Graph-Based Bit-Wise Soft Channel Estimation for Superposition Mapping

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ABSTRACT: *A novel channel estimator which performs channel estimation bit-wise instead of symbol-wise is proposed in this paper. Combined with superposition mapping (SM), the proposed algorithm is able to provide multiple channel estimates for a single channel coefficient. Numerical results indicate that bit-wise soft channel estimation (BWSCE) is able to outperform symbol-wise soft channel estimation (SWSCE) even at lower computational cost.*

Keywords: Graph-Based Receiver, Bit-wise Soft Channel Estimation, Iterative Receiver, Superposition Mapping, Monte-Carlo Simulations, Turbo Channel Coding

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1. Introduction

Channel estimation is a challenging task for wireless communication channels, especially at high vehicular speeds and/or high carrier frequencies. Iterative processing can help to improve the system performance. In the past decades, many iterative channel estimation approaches have shown to outperform non-iterative approaches, see e.g. [1]–[6]. The concept of factor graphs [7] is a powerful tool for the design of iterative estimation/detection algorithms. Several graph-based approaches have been suggested [5], [6], [8]. In [5], a graphbased soft iterative receiver (GSIR) has initially been proposed in which joint data detection, channel estimation and channel decoding are integrated in a single factor graph. Utilising the sum-product algorithm, the messages exchange in the graph is extrinsic and soft. Hence, both channel estimation and data detection can benefit from channel coding. In [8], the GSIR has been extended to higher-order modulation schemes and timevarying channels. Graph-based soft channel estimation has shown to provide a desirable performance using less training symbols than conventional channel estimation schemes. All of the above mentioned algorithms are using symbol-wise channel estimation when considering higher-order modulation. Thus, their complexities grow exponentionally with number of bits per symbol.

Superposition mapping (SM) [9] is a recently developed modulation/multiplexing/mapping scheme. SM can generate a Gaussian distributed transmit signal, which has the ability to approach the channel capacity. In this paper, an additional advantage of SM will be explored. In conjunction with graphbased iterative processing, the characteristic of SM allows to perform channel estimation on individual bits, rather than on a per symbol basis as in [8]. In this way, unlike symbolwise soft channel estimation (SWSCE) , multiple estimates of one channel coefficient will be obtained at the receiver side. Since messages are soft, exploiting multiple estimates can improve channel estimation by further exploiting coding gain. Moreover, as superposition is a very natural composition in communication systems, it is straightforward to extend this algorithm to multiple antennas and multi-user communications.

2. Fundamentals

2.1 Superposition Mapping

The procedure of superposition mapping is shown in Figure 1. The info bits are first BPSK mapped onto bipolar antipodal info symbols. Then, each info symbol is multiplied by a weighting factor, and superimposed to create a complex valued data symbol before transmission, represented in baseband notation as

$$\begin{aligned} x[k] &= \sum_{n=1}^{N} \alpha_n d_n[k] \\ &= \sum_{n=1}^{N} \alpha_n (1-2b_n[k]),\ b_n[k] \in \{0, 1\} \end{aligned} \tag{1}$$

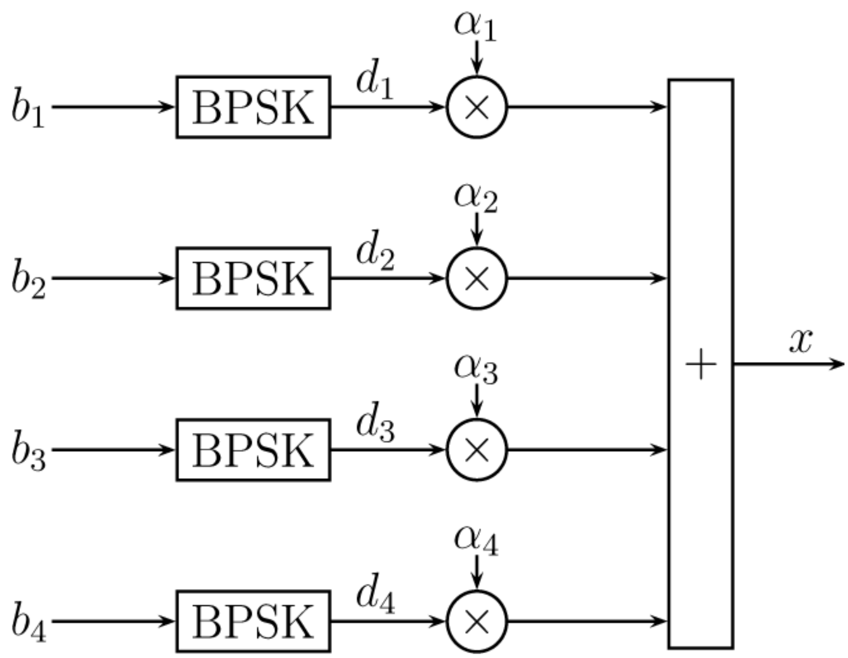

Figure 1. Block diagram of superposition mapping with 4 info bits per symbol

where $d_n[k]$ denotes the n^{th} binary antipodal symbol (chip) at time index k and α_n represents its allocated complex-valued weighting factor. N is the number of info bits per symbol.

SM provides a degree of freedom of generating the symbol constellation by properly choosing the values of α_n. For instance, if

$$[\alpha_1, \alpha_2, \alpha_3, \alpha_4] = [1/\sqrt{10}, 1/\sqrt{10}, 2/\sqrt{10}, 2/\sqrt{10}]$$

is chosen, we will have a 16-QAM constellation. If

$$\alpha_n = \frac{1}{\sqrt{N}} e^{jn\pi/N}$$

is selected, the symbol constellation of $x[k]$ is approximately Gaussian distributed. Due to Shannon's channel coding theorem, the capacity of a Gaussian channel can be achieved if and only if the channel output is Gaussian distributed. In other words, the channel input is also required to be Gaussian distributed. For this reason, SM has the potential to achieve channel capacity. In addition, the superimposing property of SM provides the foundation for bit-wise soft channel estimation.

2.2 System Model

We consider bit-interleaved coded modulation (BICM) in conjunction with superposition mapping. After channel encoder and interleaver, the signal stream is superposition mapped before transmission. Concerning a flat fading singleinput single output (SISO) channel, the relationship between the n^{th} chip $d_n[k]$ and the channel output $y[k]$ can be represented as

$$y[k] = h[k] \sum_{n=1}^{N} \alpha_n d_n[k] + w[k] \tag{2}$$

where $h[k]$ is the channel coefficient, and $w[k]$ is an additive white Gaussian noise (AWGN) sample. Based on (2), a factor graph representing the joint iterative receiver for coded transmission over a time-varying SISO flat fading channel with $N = 2$ is illustrated in Figure 2. Each column represents a different time index. A channel node $h[k]$ carries the soft information of a channel coefficient. The message exchange between neighboring channel coefficients is carried out by a transfer node Δ. An observation node $y[k]$ denotes the information of a received symbol. A chip node $d_n[k]$ represents the soft information of a transmitted code bit, which is directly connected to the observation node. A ⊞ denotes a code constraint. As the coderate is 1/3 and interleaving is taken into account, each ⊞ is randomly connected with three chip nodes. In the remainder, the time index k is dropped to simplify the notation.

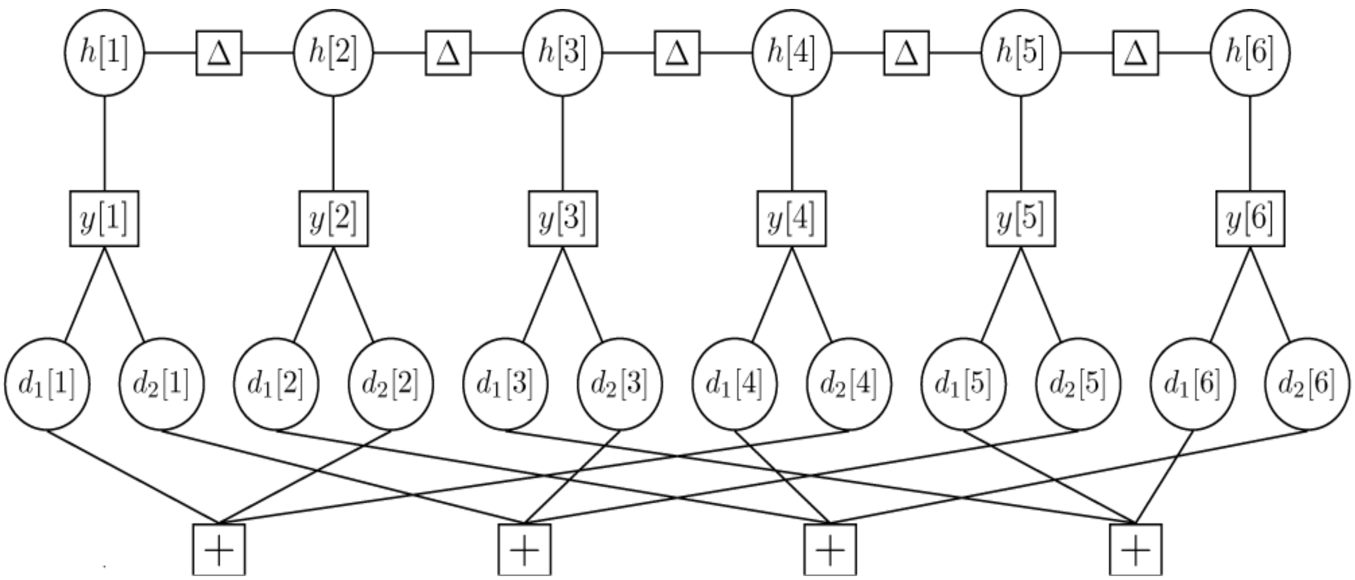

Figure 2. Factor graph for the GSIR over a time-varying channel assuming a rate 1/3 channel code and 2 info bits per symbol

3. Bit-Wise Soft Channel Estimation (BWSCE)

The meaning of soft information in the factor graph is twofolded. One is that the soft information of the binary symbols is represented by log-likelihood ratio (LLR) values. The other is that the soft information of the channel estimates is depicted by the probability density function (pdf) of h. Approximating h by a Gaussian variable, $h \sim CN(\mu_h, \sigma_h^2)$, with h denoting the hard channel estimate and σ_h^2 measuring the reliability of this estimate. The procedure of the iterative receiver works as follows. First, initial LLRs and initial pdfs of h at the pilot positions are obtained using training symbols, i.e. reference symbols known to the receiver. Afterwards, the soft information is spread to the data positions via forward/backward propagation. Later on during iterations, both training and data symbols contribute to iterative channel estimation and data detection. The message passing algorithm concerning data detection has been elaborated in [8]. Different from [8] where channel estimation is performed symbol-wise, in this work the principle of bit-wise channel estimation is proposed.

For channel estimation during iterations, the goal is to calculate the pair (μ_h, σ_h^2) (pdf of h) given the corresponding LLRs from the chip nodes obtained in the preceding iteration. Revisiting (2) and considering a certain chip d_n, we obtain

$$y = h\alpha_n d_n + \underbrace{\sum_{j=0;\, j \neq n}^{N} h\alpha_j d_j}_{\text{ICI}} + w \tag{3}$$

The received symbol y is composed of the desired chip, interchip interference (ICI), and noise w. According to the central limit theorem, the interfering part

$$v_n = \sum_{j=0;\, j \neq n}^{N} h\alpha_j d_j + w = \sum_{j=0;\, j \neq n}^{N} \gamma_j + w \tag{4}$$

can be approximated by a Gaussian random variable with mean μ_{v_n} and variance $\sigma_{v_n}^2$ if N is sufficiently large. Assuming that different values of $\gamma_j = h\alpha_j d_j$ are independent, the mean and variance of the interfering part μ_{v_n} and $\sigma_{v_n}^2$ are computed as

$$\mu_{v_n} = \sum_{j=1;\, j \neq n}^{N} \mu_{r_j} \tag{5}$$

$$\sigma_{v_n}^2 = \sum_{j=1;\, j \neq n}^{N} \sigma_{r_j}^2 + \sigma_w^2$$

where

$$\mu_{r_j} = \mu_h \alpha_j (P_{j,+1} - P_{j,-1}) \tag{6}$$

$$\sigma_{rj}^2 = \sigma_h^2 + 4P_{j,+1} P_{j,-1} |\mu_h \alpha_j|^2$$

The pair (μ_h, σ_h^2) in (6) is the soft estimate of h obtained from the previous iteration. $P_{j,\pm 1}$ stands for the probability of $d_j = \pm 1$, this can easily be computed by the LLR of d_j from the preceding iteration. After obtaining μ_{v_n} and $\sigma_{v_n}^2$, the soft estimate of h from the nth chip (defined hn here) is calculated as follows:

$$\mu_{h_n} = \frac{(\mathrm{y} - \mu_{h_n})(P_{n,+1} - P_{n,-1})}{\alpha_n} \tag{7}$$

$$\sigma_{h_n}^2 = \frac{\sigma_{v_n}^2 + 4P_{n,+1} P_{n,-1} |\mathrm{y} - \mu_{v_n}|^2}{|\alpha_n|^2}$$

Since each chip provides one estimate for one h, we will have N channel estimates for a certain channel coefficient h. According to the sum-product algorithm, $p(h)$ can be attained by multiplication of $p(h_n)$ with the formula

$$p(h) = \prod_{n=1}^{N} p(h_n) \tag{8}$$

After some derivations, the pair (μ_h, σ_h^2) is of the form

$$\mu_h = \sum_{n=1}^{N} \frac{\mu_{h_n}}{\sigma_{h_n}^2} \bigg/ \sum_{n=1}^{N} \frac{1}{\sigma_{h_n}^2}, \quad \sigma_h^2 = 1 \bigg/ \sum_{n=1}^{N} \frac{1}{\sigma_{h_n}^2} \tag{9}$$

BWSCE has two advantages. First, several observations contribute to one channel estimate. Thus, weaker estimates can be compensated by stronger estimates through interleaving and channel decoding during iterations. Second, BWSCE involves a lower computational cost. Since for SWSCE the probability of each complex-valued symbol must be know a priori, the complexity of SWSCE is exponential w.r.t. N, whereas the complexity of BWSCE is linear w.r.t. N.

4. Numerical Results

In order to examine the performance of the BWSCE, Monte Carlo simulation are conducted. A superposition mapper with $\alpha_n = \frac{1}{\sqrt{N}} e^{jn\pi/N}$ is chosen. A concatenated code composed of a rate 3/4 Turbo code and a rate 1/3 repetition code is used for

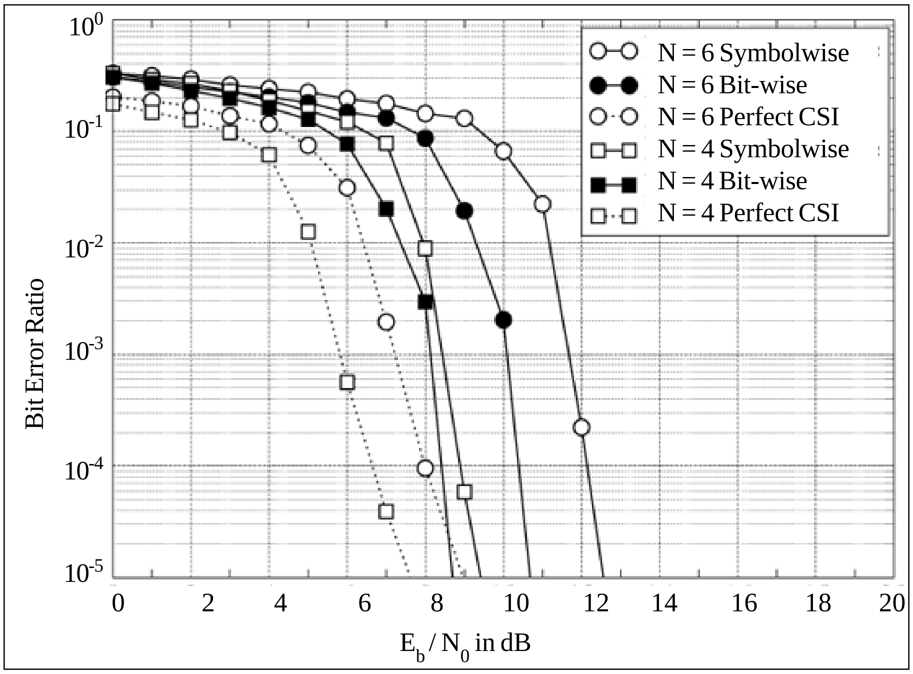

Figure 3. BER performance between SW and BW channel estimation, N = 4, 6

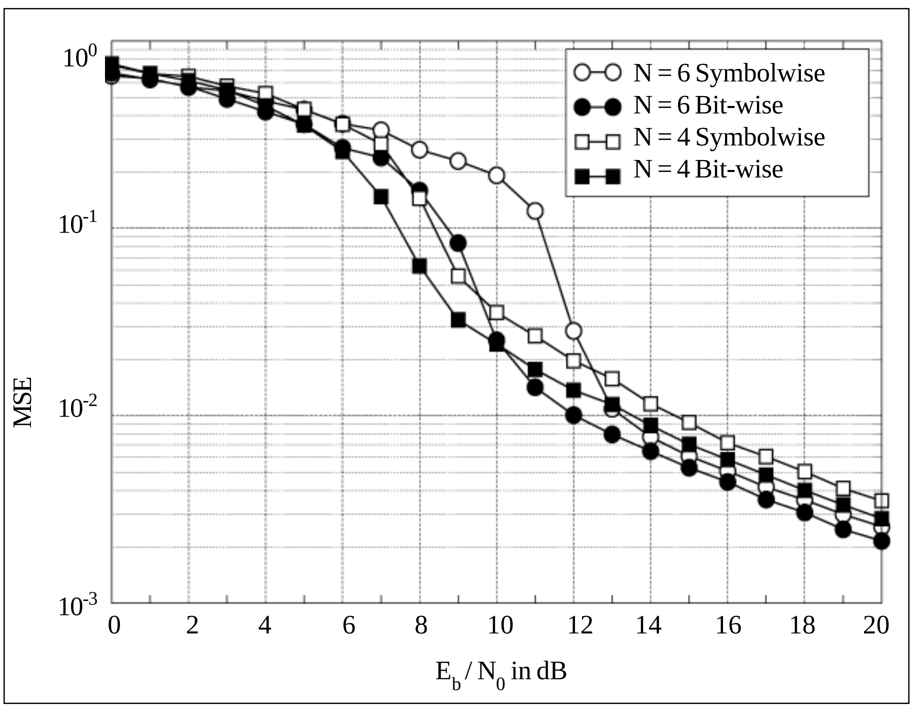

Figure 4. MSE performance between SW and BW channel estimation, N = 4, 6

channel coding. Training symbols are inserted every 8 data symbols. The symbols are transmitted over a Rayleigh time-varying channel with a maximum Doppler frequency of $f_D T_S = 0.02$. At the receiver side, a graph-based iterative receiver with 10 iterations is adopted. The performance of SWSCE [8] and the receiver knowing perfect channel state information (CSI) with the same simulation setup is also studied for comparison. From Figure 3, it can be observed that the MSE for the BWSCE converges earlier than that for SWSCE for both N = 4 and N = 6. At N = 6, there is a considerable performance gain of BWSCE over SWSCE for both

BER and MSE results. As is shown on the left hand side of Figure 2, at BER = 10^{-5}, BWSCE provide an approximately 1 dB and 2 dB gain over SWSCE in case of N = 4 and N = 6, respectively. Meanwhile, the complexity for BWSCE is lowerthan SWSCE, especially when N increases.

5. Conclusion

In this paper, a novel graph-based bit-wise channel estimation technique has been proposed. This algorithm can outperform symbol-wise channel estimation in many situations meanwhile maintaining a lower computational complexity.

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