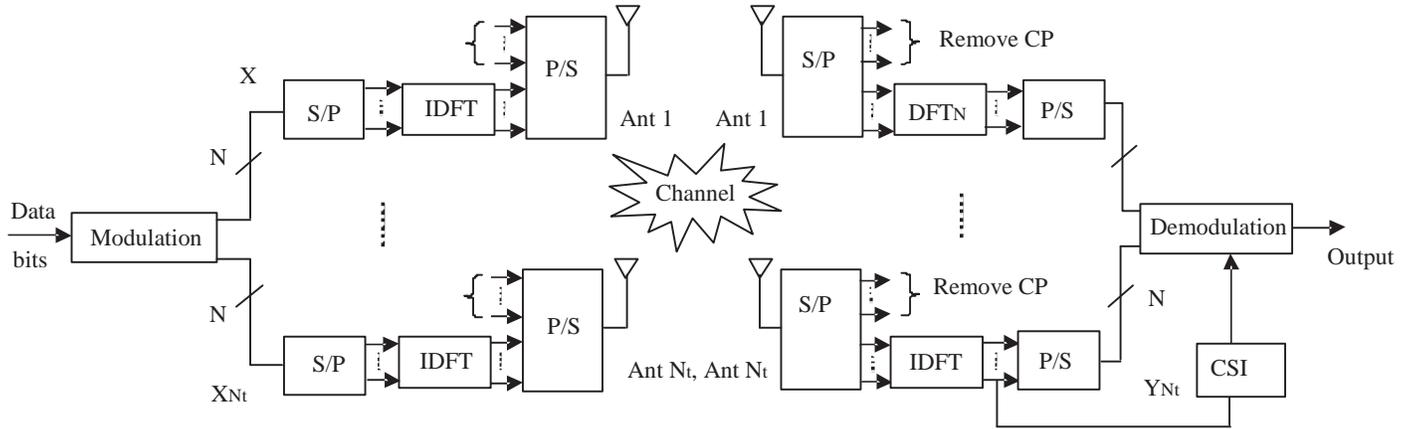


Figure 1. MIMO-OFDM transceiver

$$\begin{aligned}
 Y_k &= [Y_{1,k}, Y_{2,k}, \dots, Y_{Nr,k}] \\
 &= \begin{bmatrix} H_{1,1,k} & \dots & H_{1,Nt,k} \\ \vdots & \ddots & \vdots \\ H_{Nr,1,k} & \dots & H_{Nr,Nt,k} \end{bmatrix} X_k + \begin{bmatrix} W_{1,k} \\ \vdots \\ W_{Nr,k} \end{bmatrix} \\
 &= H_k X_k + W_k
 \end{aligned} \tag{5}$$

Let define the DFT matrix

$$F = \begin{bmatrix} F_N^{00} & \dots & F_N^{0N} \\ \vdots & \ddots & \vdots \\ F_N^{N0} & \dots & F_N^{NN} \end{bmatrix} \tag{6}$$

where, $F_N^{ik} = \frac{1}{\sqrt{N}} e^{-j2\pi\left(\frac{ik}{N}\right)}$

Let $H = DFT(g) = Fg$ and $W = Fw$

Then, (3) becomes under assumption that the interferences are completely eliminated

$$Y = XFG + W = XH + W$$

Where $X = \text{diag}(x)$ is the input matrix.

3. Channel Estimation

The two basic 1D channel estimations in OFDM systems are block-type pilot channel estimation, and comb-type pilot channel estimation. Block-type pilot channel estimation is developed under the assumption of slow fading channel, and it is performed by inserting pilot tones into all subcarriers of OFDM symbols within a specific period as illustrated in Figure 2. The second one is introduced to satisfy the need for equalizing when the channel changes even from one OFDM block to the subsequent one. It is thus performed by inserting pilot tones into certain subcarriers of each OFDM symbol, where the interpolation is needed to estimate the conditions of data subcarriers.

In this paper, we are interested in the first type, since we deal with slow fading channels.

In block-type pilot-based channel estimation, as shown in Figure 2, OFDM channel estimation symbols are transmitted periodically, and all subcarriers are used as pilots. The task here is to estimate the channel conditions, given the pilot signals and received signals, with or without using certain knowledge of the channel statistics. The receiver uses the estimated channel conditions to decode the received data inside the block until the next pilot symbol arrives. The estimation can be based on least square (LS), minimum mean-square error (MMSE), or modified MMSE.

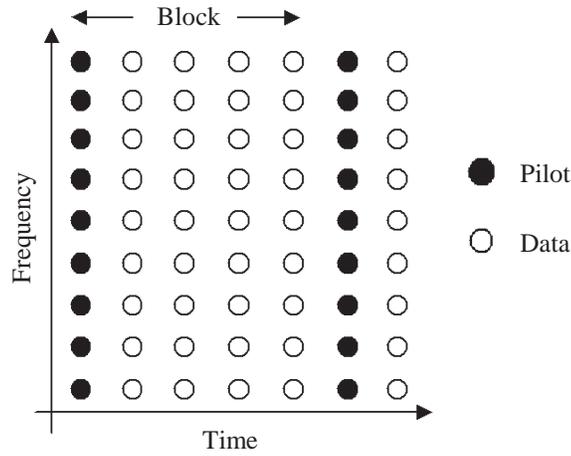


Figure 2. Block-type pilot channel estimation

3.1 SISO OFDM Channel Estimation

3.1.1 LS Estimator

The idea behind least squares is to fit a model to measurements in such a way that weighted errors between the measurements and the model are minimized. The LS estimate of the H , given the received data Y and the transmitted symbols X is [11]

$$\hat{H}_{LS} = X^{-1}Y \quad (8)$$

Without using any knowledge of the statistics of the channels, the LS estimator is calculated with very low complexity, but it suffers from a high mean-square error.

3.1.2 MMSE Estimator

The MMSE channel estimator is the best linear estimator in the mean-square error sense. It employs the second-order statistics of the channel conditions to minimize the mean-square error.

Denote by R_{gg} , R_{HH} , and R_{YY} the autocovariance matrices of g , H and Y , respectively, and by R_{gY} the cross covariance matrix between g and Y . Also denote by σ_N^2 the noise variance $E\{|W|^2\}$. Assume the channel vector g and the noise W are uncorrelated, it is derived that:

$$R_{HH} = E\{HH^H\} = E\{(Fg)(Fg)^H\} = FR_{gg}F^H \quad (9)$$

$$R_{gY} = E\{gY^H\} = E\{g(XFg + W)^H\} = R_{gg}F^H X^H \quad (10)$$

$$R_{YY} = E\{YY^H\} = XFR_{gg}F^H X^H + \sigma_N^2 I_N \quad (11)$$

Assume R_{gg} (thus R_{HH}) and σ_N^2 are known at the receiver in advance; the MMSE estimator of g is given by [11]

$$\hat{g}_{MMSE} = R_{gY}R_{YY}^{-1}Y^H \quad (12)$$

Note that if g is not Gaussian, \hat{g}_{MMSE} is not necessarily a minimum mean-square error estimator, but it is still the best linear estimator in the mean-square error sense. The MMSE estimator is given by

$$\begin{aligned} \hat{H}_{MMSE} &= F\hat{g}_{MMSE} = F \left[(F^H X^H)^{-1} R_{gg}^{-1} \sigma_N^2 + XF \right]^{-1} Y \\ &= FR_{gg} \left[(F^H X^H XF)^{-1} \sigma_N^2 + R_{gg} \right] F^{-1} \hat{H}_{LS} \end{aligned}$$

$$= R_{HH} \left[R_{HH} + \sigma_N^2 (XX^H)^{-1} \right]^{-1} \hat{H}_{LS} \quad (13)$$

The major disadvantage of the MMSE estimator is its high complexity, which grows exponentially with the observation sample.

3.1.3 Low-rank MMSE

In order to reduce the complexity of the MMSE estimator, we propose an OLR-MMSE estimator that combines the following assumptions [9], [19]:

The first simplification of MMSE estimator is to replace the term $(X[(X^H)]^{-1})$ in (13) with its expectation $E\{(X[(X^H)]^{-1})\}$. Assuming the same signal constellation on all tones and equal probability on all constellation points, we have

$$E\left\{(XX^H)^{-1}\right\} = E\left\{\left|\frac{1}{X_k}\right|^2\right\} \quad (14)$$

Defining the average SNR as $\frac{\beta}{SNR} = \frac{E\{|X_k|^2\}}{\sigma_N^2}$, and the term $\beta = \frac{E\{|X_k|^2\}}{\sigma_N^2} E\left\{\left|\frac{1}{X_k}\right|^2\right\}$. Then, the term $\sigma_N^2 (XX^H)^{-1}$ is approximated by $\left(\frac{\beta}{SNR}\right)I$, where β is a constant depending only on the signal constellation.

The second simplification is based on the low-rank approximation. Since $0 \leq \tau mTs \leq TG$ (where TG is the length of the cyclic extension), most of the energy in g is contained in, or near, the first $(L+1)$ taps, where $L = [TG/Ts]N$. Therefore, we can only consider the taps with significant energy, that is, the upper left corner of the autocovariance matrix R_{gg} . In the IEEE Std. 802.11 and IEEE Std. 802.16, $[TG/Ts]$ is chosen among $\{1/32, 1/16, 1/8, 1/4\}$, so the effective size of matrix is reduced dramatically after the low-rank approximation is used.

The third simplification is the optimal rank reduction that uses the singular value decomposition (SVD). The SVD of R_{HH} is $R_{HH} = UVU^H$, where U is a unitary matrix containing the singular vectors, and V is a diagonal matrix containing the singular values ... N on its diagonal. The SVD also dramatically reduces the calculation complexity of matrices.

The optimal rank reduction is found from the correlation matrices:

$$R_{H\hat{H}_{LS}} = E\{H\hat{H}_{LS}^H\} \quad (15)$$

$$R_{\hat{H}_{LS}\hat{H}_{LS}} = E\{\hat{H}_{LS}\hat{H}_{LS}^H\} \quad (16)$$

and the SVD

$$R_{H\hat{H}_{LS}} R_{\hat{H}_{LS}\hat{H}_{LS}}^{-1/2} = Q_1 D Q_2^H \quad (17)$$

where, Q_1 and Q_2 are unitary matrices and D is a diagonal matrix with the singular values d_1, d_2, \dots, d_N on its diagonal. The best rank- p estimator is then

$$\hat{H}_p = Q_1 \begin{bmatrix} D_p & 0 \\ 0 & 0 \end{bmatrix} Q_2^H R_{\hat{H}_{LS}\hat{H}_{LS}}^{-1/2} \hat{H}_{LS} \quad (18)$$

where D_p is the upper left corner of D , i.e., we exclude all but the p largest singular vectors.

We have $R_{H\hat{H}_{LS}} = R_{HH}$ and $R_{\hat{H}_{LS}\hat{H}_{LS}} = R_{HH} + \frac{\beta}{SNR}I$. We note that they share the UH . Thus

$$R_{\hat{H}_{LS}\hat{H}_{LS}} R_{\hat{H}_{LS}\hat{H}_{LS}}^{-1/2} = UVU^H \left(U \left(V + \frac{\beta}{SNR}I \right) U^H \right)^{-1/2}$$

$$= UV \left(V + \frac{\beta}{SNR} I \right)^{-1/2} U^H = Q_1 D Q_2^H \quad (19)$$

where, $Q_1 = Q_2 = U$

$$D = V \left(V + \frac{\beta}{SNR} I \right)^{-1/2}$$

Combining all simplification techniques, the OLR-MMSE estimator is explained as follows. The system first determines the number of ranks required by the estimator, denoted by p , which should be no smaller than $(L+1)$. Then, given the signal constellation, the noise variance and the channel autocovariance matrix R_{HH} , the receiver pre-calculates, $\frac{\beta}{SNR}$, the unitary matrix U , and the singular values k . It thus obtains the $(N \times N)$ diagonal matrix p with entries

$$\delta_k = \begin{cases} \left[\frac{\lambda_k}{\lambda_k + \frac{\beta}{SNR}} \right] & k = 1, \dots, p \\ 0 & k = p+1, \dots, N \end{cases}$$

During the transmission, using the transmitted pilots X and received signals Y , the \hat{H}_{LS} is calculated according to (8), and the OLR-MMSE estimator with rank p is given by [19]

$$\hat{H}_{OLR-MMSE} = U \Delta_p U^H \hat{H}_{LS} \quad (21)$$

The low-rank estimator can be interpreted as first projecting the LS-estimates onto a subspace and then performing the estimation. If the subspace has a small dimension and can describe the channel well, the complexity of the estimator will be low while showing a good performance. However, the low-rank estimators introduce an irreducible error floor due to the part of the channel that does not belong to the subspace. To eliminate this error floor up to a given SNR, we need to partition the tones into reasonably-sized blocks and perform the estimation independently in these blocks. For example, the 2048-tone system can be approximately described by 32 parallel 64-tone systems, and each channel attenuation can be estimated independently by OLR-MMSE estimator with rank $p = (64/4 + 1) = 17$. In the scenarios when $(L+1)$ is too large, this strategy reduces the complexity significantly at the expense of certain performance loss because it neglects the correlation between tones in different subsystems.

3.2 MIMO OFDM Channel Estimation

3.2.1 LS Estimator

Similar to the SISO scenario, the LS channel estimation for MIMO-OFDM System between the n -th ($n = 1, \dots, N_t$) transmitter and m -th ($m = 1, \dots, N_r$) receiver antenna is given by:

$$\hat{H}_{LS}^{(n,m)} = \left(X^{(n)} \right)^{-1} Y^{(m)} \quad (22)$$

3.2.2 MMSE Estimator

The MMSE channel estimator between the n -th antenna and m -th antenna is given by:

$$\hat{H}_{MMSE}^{(n,m)} = R_{HH} \left[R_{HH}^{(n,m)} + \sigma_N^2 \left(\left(X^{(n)} \right) \left(X^{(n)} \right)^H \right)^{-1} \right]^{-1} \hat{H}_{LS}^{(n,m)} \quad (23)$$

3.2.3 OLR-MMSE Estimator

The OLR-MMSE channel estimator between the n -th antenna and m -th antenna is given by

$$\hat{H}_{OLR-MMSE}^{(n,m)} = U \Delta_p U^H \hat{H}_{LS}^{(n,m)} \quad (24)$$

4. Simulation Results

In this section, we report computer simulation carried out to evaluate and compare performance of the proposed channel estimator in

terms of the instantaneous MSE and Symbol error rate (SER). In our simulation we consider a 2×2 MIMO-OFDM system over a slow fading channel, operating with a bandwidth of 1.25 MHz, divided into 256 subchannels. The two subchannels on each end are used as guard tones, and the rest (252 tones) are used to transmit data. To make the tones orthogonal to each other, the symbol duration is about 204.8 μ s. An additional 20.2 μ s guard interval is used to provide protection from ISI due to channel multipath delay spread. The employed symbol modulation is Binary Phase Shift Keying (BPSK).

The channel model for SISO OFDM with sampling interval T_s is given by

$$h(n) = \delta(n) + \delta(n - 0.5 T_s) + \delta(n - 3.5 T_s)$$

The channel models for the (2x2) MIMO-OFDM are given by

$$h_{11}(n) = \delta(n) + \delta(n - 0.5 T_s) + \delta(n - 3.5 T_s)$$

$$h_{12}(n) = \delta(n) + \delta(n - 0.4 T_s) + \delta(n - 1.1 T_s)$$

$$h_{21}(n) = \delta(n) + \delta(n - 0.4 T_s) + \delta(n - 0.9 T_s)$$

$$h_{22}(n) = \delta(n) + \delta(n - 0.6 T_s) + \delta(n - 2.2 T_s)$$

We used Monte-Carlo simulations to generate R_{gg} for this channel model. 1000 data vectors were sent through this channel.

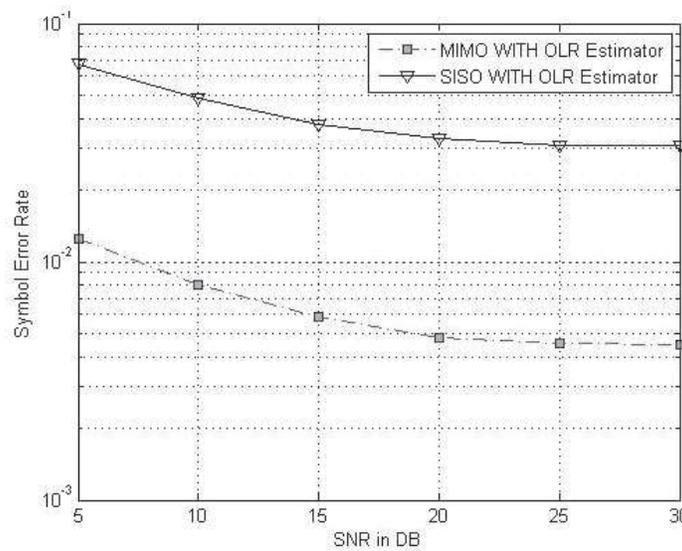


Figure 3. SER Vs SNR with OLR-MMSE estimator for MIMO-OFDM and SISO-OFDM systems (p=25)

The instantaneous MSE is defined as the average error within an OFDM block. The MSE of the rank-p estimate is given by [19]

$$MSE(p) = \frac{1}{N} \sum_{k=1}^p \left(\lambda_k (1 - \delta_k)^2 + \frac{\beta}{SNR} \delta_k^2 \right) + \frac{1}{N} \sum_{k=p+1}^N \lambda_k \quad (25)$$

Figure 3 compares the SER of the MIMO-OFDM system with that of SISO-OFDM system versus the SNR, while using the OLR-MMSE channel estimation with rank $p = 25$. It is clear that the SER performance of the MIMO-OFDM system is better than that of the SISO-OFDM system for all values of the SNR.

In Figure 4, the SER performance of the SISO-OFDM system is plotted versus SNR with the LS, MMSE and OLR-MMSE estimators. It can be observed that the LS estimator yields the worst performance, and the proposed estimator is 5 dB better than the LS estimator and closer to the performance of the MMSE estimator. The simulation results show that the performance of the proposed estimator with sufficient rank increases as SNR increase, until 25 dB, where the curves of the MMSE and OLR-MMSE estimators are identical.

Figure 5 shows the SER performance versus SNR of the estimation schemes with LS, MMSE, and OLR-MMSE for the MIMO-OFDM system. The results show that the OLR-MMSE estimator outperforms the LS estimator, and close to that of the MMSE estimator. It can achieve almost the full performance for a rank $p = 40$.

Figure 6 compares the MSE of the OLR-MMSE channel estimator with rank $p = 32$ of a MIMO-OFDM system with the OLR-MMSE of a SISO system. It can be observed that the latter presents higher mean square error than the MIMO-OFDM system

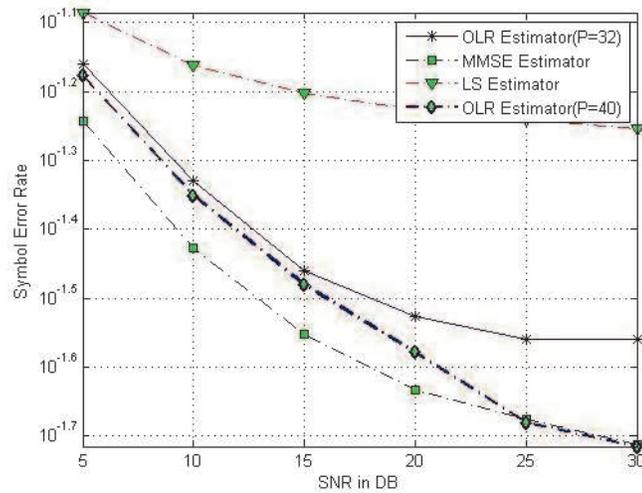


Figure 4. SER versus SNR for SISO-OFDM system with LS, MMSE, and OLR-MMSE

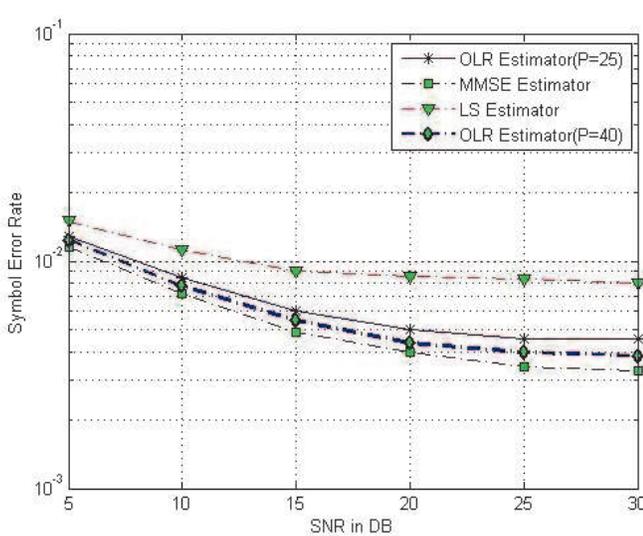


Figure 5. SER versus SNR for MIMO-OFDM system with LS, MMSE, and OLR-MMSE

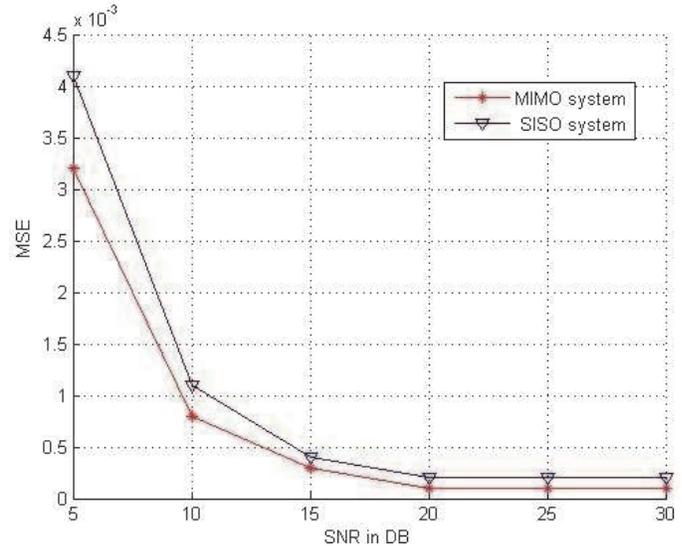


Figure 6. MSE versus SNR for MIMO and SISO-OFDM systems with OLR-MMSE ($p = 32$)

5. Conclusion

In this paper, a reduced-rank estimator based on singular value decomposition has been presented. We implement it into a (2×2) MIMO-OFDM system. We compared the performance of the proposed estimator with that of LS and MMSE estimators in both SISO and MIMO OFDM systems in terms of the symbol error rate, and the mean square error. Simulation results indicate that better channel estimation performance can be achieved through diversity at both transmitter and receiver. Also, we showed that the computational complexity can be reduced considerably by reducing the rank of the autocovariance of the channel matrix, with only a small loss in performance and this performance degradation can be limited by choosing a sufficient rank. Theoretical analysis and simulation results show that the optimal low rank MMSE estimator obtains good compromise between BER performance and computational complexity.

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