VIKOR Method for the Group Decision Making Problems with Ordinal Interval Numbers

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ABSTRACT: *In order to solve the group decision making problems with ordinal interval preference information, a new decision method is proposed based on VIKOR method. The VIKOR method of compromise ranking determines a compromise solution, providing a maximum "group utility" for the "majority" and a minimum of an "individual regret" for the "opponent", which is an effective tool in multi-attribute decision making. By integrating the operational laws of ordinal interval and the concept of VIKOR method, the detail calculation steps are developed for the group decision making with ordinal interval preference information. Because the comprehensive evaluation values are still interval numbers, we induce the possibility degree to compare these interval numbers. Finally, an example is given, the result shows the approach is simple, effective and easy to calculate.*

Keywords: Group Decision Making, Ordinal Interval, Preference Information, VIKOR Method

Received: 10 May 2015, Revised 13 June 2015, Accepted 20 June 2015

1. Introduction

Due to the fuzziness and uncertainty of the objective things, in some practical group decision making process, members in the group may provide fuzzy or uncertain preference information when they are asked to rank alternatives [1]. For example, some member of a group may sort the alternative in first, second or third, which the three order are equal in his/her opinion because of his/her limitation of knowledge or other reasons. Then the ordinal preference information will occur in this situation [2-4]. In recent years, the group decision making method deal with ordinal preference information attracts many scholars' interest. Many methods are put forward. Ref. [5] proposed a decision making method to solve group decision making problems with ordinal preference information by using integer programming method. Ref. [6] proposed a method which first transform the conversion of reciprocal judgment matrix into order interval through the comparison of two alternatives, then solve the group decision problem with the method proposed by Ref. [5]. On the basis of the traditional Borda method, Ref. [7] proposed a method of solving ordinal preference information in group decision making method, which has the advantage of clear in concept, and simple in calculation comparing with the integer programming method. On the basis of TOPSIS method, Ref.[8] presented a method to group decision making problem with ordinal interval preference information. Ref. [9] extended Grey relational analysis (GRA) method to group decision making problems with ordinal preference information. VIKOR method, firstly proposed by Opricovic [10], can provide a maximum "group utility of the majority" as well as the minimum "individual regret of the opponent". Thus in the group decision making process, it has much advantage over other method, such as TOPSIS and ELECTRE Method [11, 12]. Motivated by the concept of VIKOR method, a new group decision method is put forward to solve the group decision making problems with ordinal interval information.

The rest of the paper is organized as follows. Section 2 gives the preliminary knowledge. In Section 3, a new decision making method is proposed based on the concept of VIKOR, and to illustrate the effectiveness and feasibility of the proposed method, a practical example is given in Section 4. Finally, a conclusion is given in Section 5.

2. Preliminary Knowledge

The preliminary definitions and lemmas are given as below.

Definition 1. [7, 8] Suppose r^L and r^U be positive real numbers, where $r^L \leq r^U$, then $\tilde{r} = [r^L, r^U]$ is called ordinal interval.

In particular, when $r^L = r^U$, then the order interval $\tilde{r}$ is degraded as ordinary order value. Without loss of generality, it is assumed that the smaller value of r^L (or r^U), show that the proposed represent the actual meaning (e.g., ranking) the better. For example, $\tilde{r} = [r^L, r^U]$ = 2,4] indicates that the alternative can be ranked equivalent in the position of No. 2, 3 or 4.

Definition 2. [8] Let $\tilde{r}_1 = [r_1^L, r_1^U]$ and $\tilde{r}_2 = [r_2^L, r_2^U]$ be two any uncertain preference ordinal, then the distance between them is defined as follows:

$$d(\tilde{r}_1, \tilde{r}_2) = | r_1^L - r_2^L | + | r_1^U - r_2^U | \tag{1}$$

Obviously, the smaller of $d(\tilde{r}_1, \tilde{r}_2)$ is, the difference degree of $\tilde{r}_1$ and $\tilde{r}_2$ smaller is. In particular, when there is uncertainty $d(\tilde{r}_1, \tilde{r}_2) = 0$, we say preference $\tilde{r}_1$ is equal to $\tilde{r}_2$ (i.e.) $\tilde{r}_1 = \tilde{r}_2$.

Suppose that $\tilde{u} = (\tilde{u}_1, \tilde{u}_2, ..., \tilde{u}_n)^T$ and $\tilde{v} = (\tilde{v}_1, \tilde{v}_2, ..., \tilde{v}_n)^T$ are two any preference ordinal vector, where $\tilde{u}_i (i = 1, 2, ..., n)$ and $\tilde{v}_j (j = 1, 2, ..., n)$ are all uncertain interval ordinal; set $w = (w_1, w_2, ..., w_n)^T$ be a weight vector, where w_j represents important degree of $\tilde{u}_j$ and $\tilde{v}_j$, which satisfies, $w_j \geq 0$, $j = 1, 2, ..., n$ and $\sum_{j=1}^{n} w_j = 1$. The weighted distance of the two uncertainty preference ordinal vector $\tilde{u}$ and $\tilde{v}$, noted by, $d(\tilde{u}, \tilde{v})$ is defined as follows:

$$d(\tilde{u}, \tilde{v}) = \sum_{j=1}^{n} [w_j \, | r_1^L - r_2^L | + w_j \, | r_1^U - r_2^U |] \tag{2}$$

Definition 3. [13] Let $\tilde{a}$ = $[a^L, a^U]$ and $\tilde{b}$ = $[a^L, a^U]$ are two any interval numbers, then the the possible degree of $\tilde{a} \geq \tilde{b}$ is

$$p(\tilde{a} \geq \tilde{b}) = \frac{\max\{0, l_{\tilde{a}} + l_{\tilde{b}} - \max(b^U - a^L, 0)\}}{l_{\tilde{a}} + l_{\tilde{b}}} \tag{3}$$

where $l_{\tilde{a}} = a^U - a^L$ and $l_{\tilde{b}} = b^U - b^L$ are the length of $\tilde{a}$ and $\tilde{b}$.

Suppose that $X = \{x_1, x_2, .., x_m\}$ is a set of $m(m \geq 2)$ alternatives, where x_i is the i^{th} alternative; $E = \{E_1, E_2, .., E_n\}$ is a set of n $(n \geq 2)$ experts, where E_j is the j^{th} expert; And $w = (w_1, w_2, .., w_n)^T$ is the weight vector of experts, where w_j is the weight (or important degree) of expert E_j and w_j satisfies $\sum_{j=1}^{n} w_j = 1$, $w_j \geq 0$, $j = 1, 2, ..., n$ usually the weight vector is given by the organizers of expert group decision analysis. The uncertain preference information decision matrix can be written as $\tilde{R} = (\tilde{r}_{ij})_{m \times n}$, where $\tilde{r}_{ij} = [r_{ij}^L, r_{ij}^U]$ is ordinal interval evaluation information of the expert E_j with respect to the alternative x_i.

Without loss of generality, we assume that the smaller r_{ij}^L or r_{ij}^U is, the better the ranking position of the corresponding alternative x_i is.

3. VIKOR Method for Group decision with Ordinal interval numbers

VIKOR method is a multi-attribute decision making method based on the ideal decision-making one raised by Opricovic (1998), which defines ideal solutions and negative-ideal solutions firstly, and then the alternatives are sorted and chosen the best in the light of all values of each alternative and the approach degree of ideal alternative. It is a compromise decision-making method when evaluated, considers maximum group utility and minimum individual regret. VIKOR is a decision-making method coming from LP-metric aggregate function [14], which has the following form:

$$L_p(i) = \left\{ \sum_{j=1}^{n} \left(w_j \frac{x_j^* - x_{ij}}{x_j^* - x_j^-} \right)^p \right\}^{1/p} \tag{4}$$

Where $1 \le p \le \infty$, $j = 1, 2, \ldots, n$ with respect to the attribute, and $i = 1, 2, \ldots n$ with respect to the alternative x_i.

For alternative x_i, the evaluated value of the j^{th} attribute is denoted by x_{ij}, and n is the number of attribute. The measure $L_p(i)$ shows the distance between alternative x_i and the positive-ideal solution $x^* = (x_1^*, x_2^*, \ldots, x_n^*)$.

When $p = 1$ and $p = \infty$, respectively, then the equation

$$d_i^{p=1} = S_i = \sum_{j=1}^{n} w_j \left(\frac{x_j^* - x_{ij}}{x_j^* - x_j^-} \right) \tag{5}$$

denotes group utility;

The equation

$$d_i^{p=\infty} = R_i = \max_{1 \le j \le n} \left\{ w_j \left(\frac{x_j^* - x_{ij}}{x_j^* - x_j^-} \right) \right\} \tag{6}$$

denotes individual regret.

The sorting functions used by VIKOR method are just combined function form of (5) and (6) measurement.

Based on VIKOR method, we propose a new group decision method, and the specific calculation steps are given as follows:

Step 1. Determine the positive ideal solution (PIS) r^* and negative ideal solution (NIS) r^- as follows:

$$r^* = \{r_1^*, r_2^*, \ldots, r_n^*\} \text{ and } r^- = \{r_1^-, r_2^-, \ldots, r_n^-\} \text{ where } r_j^* = 1, r_j^- = m,\ j = 1, 2, \ldots, n.$$

Step 2. Calculate the group utility value $\tilde{S}_i = [S_i^L, S_i^U]$ and individual regret value $\tilde{R}_i = [R_i^L, R_i^U]$,

where:

$$S_i^L = \sum_j w_j \frac{r_{ij}^L - r_j^+}{r_j^- - r_j^+},\quad S_i^U = \sum_j w_j \frac{r_{ij}^U - r_j^+}{r_j^- - r_j^+} \tag{7}$$

and

$$R_i^L = \max_{1 \le j \le n}\{w_j(\frac{r_{ij}^L - r_j^+}{r_j^- - r_j^+})\},\quad R_i^U = \max_{1 \le j \le n}\{w_j(\frac{r_{ij}^U - r_j^+}{r_j^- - r_j^+})\} \tag{8}$$

Step3. Calculate the comprehensive sorting index $\tilde{Q}_i = [Q_i^L, Q_i^U]$, where

$$\tilde{Q}_i = \upsilon \frac{\tilde{S}_i - S^*}{S^- - S^*} + (1-\upsilon)\frac{\tilde{R}_i - R^*}{R^- - R^*} \tag{9}$$

by the algorithm of interval fuzzy number :

$$Q_i^L = \upsilon \frac{S_i^L - S^*}{S^- - S^*} + (1-\upsilon)\frac{R_i^L - R^*}{R^- - R^*} \tag{10}$$

and

$$Q_i^U = \upsilon \frac{S_i^U - S^*}{S^- - S^*} + (1-\upsilon)\frac{R_i^U - R^*}{R^- - R^*} \tag{11}$$

Where $S^* = \min_i S_i^L, S^- = \max_i S_i^U, R^* = \min_i R_i^L, R^- = \max_i R_i^U$ and υ is decision mechanism index, $\upsilon \in [0,1]$. If $\upsilon > 0.5$, it means making decision in the light of maximum group benefit; if $\upsilon = 0.5$, it means making decision in accordance with compromise. If $\upsilon < 0.5$, it means making decision in the light of minimum individual regret value. In the VIKOR, take $\upsilon = 0.5$generally, that is, compromise makes maximum group benefit and minimum individual regret value.

Step4. Rank the alternatives based on $\tilde{Q}_i, \tilde{S}_i$ and $\tilde{R}_i$.
Since $\tilde{Q}_i, \tilde{S}_i$ and $\tilde{R}_i$ are still interval numbers, then we use the possible degree theory to compare two interval numbers.

For a number of interval numbers $\tilde{A}_i = [A_i^L, A_i^U]$ $(i = 1,2,...,m)$, the comparison steps of these interval numbers are given as follows:

(a) According to the Eq.(3), for any two interval numbers $\tilde{A}_i = [A_i^L, A_i^U]$ and $\tilde{A}_j = [A_j^L, A_j^U]$ calculate the possible degree $p_{ij} = p(\tilde{A}_i \geq \tilde{A}_j)$ and then construct the possible degree matrix $P = (p_{ij})_{m\times m}$, which produced by comparison any two interval numbers $\tilde{A}_i = [A_i^L, A_i^U]$ and $\tilde{A}_i = [A_i^L, A_i^U]$, where $i, j = 1,2,...,m$. Xu [9] proved that the matrix satisfies $p_{ij} \geq 0, p_{ij} + p_{ji} = 1$, $p_{ii} = 0.5$ $(i, j = 1,2,...,m)$.

Thus the matrix is a fuzzy complementary judgment matrix, and then we can rank the alternatives as follow.

(b) Rank the interval numbers $\tilde{A}_i = [A_i^L, A_i^U], i = 1,2,...,m$.

The ranking formula is given as follows

$$u_i = \frac{1}{m(m-1)}(\sum_{j=1}^{m} p_{ij} + \frac{m}{2} - 1), i = 1,2,...,m \tag{12}$$

The smaller u_i is, the smaller $\tilde{A}_i = [A_i^L, A_i^U]$ is.

Then we can accord to the above steps to compare the interval numbers $\tilde{Q}_i, \tilde{S}_i$ and $\tilde{R}_i$ ($i = 1,2,...,m$), and the smaller $\tilde{Q}_i$ is, then the better alternative x_i is.

4. Case Analysis

To illustrate the effectiveness and feasibility of the proposed method, the example adopted the paper [15] is used to analysis. The example is given as follow:

Eastsoft is one of the top five software companies in China. To improve the operation and competitiveness capability in the global market, Eastsoft plans to establish a strategic alliance with a transnational corporation. After lots of consultations, four transnational corporations HP (x_1), PHILIPS (x_2), EMC (x_3), and SAP (x_4) are the alternatives, which would like to establish a strategic alliance with Eastsoft. To select the desirable strategic alliance partner, they invited five experts E_1, E_2, E_1, E_3, E_4 and E_5 to participate in the decision analysis. The five experts respectively come from the operation management department, the engineering management department, the finance department, the human resources department, and the business process outsourcing department of Eastsoft. The evaluations information given by the five experts is in the form of ordinal interval number, which is shown in Table 1.

The weight vector of experts provided by the decision maker is $w = (0.2, 0.2, 0.2, 0.2, 0.2)^T$.

	E_1	E_2	E_3	E_4	E_5
x_1	[2,3]	[3,4]	[4,4]	[1,1]	[2,3]
x_2	[1,1]	[1,2]	[2,3]	[2,3]	[1,3]
x_3	[2,4]	[1,2]	[2,3]	[4,4]	[1,2]
x_4	[3,4]	[3,4]	[1,2]	[2,3]	[4,4]

Table 1. Preference information provided by experts

In order to solve the problem of group decision making, we use the proposed VIKOR method to solve, and the step is given as follows.

Step 1. Determine the positive ideal solution (PIS) r^* and negative ideal solution (NIS) r^- as follows:

$r^* = \{r_1^*, r_2^*, r_3^*, r_4^*, r_5^*\}$ and $r^- = \{r_1^-, r_2^-, r_3^-, r_4^-, r_5^-\}$, where $r_j^* = 1, r_j^- = m, j = 1,2,3,4,5$.

Step2. According to the Eq. (6) and Eq. (7), the group utility value $\tilde{S}_i = [S_i^L, S_i^U]$ and individual regret value $\tilde{R}_i = [R_i^L, R_i^U]$ are given as follows:

$\tilde{S}_1 = [0.4667, 0.6667], \tilde{S}_2 = [0.1333, 0.4667], \tilde{S}_3 = [0.3333, 0.6667], \tilde{S}_4 = [0.5333, 0.8000]$ and

$\tilde{R}_1 = [0.2000, 0.2000], \tilde{R}_2 = [0.0667, 0.1333], \tilde{R}_3 = [0.2000, 0.2000], \tilde{R}_4 = [0.2000, 0.2000]$

Step3. Calculate the comprehensive sorting index $\tilde{Q}_i = [Q_i^L, Q_i^U]$:

$\tilde{Q}_1 = [0.75, 0.90], \tilde{Q}_2 = [0, 0.5], \tilde{Q}_3 = [0.65, 0.90], \tilde{Q}_4 = [0.80, 1.0]$

Step 4. According to the Eq.(3), the possible degree matrix $P = (p_{ij})_{m \times m}$ is

$$P = \begin{bmatrix} 0.50 & 1.0 & 0.625 & 0.2857 \\ 0 & 0.50 & 0 & 0 \\ 0.375 & 1.0 & 0.5 & 0.222 \\ 0.7143 & 1.0 & 0.7778 & 0.50 \end{bmatrix}$$

Step5. Rank the alternatives.

According to the derived ranking values and matrix P, the ranking order of the alternatives with possibility degrees is $u_1 = 0.2842, u_2 = 0.1250, u_3 = 0.2851, u_4 = 0.3327$

So, the ranking order is $x_4 \succ x_1 \succ x_3 \succ x_2$ .The ranking result above is the same as that derived by Fan and Liu [15] and Xu [16].

5. Conclusion

For the group decision making problems with ordinal preference information, a new decision method is put forward based on VIKOR method. The proposed method is easy to calculation, and has more advantage than the integer programming method. Finally, a case study is use to demonstrate and validate the application of the proposed method. The proposed method can also be extended to other multi-attribute group decision making problems in which attribute values are expressed with interval number, triangular fuzzy number and intuitionistic fuzzy number.

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