Direction of Arrival Estimation using MUSIC and ESPRIT Algorithms for Wireless Sensor Network

Ousmane Abdoulaye Oumar, Ming Fei Siyau, Tariq P. Sattar
Faculty of Engineering, Science and the Built Environment
London South Bank University
London, UK
{oumaro, siyaum, sattartp}@lsbu.ac.uk

ABSTRACT: *The accurate estimation of direction of interest which is also known as Direction of Arrival (DOA) of the incident signals is very significant to produce beam form antenna. There are several algorithms that have the ability to calculate the DOA of the incident signals. In this paper, we investigate and compare Multiple Signal Classification (MUSIC) and Estimation of Signal Parameters via Rotational Invariance Technique (ESPRIT) algorithms in terms of the Angle of Arrival (AOA), Time of Arrival (TOA), Time Difference of Arrival (TDOA) and Received Signal Strength (RSS). MUSIC and ESPRIT algorithms are attractive solution to many parameter estimation problems. The performance of DOA using MUSIC and ESPRIT algorithm are analyzed on many parameters such number of users, number of snapshots, and number of array elements, user space distribution and SNR. MATLAB simulations indicate that both algorithms provide high resolution but the MUSIC algorithm is more accurate and stable compared to the ESPRIT algorithm.*

Keywords: MUSIC, ESPRIT, DOA, Localization, WSN

Received: 24 June 2012, Revised 29 August 2012, Accepted 6 September 2012

© 2012 DLINE. All rights reserved

1. Introduction

Wireless sensors Network (WSN) are novel type of wireless network where hundreds or thousands of energy-constrained devices cooperate to monitor, localize and or accurate in a given area. Those networks allow real time monitoring of regions where humans may be restricted [13].

The strong demand on the usage of WSN has attracted larger number of developments. Nowdays the technology improvements have made the deployment of small, inexpensive, low-power, distributed devices, which are capable of local processing and wireless communication, a reality. Unlike traditional networks, sensor networks depend on dense deployment and co-ordination to carry out their tasks [1, 8].

Localization in WSN [9] has been developed in the last three decades, mainly being started as a military project and today have attracted significant research interest during the last few years. The Global Positioning System (GPS) (the result of heavy investments made by the U.S. Department of Defense in the 1970s) is an immediate solution nevertheless, there are some strong factors against the usage of GPS today [1, 7], such as indoor blind.

The most important method is flexible enough to perform localization using the most common radio measurements and various techniques for calculating the DOA, for example time of arrival (TOA), time difference of arrival (TDOA), angle of arrival (AOA), and received signal strength (RSS).

This paper studies the methods of MUSIC (Multiple Classification Signal) and ESPRIT (Estimation of Signal Parameters via Rotational Invariance Techniques).

MUSIC algorithm can estimate the signal's DOA accurately for any array geometry, but if the array's steering vector is incomplete e.g., when a circular array is mounted on a metallic cylindrical surface, for a signal with special DOA, some antenna elements cannot receive this signal [4]. Radiolocation can be implemented that are based on either signal strength or angle of arrival AOA measurements [5]. MUSIC based and ESPRIT based algorithms, when used on uniform liner array with Omni-directional antenna elements, always have high performance [2, 3]. The accurate estimation of direction which is also known as DOA of the incident signals is very significant to produce beam form antenna. The DOA estimation techniques with array antennas are applied in wide areas of research fields and have received considerable attention in literature [11, 12].

The rest of this paper is organized as follows. In Section II describes the DOA estimation methods. In Section III explains MUSIC and ESPRIT, in section IV explains in detail with numerical simulation examples and compared MUSIC and ESPRIT algorithms. Finally, section V gives conclusions.

2. DOA Estimation Methods

The estimation methods in WSN can be classified into: Received Signal Strength (RSS), Time based methods (Time of Arrival (TOA), Time Difference of Arrival (TDOA), and Angle of Arrival (AOA), etc...

2.1 Received signal strength (RSS)

Positioning techniques based on the Received Signal Strength (RSS) [16, 17, 18] have been extensively studied. The distance between the transmitter and receiver can be estimated from the received signal strength associated with a propagation model in the environment. Three receivers are needed to define the position in two dimensional (a GPS receiver needs at least three satellites to plot a rough, two dimensional positions). By collecting multiple received signal strength measurements at different positions, the location of the target can be determined by the intersection of at least three areas possible for the position of the moving object is deduced by trilateration, a process described in a later section.

This technique is easily suitable in the case of WLAN and cellular networks; the signal strength is available at the receivers and transmitter. The validity of the propagation model corresponding to the work environment plays an important part in the accuracy of the location.

Depending on the configuration of the environment, multipath leading to slight difference in condition in signal level that can reach 15-25 dB beyond a distance of about a fraction of a wavelength. These random variations generate very large errors in estimating the distance. One possibility to improve results is to average the measurements over time or frequency and the measurement of power can also be combined with mapping techniques.

2.2 Time of arrival (TOA)

The most important parameter for accurate localization systems in the current is the TOA. The signal's travel time between the target and observer can be expressed in a completely synchronized network as an outcome the distance can be calculated from the time of arrival as signals travel with a known velocity which in this case is the speed of light.

The distance between the target and observer, calculated from the measured propagation time, supply a circle centered at the spotter position on which the target must lie. Placing the observers at three different locations, the target's position is given by the intersection of its corresponding circles [6]. This requires that the observers know the exact time at which the target will transmit, and that the observers have a very stable and accurate clock.

However, the measurement of TDOA can be easily obtained after differentiation from TOA Measurement made at each receiver. The latter method, fairly intuitive, although it also suffers the problems of multipath and non-visibility, overcomes most of the ambiguity mentioned above, the objective being the systematic identification of the first trip observed on each of the observed responses.

2.3 Time difference of arrival (TDOA)

The measurement of TDOA is to estimate the path difference of a signal observed at multiple receivers occurring during same time, consideration of the instant of transmission of this signal.

Each TDOA measurement determines that the transmitter must lie on a hyperboloid. At least three receivers are needed for a 2D location estimate (target and receivers are co-planar) and four receivers for a full 3D location estimation [15, 23, 24].

This can be achieved by finding the value of time shift that maximizes the cross correlation function between the signals received at a pair of references strictly synchronized. This generalized correlation method requires that the analog signal is received at first digitized and then transmitted to a central processing location.

The temporal resolution of a conventional cross-correlation can be approximated by the problem of resolving closely spaced sinusoids [25].

One way to access the TDOA is to measure the delay of signals arriving at each receiver and perform their difference. As commonly in the case of systems using this technique, the receivers are not synchronized with the transmitter but only them, the TDOA measured and will include an offset which will be similar, however, because of the timing between the receivers. Other technique to evaluate the TDOA is the correlation between signals received at different receivers.

Nevertheless, this measure TDOA can be easily established after differentiation from pre TOA measures made at each receiver. The ultimate method, fairly intuitional, although it also affects the problems of multipath and non visibility, overcomes most of the ambiguity mentioned above, the objective being the systematic identification of the first trip observed on each of the observed responses. The measure of TOA is the most adaptable and the most appropriate solutions for the measurement of relative distances and TDOA measurements can be easily obtained from TOA measurements.

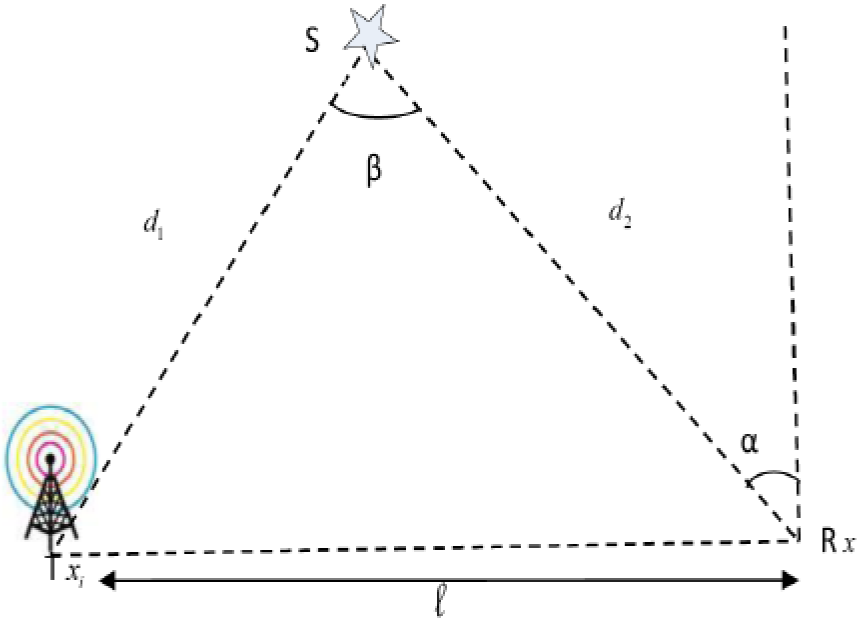

Figure 1. Illustrates TDOA system

Where T_{xi} is the transmitter, S is the target, R_x is the receiver, d_1 is the distance between the target and transmitter, d_2 is the distance between the target and receiver and l is the distance between the transmitter and receiver.

3.4 Angle of arrival (AOA)

This technique is based on the use of angles of signals from the moving object at least two points of reception.

One common approach to obtain AOA measurements is to use an antenna array on each sensor node. Other techniques to detect the angles between nodes are discussed in [26, 27]. The location of the unknown is located by the intersection of lines passing over each receiver and the angle AOA estimated relative to an arbitrary reference. Frequently a small of error of DOA estimation is introduced, conducting to the intersection of the beams, which defines an area as potential to the position of the issuer. The dimension of each beam augments with distance from the transmitter leading to major errors. In the case where the transmitter is on the line joining the two receivers, the location estimate is not any more possible. The presence of an extra receiver is needed and the major drawback of this technique is related to the need for antenna arrays that increase the size of the equipment used and involves additional costs.

This method also requires a relatively complex AOA algorithm. Although there are exceptions, these algorithms tend to be highly complex because of the need for measurement, storage and usage of array calibration data and their computationally intensive nature [14, 15].

3. The MUSIC and ESPRIT Algorithms

The performance is examined and studied carefully on many parameters such number of array elements, number of users, user space distribution, signal to noise ratio and number of snapshots. The methods are based on simultaneous search parameters of interest and give as an estimate of point values. Complexity is higher but the results are more accurate.

3.1 Music algorithm

The MUSIC (Multiple Signal Classification) [10] method is one of the most used today in various treatment such as array processing, it uses the signals received by the M antenna sensors; these are the components signal X (t) from sources that are supposed to point and spatially coherent. This property implies that the various components of each vector sm (t), emitted by the N sources are the same processes by linear filtering.

MUSIC method is an estimation algorithm based on eigenvector decomposition, which is also called based on spatial structure method [22]. MUSIC method is derived from the decomposition of specific elements of the covariance matrix is defined, nonnegative and hermitian. The following equation shows this decomposition.

$$R = \sum_{m=1}^{M} \lambda_1 v_s V_m^H = V \Lambda V^H \tag{1}$$

Where the values of λ_m are real and positive, arranged in descending order, and the associated eigenvectors are orthonormal V_m. The absence of noise, the covariance matrix R is written:

$$R = ASA^H \tag{2}$$

Where the matrices A and S being the respective dimension (M, N) and (N, N), with N < M, the matrix R of dimension (M, M) has rank at most equal to N. In the preceding theorem we assume that A and S are full rank; hence R is of rank N. Moreover, R is a non-negative hermitian matrix defined, its eigenvalues are real and positive or zero. The assumption N < M implies that R M - N zero eigenvalues and N eigenvalues strictly positive.

Notes the subspace source subspace $\{V_s\}$ spanned by the eigenvectors associated with the N largest eigenvalues of R and the noise subspace $\{V_b\}$ subspace spanned by the eigenvectors associated with smallest eigenvalue σ^2. It thus comes as not fully correlated sources, it is the condition that S is full rank, the source vectors $a(\theta_n)$ are orthogonal to noise subspace, ie:

$$V_b^H a(\theta_n) = 0, n = 1, \ldots N \tag{3}$$

Where the vectors corresponding to the DOA sources sought are the vectors of the variety that are orthogonal toV_b.

To solving the nonlinear method gives uniquely the DOA Sought, that is, there exist number N of vectors of the form as an independent in N-dimensional space of M.

These measurements form the basis of the MUSIC system. The outline of the algorithm is as follows: Firstly, determining the minimum eigenvalue and the number of eigenvalues equal to this value and then, as the noise subspace eigenvectors associated with the minimum eigenvalue then form the estimator:

$$P_{MUSIC}(\theta) = \frac{1}{a^H(\theta)\, V_b V_b^H a(\theta)} \tag{4}$$

This estimator takes the maximum value theoretically infinite as the denominator measures the orthogonality between the noise subspace and vector candidates of the variety of the antenna to the true values of the DOA.

The MUSIC method is known for its higher performance and owes its reputation in part due to its use very general. Without restriction, it can be used for networks with arbitrary geometry known to calculate parameters from multiple sources both in terms of delay, AOA, etc. In theory, if all conditions are met than MUSIC allows parameter calculation asymptotically unbiased, the estimation error tends to zero if the number of observations tends to infinity.

3.2 Esprit algorithm

Estimation of ESPRIT approximates signal arrival directions by exploiting the rotational invariance of the signal subspaces of subsets of the array receivers [21].

The ESPRIT (Estimation of Signal Parameters via Rotational Invariance Techniques) [9] method, introduced for the first time, is fast, efficient and robust parameter estimation can be used in determining the directions of incidence of multiple sources level of an antenna array. With the simplicity and performance, ESPRIT has become a very well known method. The algorithm applies the same model of the signal that the MUSIC algorithm, but it has the advantage of significantly reducing the computing power and memory needed for storage. This is because we impose at the antenna array structure of translational invariance (ie sensors arranged in pairs with the same separation), which can be obtained easily in practice. ESPRIT estimates the DOA and the time delay of'd' incoming signals by using their eigenvalues [9].

ESPRIT eliminates the search procedure & produces the DOA estimation directly in terms of the eigenvalues without much computational and storage requirements [19]. The ESPRIT method estimates signal DOA by finding the roots of two independent equations closest to the unit circle. This method does not require using a scan vector to scan over all possible directions like the MUSIC (Multiple Signal Classification) algorithm [20]. Consider a network of 2M sensors fewer than two antennas deducted from each other by a translation of vector $\tilde{\Delta}$ known, translating the first sensor in the first antenna to the first sensor in the second antenna, this vector as module $|\Delta| = \Delta$. X_1 and X_2 are vectors of observation output of these two sub branches and let the vector X observing complete output of the antenna, and consequentially:

$$X = [\mathrm{X}_1 \ \ \mathrm{X}_2]^T \tag{5}$$

The spectral matrices N x N and M x M sources of noise could be written as:

$$\Lambda = diag\left\{ e^{2\pi V_0 \Delta \frac{\sin\theta_1}{c}} + e^{2\pi V_0 \Delta \frac{\sin\theta_2}{c}} ,\ldots, e^{2\pi V_0 \Delta \frac{\sin\theta_n}{c}} \right\} \tag{6}$$

$\lambda_1 \geq \lambda_2 \geq ,\ldots, \lambda_N \geq \lambda_{N+1} = \ldots = \geq \lambda_{2M} = \sigma_b^2$

The N largest eigenvalues correspond to the signal space spanned by the N sources.

Once the eigenvalues of $\lambda_1, \lambda_2 \ldots \lambda_D$ are calculated, we can estimate AOA as:

$$\theta_i = \arcsin\left[-\frac{\lambda}{2\pi\Delta}\right] \tag{7}$$

4. Simulation Results

In this section we investigate the performances of the MUSIC and ESPRIT algorithm using different parameters. A uniform linear array with M elements has been considered throughout our simulation experiments. It is assumed that the spacing between elements is 0.5λ which is required by these DOA algorithms to be effective. In case the distance is greater than the information

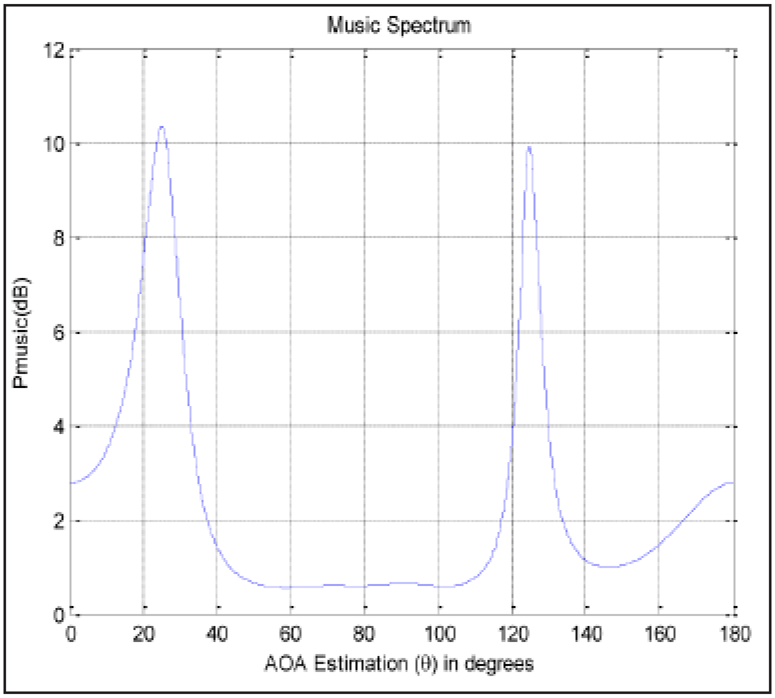

Figure 2. MUSIC spectrum for 64 snapshots, SNR = 5dB and 5 array elements

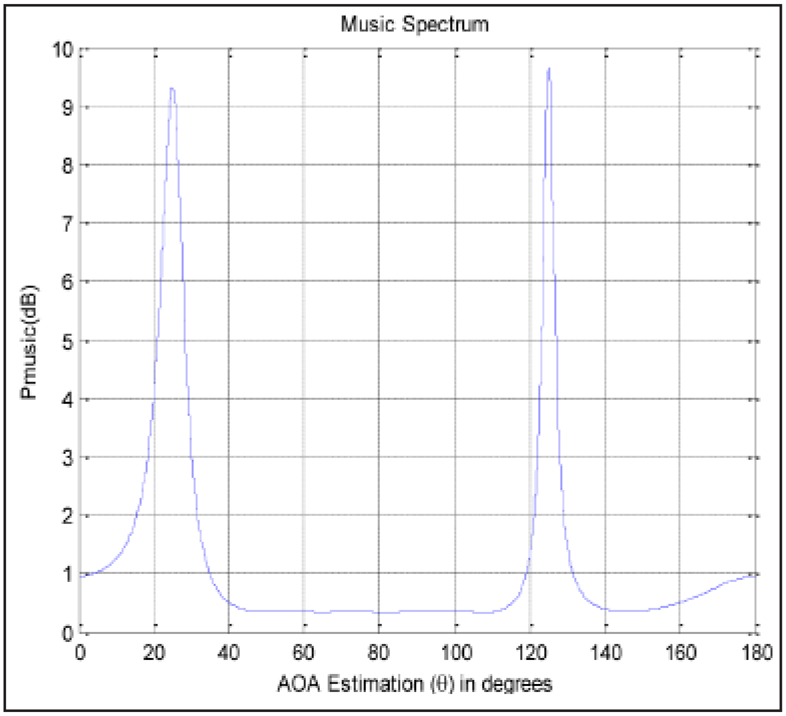

Figure 3. MUSIC spectrum for 64 snapshots, SNR = 5dB and 8 array elements

contained in the phase shifts cannot deduct the angles accurately. The noise is considered as Gaussian white noise.

The simulation results of the MUSIC algorithm on two signals coming from two different AOA angles (25°, 125°) with 5 array

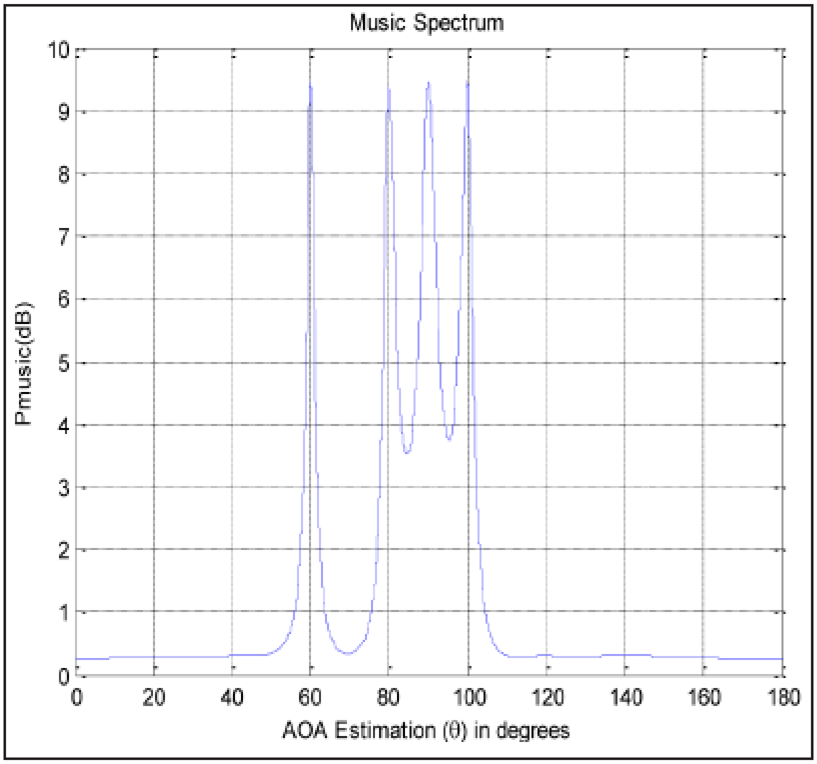

Figure 4. MUSIC spectrum for 64 snapshots, SNR = 20dB & 10 array elements

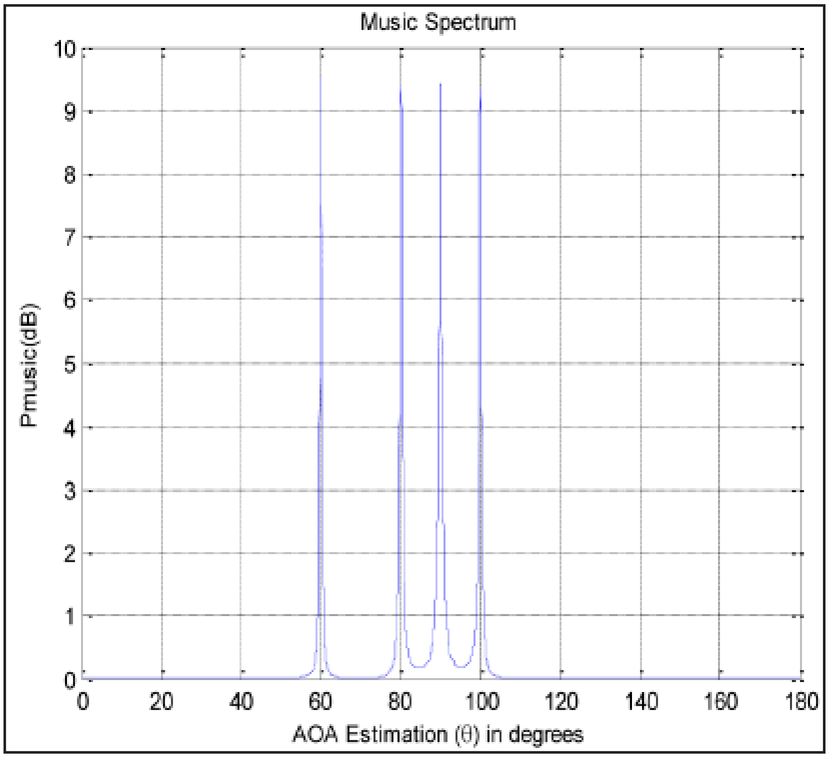

Figure 5. MUSIC spectrum for 100 snapshots, SNR = 20dB, 10 array elements

elements (Figure 2) or 8 array elements (Figure 3) are given respectively. The results indicate clearly that if array size increases, the peaks for the spectrum become sharp and the resolution capacity of MUSIC increases.

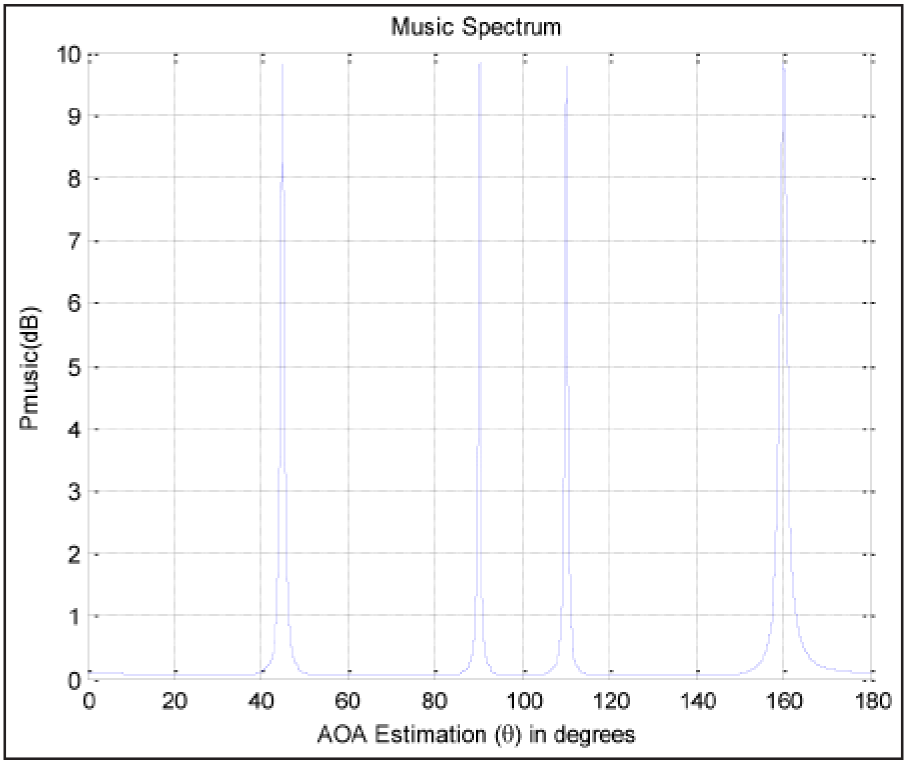

Figure 6. MUSIC spectrum for snapshots 512, SNR = 10dB and 16 array elements

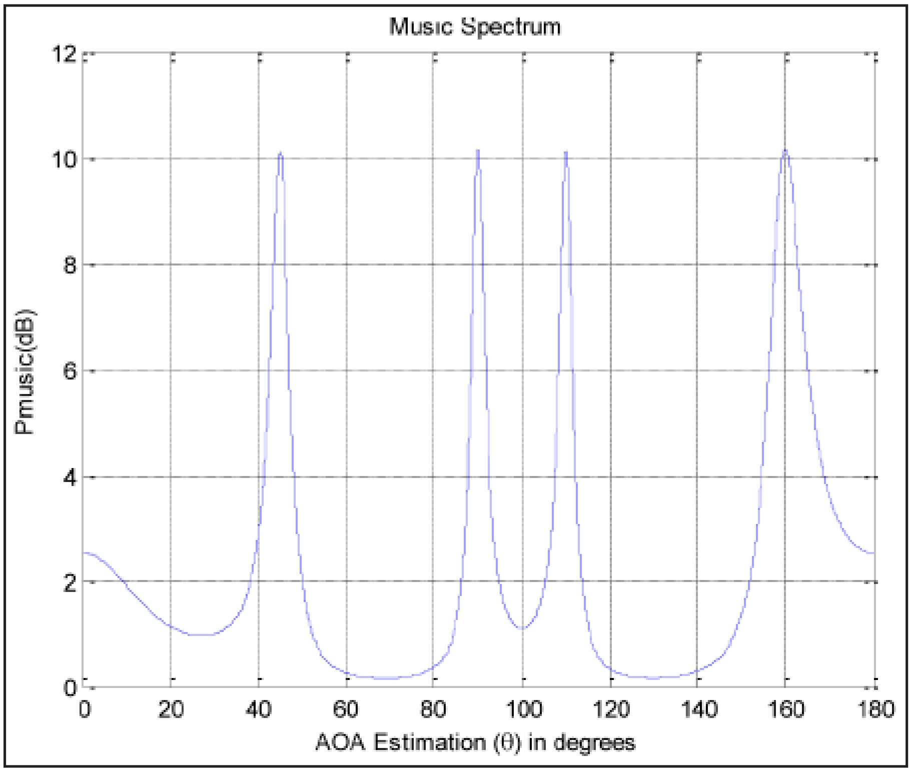

Figure 7. MUSIC spectrum for snapshots 1024, SNR = 10dB and 6 array elements

The simulation results of the MUSIC algorithm on four signals coming from four different angles (60°, 80°, 90°, 100°) with 64

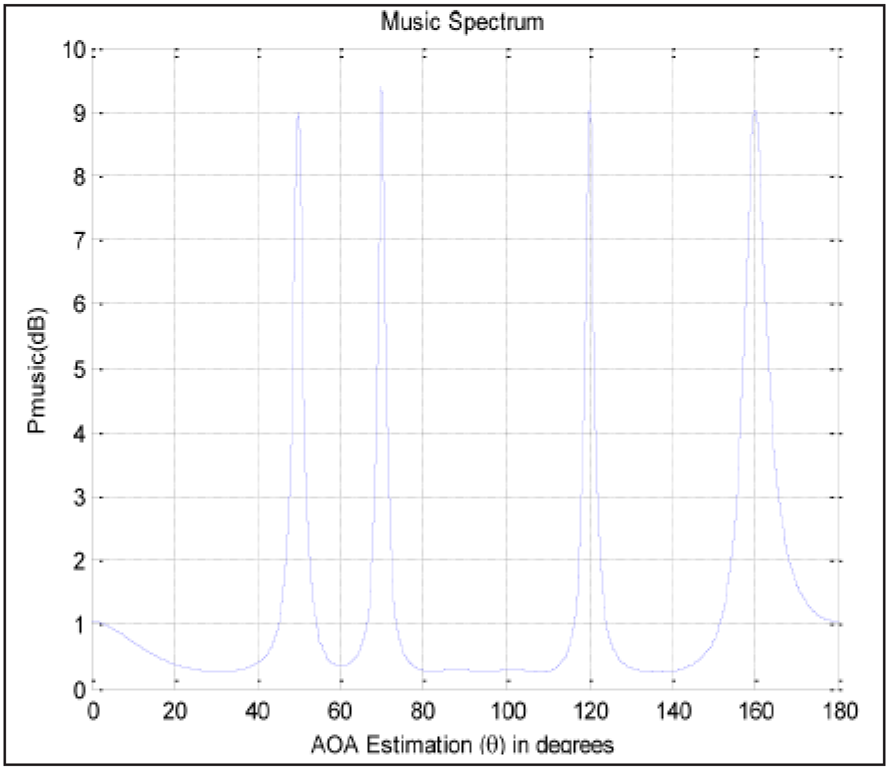

Figure 8. MUSIC spectrum for snapshots 64, SNR = 5dB and 10 array elements

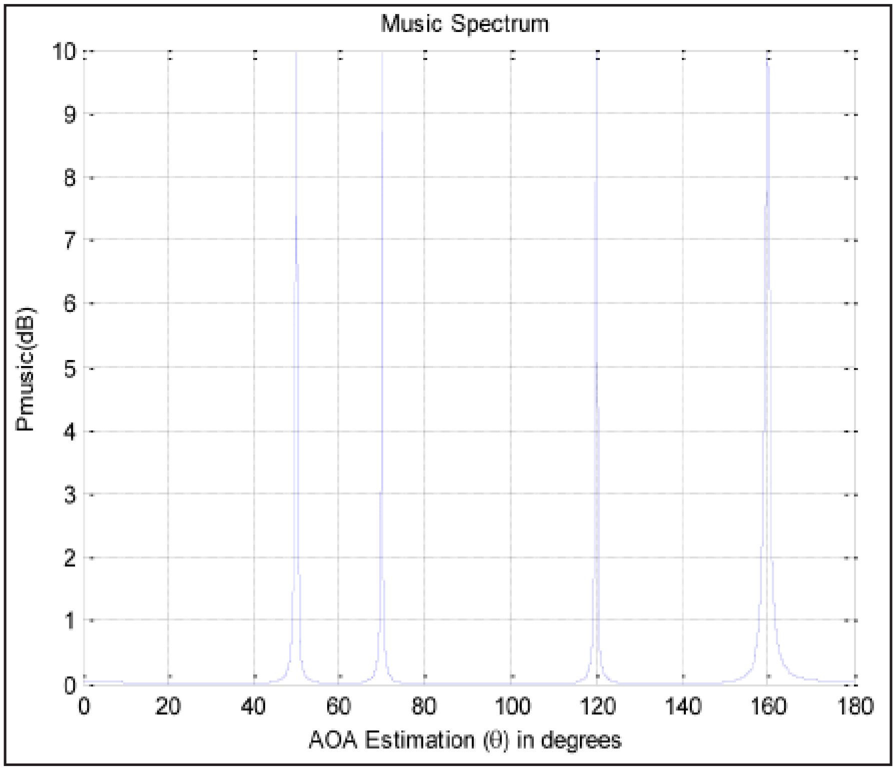

Figure 9. MUSIC spectrum for snapshots 1024, SNR=20dB and 10 array elements

snapshots (Figure 4) or 100 snapshots (Figure 5) are given respectively. The results indicate clearly that if the number of

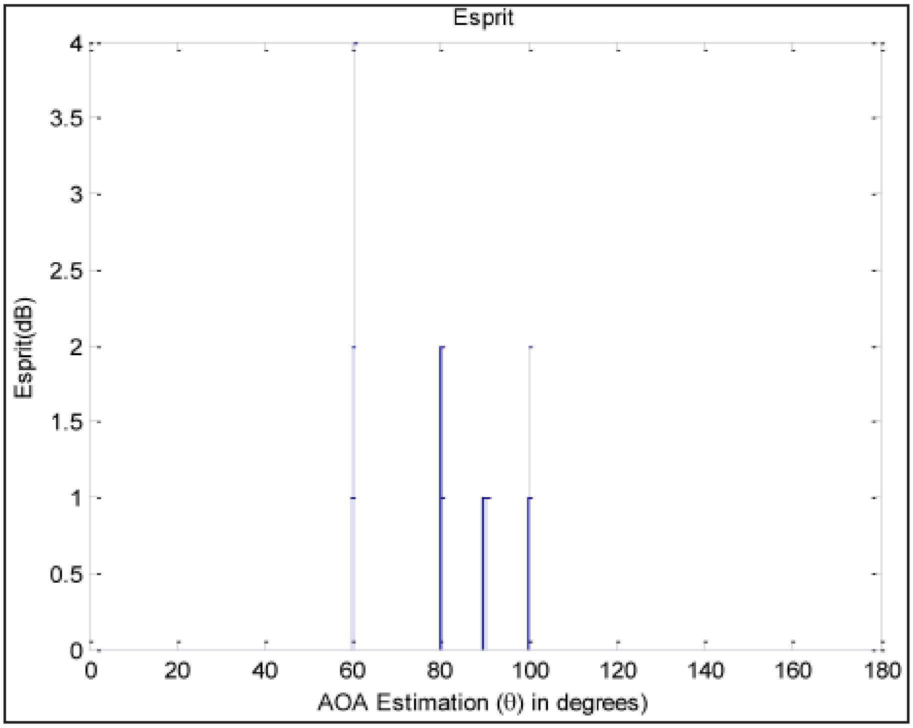

Figure 10. ESPRIT spectrum for 64 snapshots, SNR = 20dB and 10 array elements

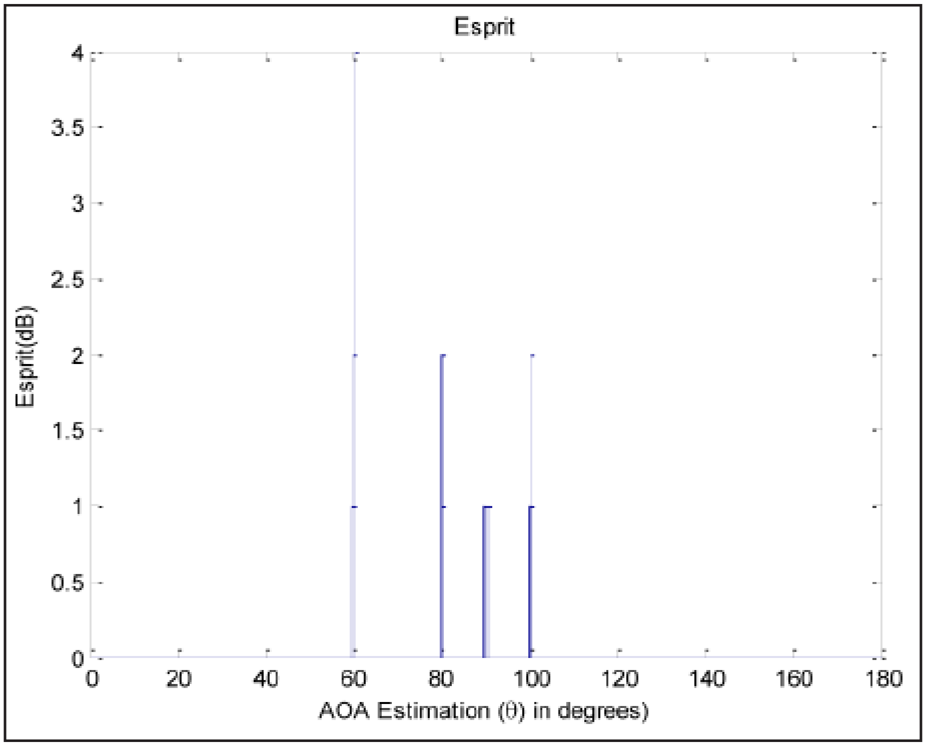

Figure 11. ESPRIT spectrum for 64 snapshots, SNR = 5dB and 10 array elements

snapshots increases (from 64 to 100), the resolution capacity of MUSIC increase and the four signals are clearly identifiable.

The simulation results of the MUSIC algorithm on four signals coming from four different AOA angles (45°, 90°, 110°, 160°),

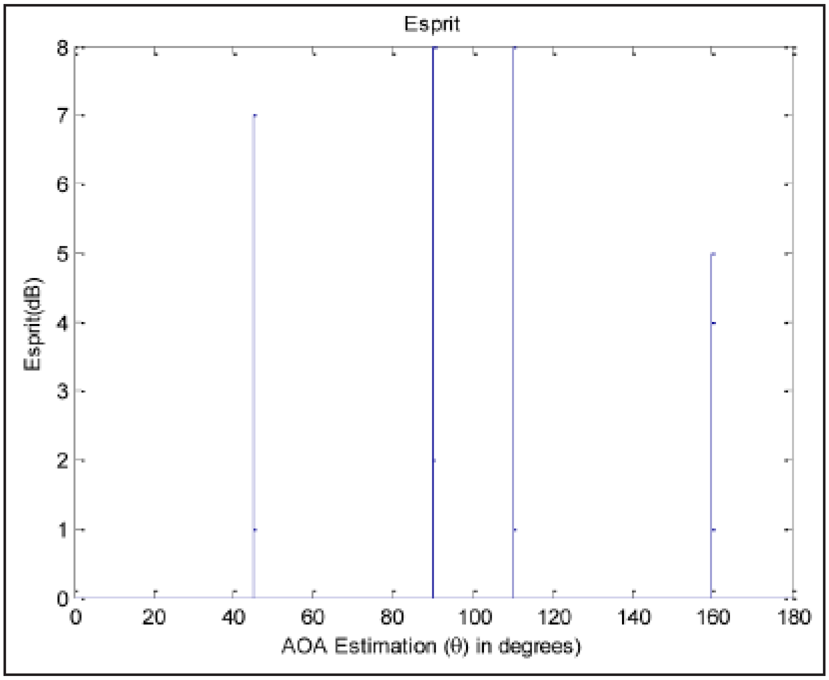

Figure 12. ESPRIT spectrum for 512 snapshots, SNR = 10dB & 16 array elements

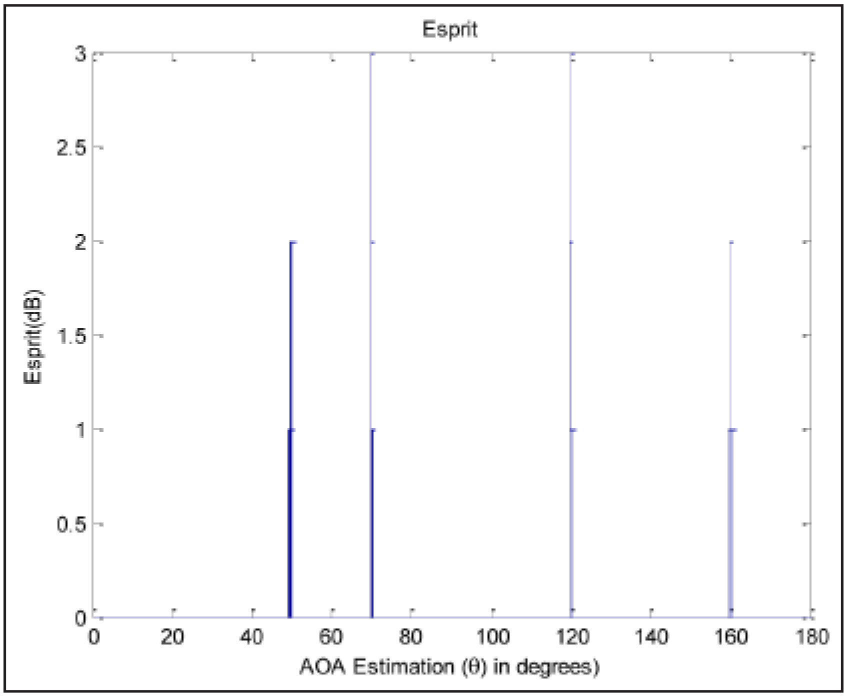

Figure 13. ESPRIT spectrum for snapshots 64, SNR = 5dB &10 array elements

SNR = 10dB, for snapshots 512, 16 array elements (Figure 6) and for snapshots 1024, 6 array elements (Figure 7) are given respectively. Also indicated clearly in the results that if the number of snapshots increases (from 512 to 1024), the resolution capacity of MUSIC increase and the four signals are clearly identifiable. Another simulation results of the MUSIC algorithm on four signals coming from four different AOA (50°, 70°, 120° and 160°) with SNR = 5dB (Figure 8) or SNR = 20dB (Figure 9) and are given respectively.

The simulation results of the ESPRIT algorithm on four signals coming from four different AOA (60°, 80°, 90° and 100°) with SNR = 20dB (Figure10) or SNR = 5dB (Figure 11) and are given respectively. Another simulation results of the ESPRIT algorithm on four signals coming from four different AOA (50°, 70°, 120° and 160°) with SNR = 5dB (Figure13) or SNR = 20dB (Figure 14) and are given respectively. The results indicate clearly that the pictures expand and fade for low values of SNR and also that the accuracy of estimation deteriorates also when the noise dominates. The estimation error increases as the SNR decreases but remains less than 0.06° for the average levels of noise.

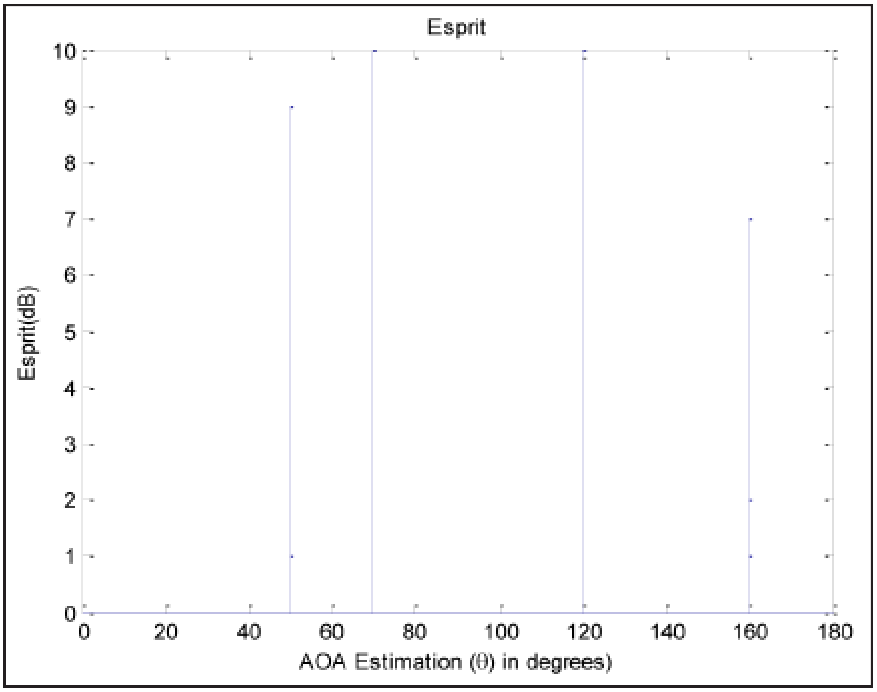

Figure 14. ESPRIT spectrum for snapshots 1024, SNR = 20dB &10 array elements

The AOA estimation results by MUSIC and ESPRIT for snapshots = 1024, M = 10, SNR = 20dB (Table 1) and for snapshot = 64, SNR = 5dB, M = 10 (Table 2) are given respectively. In MUSIC & ESPRIT more antennas results in a higher spatial resolution, less antennas will results in the reduction of the spatial resolution and using less snapshots the signals are more correlated and increased snapshots results to MUSIC spectrum more accurate detection and better resolution than low snapshots. If the signals are in low level of correlation it will result in high peaks in the spectrum and in high level of correlation will result in small peaks in the spectrum. We see that the MUSIC is more accurate and high resolution than ESPRIT.

θ Input(degrees)	MUSIC(θ)	ESPRIT(θ)
50	50.0000	49.9987
60	60.0000	60.0027
70	70.0000	70.0030
80	80.0000	80.0029
90	90.0000	89.9906
100	100.0000	99.9815
120	120.0000	119.9964
160	160.0000	160.0043

Table 1. DOA estimations (snapshots = 1024, M = 10, SNR = 20dB)

θ Input(degrees)	MUSIC(θ)	ESPRIT(θ)
50	49.9999	49.8935
60	60.0002	59.0027
70	70.0000	69.0159
80	80.0000	80.0543
90	90.0000	90.0141
100	100.0000	100.0177
120	120.0000	119.9915
160	160.0000	159.9520

Table 2. DOA estimations (snapshot = 64, SNR = 5dB, M = 10)

5. Conclusion

This paper presents the results based on the DOA by using MUSIC and ESPRIT. Attentions have been paid to their performance in terms of SNR, snapshots and the number of antennas. The simulation results show that there are errors in ESPRIT compared to MUSIC. The MUSIC method is more accurate, stable and gives better resolution than the ESPRIT method.

References

[1] Bharathidasan, A., Anand, V., Ponduru, S. Sensor networks: An Overview, University of California, Davis, CA 95616.

[2] Hightower, J., Gaetano Borriello. (2001). Location Systems for Ubiquitous Computing, *IEEE Computer*, August.

[3] Roy, R., Kailath, T., ESPRIT. (1989). Estimation of signal parameters via rotational invariance techniques, *IEEE Trans. Acoust., Speech, Signal Process*., 37 (7), Jul.

[4] Yang, P., Yang, F., Nie, Z.-P. (2010). DOA estimation with sub-array divided technique and interpolated esprit algorithm on a cylindrical conformal array antenna, *Progress In Electromagnetics Research*, 103, 201-216.

[5] James, J., Jr. Caffery, Stuber Gordon, L. (1998). Overview of Radioloca-tion in CDMA Cellular Systems, *IEEE Communications Magazine*, 36, 38 – 45

[6] Schmidt, R. O. (1986). Multiple emitter location and signal parameter estimation, *Journal*, *IEEE Trans. Antennas Propagat*., 34, 276–280, Mar.

[7] Hill, J., Szewczyk, R., Woo, A., Hollar, S., Culler, D., Pister, K. (2000). System Architecture Directions for Networked Sensors, *In*: Proceedings of the 9th International Conference on Architectural Support for Programming Languages and Operating Systems, November.

[8] Yick, J., Bharathidasan, K., Pasternack, G., Mukherjee, B., Ghosal, D. Optimizing Placement of Beacons and Data Loggers in a Sensor Network – A Case Study, Department of Computer Science, University of California Davis and Department of Land, Air, and Water Resources, University of California Davis.

[9] Mao, G., Fridan, B., Anderson, B. (2007). Wireless sensor network localization techniques, Computer Networks, *the International Journal of Computer and Telecommunications Networking archive*, 51(10) 2529-2553, July.

[10] Haardt, M., Nossek, J. A. (1998). Simultaneous Schur Decomposition of Several Nonsymmetric Matrices to Achieve Automatic Pairing in Multidimensional Harmonic Retrieval Problems, *IEEE Transaction on Signal Processing*, 46 (1), Jan.

[11] Godara, L.C. (1997). Applications of antenna arrays to mobile communications, *In*: Proceedings of the IEEE, 85, 1031-1060, July 1997 and Part II. Proc. IEEE, 85 (8) 1195-1244, August.

[12] Liao, B., Chan, S.C. (2011). DOA Estimation of Coherent Signals for Uniform Linear Arrays with Mutual Coupling, *Symposium on Circuits and Systems* (ISCAS), *IEEE International*, May, Rio de Janeiro, Brazil.

[13] Correia, L. H. A., Macedo, D. F., dos Santos, A. L., Nogueira, J. M. (2005). Issues on QoS Schemes in Wireless Sensor Networks, Technical Report: RT.DCC.004/2005, Federal University of Minas Gerais, Brazil and Federal University of Lavras, Brazil.

[14] Loomis, H. (1996). Geolocation of Electromagnetic Emitters, October 3.

[15] Rappaport, T., Reed, J., Woemer, B. (1996). Position Location Using Wireless Communication on Highways of the Future, *IEEE Communications Magazine*, p. 33-41, October .

[16] Elnahrawy, E., Li, X., Martin, R. P. (2004). The limits of localization using signal strength: a comparative study, IEEE SECON, Oct. 4-7, p. 406-414.

[17] Bahl, P., Padmanabhan, V. N. (2000). RADAR: An In-Building RF-based User Location and Tracking System, INFOCOM , Mar. 26-30, 2, 775-784.

[18] Sayrafian-Pour, K., Kaspar, D. (2006). Application of Beamforming in Wireless Location Estimation, *EURASIP Journal on Applied Signal Processing*, Article ID 51673, p. 1-13.

[19] Dongarsane, C., Jadhav, A., Hirikude, S. M. Performance Analysis of ESPRIT Algorithm for Smart Antenna System , *International Journal of Communication Network and Security* (IJCNS), Vol-1, Issue-3 ISSN: 2231-1882

[20] Roy, R. ESPRIT-Estimation of Signal Parameters via Rotational Invariance Techniques, Ph.D. thesis, Stanford Univ., Stanford. CA,

[21] Gray, D. An Introduction to Subspace Methods for Arrays, Cooperative Research Centre for Sensor Signal and Information processing, Technology Park Adelaide, South Australia, 5095.

[22] DeGroat, R., Dowling, E., Linebarger, D. (1993). The Constrained MUSIC Problem, *IEEE Transactions on Signal Processing*, 41 (3) 1445-1449, Mar.

[23] Turin, G., Jewell, W. S., Johnston, T. L. (1972). Simulation of Urban Vehicle-Monitoring Systems, *IEEE Transactions on Vehicular Technology*, VT-12 (1) 9-16, February.

[24] Knapp, C. H., Carter, G. C. (1976). The Generalisation Correlation Method for Estimation of Time Delay, *IEEE Transactions on Acoustics, Speech and Signal Processing, ASSP*-24 (4) 320-327, August.

[25] Krizman, K. J., Biedka, T. E., Rappaport, T. S. (1997). Wireless Position Location: Fundamentals, Implementation Strategies and Sources of Error, *IEEE Vehicular Technology Conference, Phoenix Arizona*, May 5-7, p. 919-923.

[26] Niculescu, D., Nath, B. (2003). Ad hoc positioning system (APS) using AOA, *In*: IEEE INFOCOM, Apr.

[27] Nasipuri, A., Li, K. (2002). A directionality based location discovery scheme for wireless sensor networks, *In*: First ACM International Workshop on Wireless Sensor Networks and Applications, Atlanta, GA, Sept.