Trapezoidal Fuzzy Number Attitude Indicators Group Decision Making Approaches Based on Fuzzy Language

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ABSTRACT: *The paper proposed a trapezoidal fuzzy numbers group decision making method based on attitude indicators, in order to solve the multi- attribute group decision making problem for the evaluation of the information given in the form of fuzzy language. Given the weights determining method of trapezoidal fuzzy number complementary judgment matrix under the case of the weight of each attribute weight and decision-makers weight information are not entirely, and solved each attribute weight information according to this method; then introduced attitude indicators and put fuzzy language trapezoidal fuzzy number decision matrix into a decision matrix with attitude indicators. Use the incomplete decision making information to build target programming model to get the decision-makers weight to meet the objective function. Eventually, get the groups risk attitude and the programs comprehensive sorting situation by integrate the attitude indicators of decision-makers. And also, the paper verified the feasibility and effectiveness of the proposed method by a numerical example.*

Keywords: Fuzzy Language, Attitude Indicator, Trapezoidal Fuzzy Number, Group Decision Making, Decision Matrix, Project Evaluation

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1. Introduction

In the multi-attribute decision-making process, as the human thought affected by its inherent ambiguity and complexity and uncertainty of objective things, the decision-makers were always difficult to give exact numbers for the things need to be evaluated in the actual judging process, but often evaluate it in forms of uncertain information as language variables or language interval value etc. Therefore, the multi-attribute decision making problems based on linguistic information or uncertain linguistic information has become a hot topic in the field of multi-attribute decision making, and attracted much attention in experts and scholars at home and abroad. Such as Han Zuo-sheng et al. (2011) proposed the distance formula between variable value of trapezoidal fuzzy language, and established extended VIKOR method based on trapezoid fuzzy language variable. Liang X C et al. (2008) proposed trapezoid fuzzy language weighted average operator for the trapezoid fuzzy language variables, and give the program sort method for trapezoidal fuzzy language variables multi-attribute decision based on possibility degree formula. Gong Zai-wu (2007) gave the transformation relationship of trapezoidal fuzzy numbers and tuple linguistic, and proposed tuple linguistic sorting method. Herrera F et al. (2000) transformed the language information into tuple constitute in language terminology and value in [-0.5, 0.5], and also gave some language integrated operators based on tuple. Li D F (2010) utilized the decision-making forms of use fuzzy language values indicate the program attribute values and use real numbers indicate the weights to deal with multi-attribute decision making problems. At present, the above method provided a good research idea for solving trapezoidal fuzzy numbers multi-attribute decision making problems, but these existing research on the multiple attribute group decision making problems that the attribute weights and decision-makers weight information is not complete and the property value in the form of trapezoidal fuzzy numbers and also consider group decision-makers mentality indicators.

Based on the above analysis, transforms the vague language into trapezoidal fuzzy number, taking group decision-making into account with the introduction of attitude indicators, proposed a trapezoidal fuzzy number group decision making method based on attitude indicators, and verified the feasibility and effectiveness of the proposed method by a numerical example.

2. Basic Theories

2.1 Related Definitions of Trapezoidal Fuzzy Numbers

Definition 1 Assume R is the real number set,

if $\widetilde{A}=(a,b,c,d)$, $-\infty < a \le b \le c \le d < +\infty$, call: $\widetilde{A}$ fuzzy numbers.When $a > 0$, call : $\widetilde{A}$ positive trapezoidal fuzzy number; when $b = c$ the trapezoidal fuzzy number degenerate into triangular fuzzy numbers. Its membership function is $\mu_{\widetilde{A}}(x): R \to [0,1]$ meet (Wang Jian-qiang et al., 2012):

$$\mu_{\widetilde{A}}(x)=\begin{cases} \dfrac{x-a}{b-a} & a \le x < b \\ 1 & b \le x \le c \\ \dfrac{x-d}{c-d} & c < x \le d \\ 0 & other \end{cases} \tag{1}$$

Definition 2 Assume $\widetilde{A}=(a,b,c,d)$ is trapezoidal fuzzy number, call:

$$I^L(\widetilde{A})=\frac{1}{2}(a+b),\quad I^R(\widetilde{A})=\frac{1}{2}(c+d) \tag{2}$$

are the left and right expectation values of trapezoidal fuzzy numbers.

Definition 3 For the trapezoidal fuzzy numbers $\widetilde{A}$, take any $\alpha \in [0,1]$, expectation value of $\widetilde{A}$ is: $I(\widetilde{A})=\alpha I^L(\widetilde{A})+(1-\alpha)I^R(\widetilde{A})$ it call: α is the attitude indicator of $\widetilde{A}$. When $0 \le \alpha < 0.5$, the decision-makers hold cautious or pessimism attitude; when, $\alpha = 0.5$, $I(\widetilde{A})=\frac{1}{4}(a+b+c+d)$ the decision-makers hold golden mean or mild attitude; when $0.5 < \alpha \le 1$, the decision-makers hold aggressive or optimism attitude.

2.2 Algorithm of Trapezoidal Fuzzy Numbers

Assume $\widetilde{A}_1=(a_1,b_1,c_1,d_1)$, $\widetilde{A}_2=(a_2,b_2,c_2,d_2)$ are two positive trapezoidal fuzzy numbers, its algorithm is as follows:

① $\widetilde{A}_1+\widetilde{A}_2=(a_1+a_2,b_1+b_2,c_1+c_2,d_1+d_2)$

② $\widetilde{A}_1-\widetilde{A}_2=(a_1-d_2,b_1-c_2,c_1-b_2,d_1-a_2)$

③ $\widetilde{A}_1\times\widetilde{A}_2=(a_1a_2,b_1b_2,c_1c_2,d_1d_2)$

④If $\lambda \ge 0$, $\lambda\widetilde{A}_1=(\lambda a_1,\lambda b_1,\lambda c_1,\lambda d_1)$

2.3 Transformation of Fuzzy Language and Trapezoidal Fuzzy Numbers

In real life, decision-makers often use fuzzy language to express the evaluation information based on their own knowledge and experience, and in the process of academic research need to transform this qualitative data into quantitative data, it requires the quantitative data which can accurately substitute the fuzzy language values of the experts (Chen Xiao-hong et al., 2012). By comparing the advantages and disadvantages of different quantitative data in terms of represent the experts language values, select trapezoidal fuzzy number to represent the expert preference information, and the language evaluation of property values can represent by trapezoidal fuzzy number, the specific as shown in Table 1.

Language Evaluation Items	Trapezoidal Fuzzy Number
Very High	(0.8,0.9,0.9,1)
High	(0.6,0.7,0.7,0.8)
Medium	(0.4,0.5,0.5,0.6)
Low	(0.2,0.3,0.3,0.4)
Very Low	(0,0.1,0.1,0.2)

Table 1. The transformation relationship between language evaluation level and trapezoidal fuzzy numbers

3. Decision Making Method

3.1 Decision Making Problem Description

For a multi-attribute group decision making problem, assume it has p decision-makers $E = \{e_1, e_2, \cdots, e_P\}$ participate in decision-making, and its weight vector is $\lambda = (\lambda_1, \lambda_2, \cdots, \lambda_P)$. The alternatives set $X = \{x_1, x_2, \cdots, x_m\}$ ($m \geq 2$), which x_i represents the i-th decision alternatives. The attribute sets $C = \{C_1, C_2, \cdots, C_n\}$ ($n \geq 2$), the attribute weights vector $W = (w_1, w_2, \cdots, w_n)$, and meet $w_j \geq 0$ and $\sum_{j=1}^{n} w_j = 1$. To ensure the scientific of evaluation, invite p decision-makers participate in the evaluation, the decision-makers e_k measure the program x_i based on attribute c_j, to get the language information evaluation matrix of x_i about attribute c_j, use Table 1 to transform it into trapezoidal fuzzy number decision making matrix $X^{(k)} = (x_{ij}^{(k)})_{m \times n}$, in which the attribute value of $X^{(k)}$ is trapezoid fuzzy number $x_{ij}^{(k)} = [a_{ij}^{(k)}, b_{ij}^{(k)}, c_{ij}^{(k)}, d_{ij}^{(k)}]$, and $i = 1,2, ..., m; j = 1,2, ..., n; k = 1,2, ..., p$.

3.2 The Algorithm Steps

Step 1 Given the trapezoidal fuzzy numbers evaluation information of all the decision-makers about the programs in each attribute to construct their group decision making matrix $\widetilde{A}^{(k)} = (\widetilde{a}_{ij}^{(k)})_{m \times n}$.

Step 2 Standardized the trapezoidal fuzzy number original decision making matrix. In order to eliminate the effects generated by the decision-making results with different dimensions between the attribute, use the formulas of standardized fuzzy decision making matrix, standardized the trapezoidal fuzzy number decision making matrix $\widetilde{A}$. Process the efficiency attributes and cost attributes according to formula (3), (4) respectively, to obtain the standardization fuzzy decision matrix $\widetilde{R}^{(k)} = (\widetilde{r}_{ij}^{(k)})_{m \times n}$.

$$r_{ij} = (\frac{a_{ij}}{u_{ij}}, \frac{b_{ij}}{u_{ij}}, \frac{c_{ij}}{u_{ij}}, \frac{d_{ij}}{u_{ij}}), \quad j \in L_1 \tag{3}$$

$$r_{ij} = (\frac{v_{ij}}{d_{ij}}, \frac{v_{ij}}{c_{ij}}, \frac{v_{ij}}{b_{ij}}, \frac{v_{ij}}{a_{ij}}), \quad j \in L_2 \tag{4}$$

Where, $u_{ij} = \max\{d_{ij}\}$, $j \in L_1$, L_1 represents efficiency type; $v_{ij} = \min\{a_{ij}\}$, $j \in L_2$, L_2 represents cost type.

Step 3 Determine the weights of each trapezoid fuzzy number decision making matrix $\widetilde{R}^{(k)}$.

1) Calculate the attribute value of trapezoidal fuzzy number decision making matrix to get their weight vector $\widetilde{V} = (\widetilde{v}_1, \widetilde{v}_2, \cdots, \widetilde{v}_n)$.

$$\widetilde{v}_i = \left[\sum_{j=1}^{n} a_{ij}, \sum_{j=1}^{n} b_{ij}, \sum_{j=1}^{n} c_{ij}, \sum_{j=1}^{n} d_{ij}\right] \otimes \left[\sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij}, \sum_{i=1}^{n}\sum_{j=1}^{n} b_{ij}, \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}, \sum_{i=1}^{n}\sum_{j=1}^{n} d_{ij}\right]^{-1} \approx \left[\frac{\sum_{j=1}^{n} a_{ij}}{\sum_{i=1}^{n}\sum_{j=1}^{n} d_{ij}}, \frac{\sum_{j=1}^{n} b_{ij}}{\sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}}, \frac{\sum_{j=1}^{n} c_{ij}}{\sum_{i=1}^{n}\sum_{j=1}^{n} b_{ij}}, \frac{\sum_{j=1}^{n} d_{ij}}{\sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij}}\right] \tag{5}$$

2) As $\widetilde{v}_i$ $(i \in N)$ is fuzzy number, calculate the expected value of $\widetilde{v}_i$ (Wu Yeke et al., 2011).

$$I(\widetilde{v}_i) = \alpha I^L(\widetilde{v}_i) + (1-\alpha) I^R(\widetilde{v}_i) \tag{6}$$

Where, α is the attitude indicator of decision-makers, $0 \le \alpha \le 1$, the paper takes $\alpha = 0.5$. Following this, the formula (6) can be converted to:

$$I(\widetilde{v}_i) = \frac{1}{4}(a_i + b_i + c_i + d_i) \tag{7}$$

3) Calculate the weights of each evaluation index (attributes).

$$w_i = \frac{I(\widetilde{v}_i)}{\sum_{i=1}^{n} I(\widetilde{v}_i)}, \quad i \in N \tag{8}$$

Where, w_i is the weight of attributes C_i.

Step 4 Integrate groups program and determine the expert weight.

For the decision making program x_i, the definition:

(9)

It can be seen $Z(x_i)$ is trapezoidal fuzzy number, the weight information of the decision making is $\lambda = \{\lambda_1, \lambda_2, \cdots, \lambda_p\}$, and establish the optimization model as follows:

$$\max Z(x_i) = \sum_{i=1}^{p} \lambda_i \sum_{i=1}^{m} \sum_{j=1}^{n} f_{ij}^{(k)}(a_k) w_j^{(k)}$$
$$\text{s.t.} \begin{cases} \sum_{i=1}^{p} \lambda_i = 1 \\ \lambda_i \ge 0 \end{cases} \tag{10}$$

This model can calculate the optimal solution through Lingo and Visual C++ mixed programming, which is obtain the weight information λ to satisfy the objective function $\max Z(x_i)$.

Step 5: The integration of group risk attitude and the program sort.

To the decision-makers that participate in the evaluation, they have their respective attitude indicator α_k, and the corresponding weight λ_k that determine the relative importance of the decision-makers, pursuant this to calculate the groups risk attitude value $\eta = \sum_{k=1}^{p} \lambda_k \alpha_k$. Finally, integrate $Z(x_i)$ and group risk attitude η, the calculation formula is:

$$F_\eta(Z(x_i)) = I(Z(x_i)) + (2\eta - 1)D(Z(x_i)) \quad (11)$$

Sort the alternative programs base on the $F_\eta(Z(x_i))$ value, and obtain the comprehensive sort of the programs.

4. The Numerical Example

A technology company want to enhance its competitiveness in the market, it intend to choose a best company that reach the cooperation union to achieve its goals from the three peers companies {A, B, C}. Screening program selected three indicators attributes: production capacity C_1, research and development capabilities C_2 and liquidity ability C_3, and all attributes are efficiency attributes. The company invited two experts to evaluate three alternative programs, its fuzzy language evaluation information as shown in Table 2 and Table 3 (Gao and Liu, 2012), and choose the best enterprise based on those information.

	C_1	C_2	C_3
A	*High*	*High*	*Medium*
B	*High*	*High*	*High*
C	*High*	Very High	*High*

Table 2. The fuzzy language evaluation information of expert1

	C_1	C_2	C_3
A	*High*	*Medium*	*Low*
B	Very High	*High*	*High*
C	*High*	Very High	*High*

Table 3. The fuzzy language evaluation information of expert2

(1) Transform the fuzzy language evaluation information matrix into trapezoidal fuzzy number decision making matrix.

Transform the fuzzy language evaluation information of expert1 and expert2 into trapezoidal fuzzy number refer to Table 1, and follow the group decision **Step 2** to standardized the trapezoidal fuzzy number decision making matrix of two experts according to the formulas (3) and (4), and get the normalized fuzzy decision making matrix as shown in Table 4 and Table 5.

	C_1	C_2	C_3
A	(0.75,0.875,0.875,1.0)	(0.6,0.7,0.7,0.8)	(0.50,0.625,0.625,0.75)
B	(0.75,0.875,0.875,1.0)	(0.6,0.7,0.7,0.8)	(0.75,0.875,0.875,1.00)
C	(0.75,0.875,0.875,1.0)	(0.8,0.9,0.9,1.0)	(0.75,0.875,0.875,1.00)

Table 4. The standardized fuzzy decision making matrix of expert1

	C_1	C_2	C_3
A	(0.6,0.7,0.7,0.8)	(0.4,0.5,0.5,0.6)	(0.25,0.375,0.375,0.5)
B	(0.8,0.9,0.9,1.0)	(0.6,0.7,0.7,0.8)	(0.75,0.875,0.875,1.0)
C	(0.6,0.7,0.7,0.8)	(0.8,0.9,0.9,1.0)	(0.75,0.875,0.875,1.0)

Table 5. The standardized fuzzy decision making matrix of expert2

Where, the attitude indicators of decision-makers are $\alpha_1 = 0.4$, $\alpha_2 = 0.7$, the weight information of decision-makers are $\lambda_1 \le \lambda_2, 0.3 \le \lambda_2 \le 0.58, \sum_{i=1}^{2} \lambda_i = 1$

(2) In accordance with **Step 3**, calculated each property values for the normalized fuzzy decision making matrix of erpert1 based on formula (5), and get their weight vector matrix R_1:

$$R_1 = \begin{bmatrix} 0.222 & 0.301 & 0.301 & 0.408 \\ 0.251 & 0.336 & 0.336 & 0.448 \\ 0.275 & 0.363 & 0.363 & 0.480 \end{bmatrix}$$

According to formulas (7) - (8), calculated each attribute weight vectors in decision making matrix of expert1: $w^{(1)}$ = (0.302,0.336,0.363).

Integrated the programs of expert1 as follow:

A: (0.610, 0.726, 0.726, 0.843)

B: (0.700, 0.817, 0.817, 0.934)

C: (0.768, 0.884, 0.884, 1.001)

Similarly, get the weight vector matrix R_2 of expert2, and obtained each attribute weight vectors of expert2: $w^{(2)}$ = (0.242, 0.379, 0.379).

Integrated the programs of expert2 as follow:

A: (0.392, 0.501, 0.501, 0.611)

B : (0.705, 0.815, 0.815, 0.924)

C : (0.733, 0.842, 0.842, 0.952)

(3) In accordance with **Step 4**, established optimization model according to formula (10). Solved the model by Lingo and Visual C++ mixed programming, get the objective function value met the model was *max* (Z (x_i) = 2.3081, meanwhile, obtained the weight vector of the experts was λ = 0.5, 0.5).

Integrated the group programs as follow:

A : (0.484,0.595,0.595,0.708)

B : (0.703,0.816,0.816,0.928)

C : (0.748,0.860,0.860,0.973)

(4) In accordance with **Step 5,** obtained the group risk attitude value $\eta = \sum_{k=1}^{2} \lambda_k \alpha_k = 0.55$, and calculated the group risk attitude and group programs integrated as *A* = 0.601, *B* = 0.821, *C* = 0.866 according to formula (11), thereby obtained the comprehensive order were: $C \succ B \succ A$ the optimal cooperation program was *C*. The results were fully consistent with literature [9], they can effectively illustrate the feasibility and effectiveness of this method.

5. Conclusions

The paper introduce the trapezoidal fuzzy numbers to determine each attribute weight for the problem on traditional AHP determining the factor weight, and enumerated the weight determination method about trapezoidal fuzzy number complementary judgment matrix. Introduced the attitude indicators to transform the trapezoid fuzzy number decision matrix of fuzzy language into the decision matrix with attitude indicators, in cases of the attribute weights and the decision-makers weight information were not complete. In addition, the use of group decision making method not only absorbed the expert opinions of individuals, but also played a role in balancing the views of the experts. Built objective programming model for different target programming model makers for different decision-makers of each program on the use of incomplete information of decision-makers, obtained the comprehensive order of the programs set by integrated the groups risk attitudes and groups evaluation values of the programs. This paper proposed a decision making method with clear concept, simple calculation process and easy to understand. It had good application and actual decision making value, and provided a new decision making methods and ideas to solve fuzzy multi-attribute group decision making problems consider the psychological behavior of decision-makers.

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