Effectiveness Evaluation of Information-Oriented Missile Defense Based on Gray Theory

Ang Gao, Huibo Zhao, Quan Pan, Yansu Hu
School of Automation
Northwest Polytechnical University
Xi'an , China
snailgao@gmail.com

ABSTRACT**:** *Contesting information-superiority is crucial for missile defense in network centric warfare, but there is no unified effectiveness evaluation model to assess ability of this, including evaluation index and approach. This paper focuses on the information inter-operation in missile defense from the domains of acquisition, delivery and processing, establishes an evaluation index system for these three domains form view of information integrity, accuracy and timeliness. By using multi-level gray evaluation approach, the example verifies the feasibility and rationality of this effectiveness evaluation model for information-oriented missile defense.*

Keywords: Information-Oriented, Network Centric Warfare, C4ISR, Gray Theory

Received: 1 March 2012, Revised 18 April 2012, Accepted 22 April 2012

1. Introduction

Effectiveness evaluation as one of the most important parts of the research on C4ISR (Command, Control, Communication, Computer, Intelligence, Surveillance, Reconnaissance), has an extensive application in the verification, optimization and the tactical guideline reasoning. At the every section of the system design, the solution and its consequence should be evaluated phased to guarantee the optimum design.

BMDS (Ballistic Missile Defense System) is a complex giant C4ISR system and NCW (Network Centric Warfare) intensified its information-oriented property. Establishment of evaluation index is the premise for system evaluation. However, there is no unified specification for the evaluation index design of BMDS. The current methods mainly focus on the missile intercepting rate, which has no attention on sub-system and sub-function and the evaluation index is simply comprehensive weighted of tactic-technical performance [1].

In the condition of NCW, the information superiority is the determinants of system opposability [2]. This paper focuses on the information inter-operation in missile defense from the domains of acquisition, delivery and processing, establishes a evaluation index for these three domains form view of information integrity, accuracy and timeliness. By using multi-level gray theory, the example verifies the feasibility and rationality of this effectiveness evaluation index for information-oriented missile defense.

2. A Brief Procedure Of Missile Defense

Network-centric warfare has been identified as one of the key concepts for future missile defense battle. According military doctrine, the conception of NCW refers to the "*linking of sensors, engagement systems and decision-maker into an effective and responsive whole*" and is achieved through "*shared situation awareness, clear procedures and the information connectivity needed to synchronism the actions of the defense force to meet the commanders*" intent [3]. Although the BMDS architecture in NCW is similar to that in platform centric warfare, with the rapid development of the information technique, modern war is turning from the platen-centric war which based on resources to the network-centric war which worked in coordination, and the core of the later is information inter-operation. Figure 1 shown is the conception of information flow of missile defense.

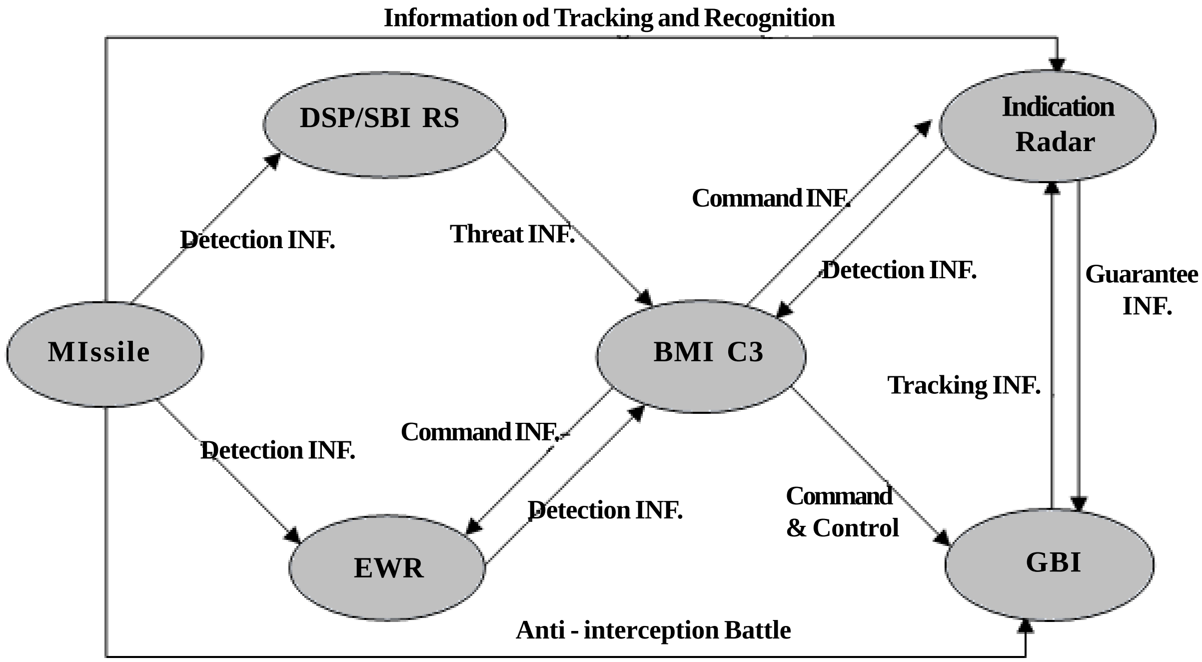

Figure 1. Conception of Information Flow of Missile Defense

The brief procedure of missile defense is shown as follows:

1) DSP or SBIRS including geostationary and highly elliptic orbit statistics can find missile by infrared radiation characteristics of ballistic missile in boosting phase. Position, velocity and other information can also be calculated from the relative position between satellite and missile. DSP/SBIRS keeps tracking the missile until its booster shuts down.

2) Information about missile target including tracking parameter and treatability are sent to BM/C3 which commands EWR (Early Warning Radar) including x-band and p-band radar to start detecting and searching in the specific airspace. If the missile is captured, the corresponding target propriety information is reported to BM/C3.

3) According to the tracking and assistant recognition information from DSP/SBIRS and EWR, with aided of the information fusion result of which, threaten sequence and predicted ballistic trajectory, BM/C3 plans detection mission assignments.

4) According to the detection mission, sets of indication radar launch by the command of BM/C3, which detect and search target in specific airspace. The target information is reported to BM/C3 continuously. After the confirmation of trajectory and fire distribution, interception command affirmed by BM/C3, GBI (Ground-Based Interception) is launched .

5) The indication radar and GBI establish information cross link, and the target information is renewed and corrected in real-time by indication radar. According to this information, GBI adjusts its flight path to hit and destroy the target.

3. Effectiveness Evaluation Index System

Information-oriented C4ISR system can be described from the domains of information acquisition, delivery and processing. From the view of set theory, the set of effectiveness **E** is

$$\mathbf{E} = \mathbf{A} \,|\, \mathbf{D} \,|\mathbf{P} \tag{1}$$

where **A** presents the ability of information acquisition, **D** presents the ability of information delivery and **P** presents that of processing. So the probability evaluation of the system R **(E)** is [4]

$$\mathbb{R}\,(\mathbf{E}) = \mathbb{R}\,(\mathbf{A} \,|\, \mathbf{D} \,|\, \mathbf{P}\,) = \mathbb{R}\,(\mathbf{ADP}) \tag{2}$$

From Equation 2, the effectiveness is the mix of three domains. So the evaluation index should be the combination of them.

In NCW, the fundamental requirement for the ability of information system is integrity, accuracy and timeliness. Integrity focuses on the accomplishment of ways used to acquire information and the dimension used to describe the battle states. Accuracy focuses on the correctness of information producing and consuming such as detection, coding and decoding, fusion etc. Timeliness means "*right information*" must be transferred to "*the right place*" at "*right time*".

3.1 Information Acquisition

The domain of information acquisition describes the ability of information collection.

	Top Level	Bottom Level
Information-Prioritied evaluation assessment	Information Acquisition	Integrity
		Accuracy
		Timeliness
	Information Processing	Integrity
		Accuracy
		Timeliness
	Information Delivery	Integrity
		Accuracy
		Timeliness

Figure 2. Effectiveness Evaluation Index

3.1.1 Integrity

Assuming the number of threaten target is *M* and the number of confirmed is *N*, so the integrity probability can be [2],

$$P_{A_I} = N / M = \lambda A_1 \,(1 - e^{-A_2}) \tag{3}$$

where A_2 is Coverage Probability, A_1 is detection probability and λ is usage probability of sensors.

3.1.2 Accuracy

For the propriety of ballistic missile, the accuracy includes two aspects, i.e. the detection for targets and the calculation for shutdown time, launching and landing position. So the corresponding index has two sub-domain, detection sub-domain and prediction sub-domain.

$$P_{A_A} = \omega_1 \sigma_D + \omega_2 \tau_P \tag{4}$$

Where σ_D is detection precision and τ_P is prediction precision, both of them are also the weighted sum of detailed sub-standards. Detection precision has the sub-standards like distance precision, angle precision and velocity prescient etc, and prediction precision with shutdown time, launching and landing location precision as well.

3.1.3 Timeliness

Index for timeliness contains the most early finding time and warning time. We use evaluation function T_f and T_w to approximate them,

$$T_f\,(x) = e^{-kx}$$

$$T_w(x) = \begin{cases} 1 - e^{-kx} & (0 \le x \le s) \\ 1 & (x > s) \end{cases}$$

so the timeliness measurement is,

$$P_{A_T} = \omega_f P_{T_f} + \omega_w P_{T_w} = \omega_f e^{-T_f} + \omega_f e^{-T_w} \quad (5)$$

where ω_f and ω_w are the weight of finding time and warning time respectively.

3.2 Information Processing

Concerned information can be abstracted by fusing and mining data from different seniors in different areas. The domain of information processing describes ability of this.

3.2.1 Integrity

Assuming the efficiency of information processing and fusing for the i^{th} senor is

$$f_{P_{I_i}} = \alpha + \beta (1 - e^{\varpi t})$$

where α and β are the capability of information processing and fusion respectively. $0 < \alpha < 1$ and $0 < \alpha + \beta \le 1$ esent the maximum value of processing and fusion. ϖ and t are the accuracy and processing time. According to reliability theory, the integrity of information processing is

$$P_{P_I}(t) = 1 - \prod_{i=1}^{k} [1 - f_{P_{I_i}}] \quad (6)$$

3.2.2 Accuracy

Accuracy means the property of targets obtained from information processing should fit the reality.

$c_i = e^{-s_i}$, $0 < c_i \le 1$, where s_i is the i^{th} standard deviation from the m characteristics. The smaller deviation, the bigger c_i.

$$f_{P_{I_i}} = \sum_{i=1}^{m} \omega_i c_i$$

where ω_i is the weight of i^{th}characteristic, $\sum_{i=1}^{m} \omega_i = 1$, so the accuracy measurement of information processing is

$$P_{P_A} = \sum_{j=1}^{n} \mu_j f_{P_{A_j}} = \sum_{j=1}^{n} \sum_{j=1}^{m} \mu_j \omega_i c_i \quad (7)$$

3.2.3 Timeliness

In the domain of information processing, measurement of timeliness is

$$P_{P_T} = e^{-t_P} \quad (8)$$

where t_p is the time that used to complete whole processing mission.

3.3 Information Delivery

Information delivery links the separated anti-missile senors and BM/C3 node into network. To establish and hold information-superiority, data should be transferred correctly, accurately and in time to BM/C3, as well as commands to the sensors and actuators.

3.3.1 Integrity

Assuming P_{P_I} is the probability that the usable information from the i^{th} sensor could be sent to the right place. Data sources, i.e. anti-missile sensors, are independent in communication, so the measurement of integrity is

$$P_{D_I} = 1 - \prod_{i=1}^{n} p_{I_i} \quad (9)$$

3.3.2 Accuracy

Assuming P_{A_i} is the probability that the i^{th} node could get its concerned information correctly. So the measurement of accuracy is

$$P_{A_I} = 1 - \prod_{i=1}^{n} p_{A_i} \tag{10}$$

3.3.3 Timeliness

Timeliness of information transmission is related to the bandwidth of communication link. From the view of queuing theory, the average delay ℓ_T is the sum of information processing time χ and queuing time ω[5], i.e. $\ell_T = \chi + \omega$. So the measurement of timeliness is

$$P_T = e^{-\ell_T} \tag{11}$$

4. Multi-level Gray Evaluation Approach

Gray system theory applies to forecast under poor information condition. Because of the uncertainty and incompleteness of information-oriented missile [6], we apply multi-level gray evaluation approach [7] to evaluation the effectiveness, specific steps are as follows:

Establishing evaluation index which has described in section 3. **U** represents a set which is composed by top level order evaluation indexes U_i, $i = 1, ..., N$, while V represents a set which is composed by bottom level 1) evaluation indexes $V_{i,j}$, $V_i = (V_{i,1}, ..., V_{i,j}, ..., V_{i,M})$, $j = 1,...M$.

2) Determine $V_{i,j}$ be the weight of index. The contribution of U_i and $V_{i,j}$ to the effectiveness is different and their weights are also different. Generally, there are three typical methods to determine the weight of these attributes. they are expert evaluation method, entropy method[8] and AHP (Analytical Hierarchy Process).

3) Assuming the sample matrix **D** of the evaluating objects is

$$\mathbf{D} = \begin{bmatrix} d_{1,1,1} & d_{1,1,2} & \cdots & d_{1,1,p} \\ \vdots & \vdots & \ddots & \vdots \\ d_{1,M,1} & d_{1,M,2} & \cdots & d_{1,M,p} \\ \vdots & \vdots & \ddots & \vdots \\ d_{i,j,1} & d_{i,j,2} & \cdots & d_{i,j,p} \\ \vdots & \vdots & \ddots & \vdots \\ d_{N,M,1} & d_{N,M,2} & \cdots & d_{N,M,p} \end{bmatrix}$$

where p is the number of the evaluators.

1) Determine the number of gray class and white weight function of corresponding classes. Supposing there are e, $e = 1, ...,G$. evaluating gray classes.

2) Calculating gray evaluation factor.

$$x_{i,j,e} = \sum_{i=1}^{p} f_e(d_{i,j,k}) \tag{12}$$

so the total gray evaluation of $V_{i,j}$ is

$$x_{i,j} = \sum_{e=1}^{G} x_{i,j,e} \tag{13}$$

6) Weigh vector for gray evaluation is

$$r_{i,j} = [\frac{x_{i,j,1}}{x_{i,j}}, ..., \frac{x_{i,j,e}}{x_{i,j}}], e = 1, ..., G \tag{14}$$

so we can obtain the weight matrix of gray evaluation:

$$\mathbf{R}_i = \begin{bmatrix} r_{i,1} \\ r_{i,j} \\ \vdots \\ r_{i,M} \end{bmatrix} = \begin{bmatrix} r_{i,1,1} & r_{i,1,e} & \cdots & r_{i,1,G} \\ r_{i,j,1} & r_{i,j,e} & \cdots & r_{i,j,G} \\ \vdots & & & \\ r_{i,M,1} & r_{i,M,e} & \cdots & r_{i,M,G} \end{bmatrix}$$

7) Comprehensive evaluation result for index V_i is $\mathbf{B}_i$,

$$\mathbf{B}_i = \mathbf{A}_i \times \mathbf{R}_i$$

8) Comprehensive evaluation result for top level index **U** is **B**, because of the Equation 15, the weight matrix for gray evaluation **R** is

$$\mathbf{R}_i = [\,\mathbf{B}_1 \ \mathbf{B}_i \ldots \mathbf{B}_N\,]^{\mathrm{T}}$$

so there is

$$\mathbf{B} = \mathbf{A} \times \mathbf{R} = [\,\mathbf{A}_1 \times \mathbf{R}_1 \ \mathbf{A}_i \times \mathbf{R}_i \ \ldots \ \mathbf{A}_N \times \mathbf{R}_N]^{\mathrm{T}} \quad (17)$$

9) Supposing the grading value vector is $C = (d_1, \ldots, d_e, \ldots, d_G)$.

$$\mathbf{W} = \mathbf{B} \times \mathbf{C}^{\mathrm{T}}$$

where **W** is the comprehensive evaluation value and evaluating level could be determined according to **W**.

5. Case Simulation

This paper divided a certain missile defense battle into three stage according to the brief procedure in Section 2, i.e. Procedure 1) is the detection and tracking for active flying targets, Procedure 2) with 3) and 4) with 5) are those of free flight phase and reentry of the trajectory respectively.

Generally speaking, the granularity of the division depends on the complexity of the mission[9]. According to the effectiveness evaluation index in Section 3, the evaluation matrix **D** for this information-oriented missile defense battle is

$$\mathbf{D} = [\,\mathbf{D}_1 \mathbf{D}_2 \mathbf{D}_2]^{\mathrm{T}} = \begin{bmatrix} 0.9 & 0.6 & 0.7 & 0.6 & 0.7 & 0.7 & 0.8 & 0.6 & 0.7 \\ 0.7 & 0.4 & 0.8 & 0.6 & 0.5 & 0.8 & 0.9 & 0.5 & 0.5 \\ 0.8 & 0.7 & 0.6 & 0.9 & 0.6 & 0.9 & 0.9 & 0.4 & 0.6 \end{bmatrix}^{\mathrm{T}}$$

Considering the importance of top level indexes are same, as well as the importance of bottom level, so AHP gives the corresponding weigh of every index,

$$\mathbf{A} = [\,0.33, 0.33, 0.33\,];$$
$$\mathbf{A}_1 = [\,0.33, 0.33, 0.33\,];$$
$$\mathbf{A}_2 = [\,0.33, 0.33, 0.33\,];$$
$$\mathbf{A}_3 = [\,0.33, 0.33, 0.33\,];$$

There are five evaluation of gray category, i.e. G = 5; $e = 1, \ldots G$, the albino functions of every gray category are shown in Figure 3.

The gray evaluation factors are calculated according to Equation 12. For the indicator $V_{1,1}$ at the time $e = 1$,

$$x_{1,1,1} = \sum_{e=1}^{3} f_1(d_{1,1,e}) = f_1(d_{1,1,1}) + f_1(d_{1,1,2}) + f_1(d_{1,1,3})$$
$$= f_1(0.9) + f_1(0.7) + f_1(0.8) = 2.6667$$

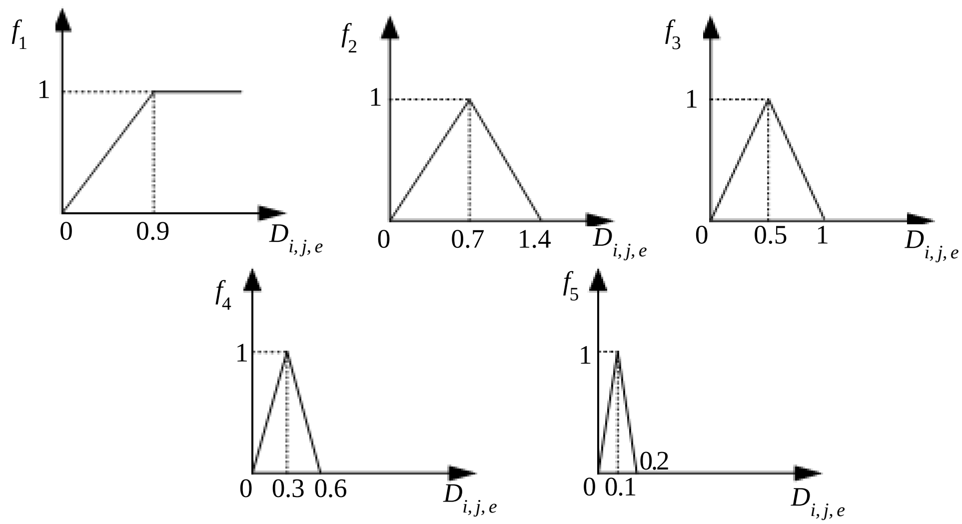

Figure 3. Albino function

By the same,

$$e = 2, x_{1,1,2} = 2.5714;$$
$$e = 3, x_{1,1,3} = 1.2000;$$
$$e = 4, x_{1,1,4} = 0;$$
$$e = 5, x_{1,1,5} = 0$$

So the total gray evaluation of $V_{1,1}$ is

$$x_{1,1,1} = \sum_{t=1}^{3} x_{1,1,t} = x_{1,1,1} + x_{1,1,2} + x_{1,1,3} + x_{1,1,4} + x_{1,1,5} = 6.4381$$

According to Equation (14), the weight vector of integrity in the domain of information acquisition is

$$r_{1,1} = r_{1,1,1} + r_{1,1,2} + r_{1,1,3} + r_{1,1,4} + r_{1,1,5} = (0.4142, 0.3994, 0.1864, 0,0)$$

Similarity, we can get the weight matrix of every top level evaluation index,

$$\mathbf{R}_1 = \begin{bmatrix} r_{1,1} \\ r_{1,2} \\ r_{1,3} \end{bmatrix} = \begin{bmatrix} 0.4142 & 0.3994 & 0.1864 & 0 & 0 \\ 0.2628 & 0.3380 & 0.3062 & 0.0928 & 0 \\ 0.3407 & 0.3964 & 0.3639 & 0 & 0 \end{bmatrix}$$

$$\mathbf{R}_2 = \begin{bmatrix} r_{2,1} \\ r_{2,2} \\ r_{2,3} \end{bmatrix} = \begin{bmatrix} 0.3556 & 0.3701 & 0.2743 & 0 & 0 \\ 0.2738 & 0.3520 & 0.3286 & 0.0456 & 0 \\ 0.4142 & 0.3994 & 0.1864 & 0 & 0 \end{bmatrix}$$

$$\mathbf{R}_3 = \begin{bmatrix} r_{3,1} \\ r_{3,2} \\ r_{3,3} \end{bmatrix} = \begin{bmatrix} 0.4003 & 0.3804 & 0.2193 & 0 & 0 \\ 0.2249 & 0.2892 & 0.3509 & 0.1350 & 0 \\ 0.3139 & 0.4306 & 0.2825 & 0 & 0 \end{bmatrix}$$

According to Equation 15, the comprehensive evaluation $\mathbf{B}_1$ for information acquisition V_1, $\mathbf{B}_2$ for information processing V_2

and $\mathbf{B}_3$ for information delivery V_3 are

$$\mathbf{B}_1 = (0.3389, 0.3776, 0.2516, 0.0309, 0)$$
$$\mathbf{B}_2 = (0.3475, 0.3735, 0.2628, 0.0152, 0)$$
$$\mathbf{B}_3 = (0.3127, 0.3664, 0.2839, 0.0450, 0)$$

By Equation 16, we know the comprehensive evaluation result for this missile defense mission is

$$\mathbf{B} = \mathbf{A} \times \mathbf{R} = (0.3327, 0.3721, 0.2658, 0.0303, 0)$$

The gray grade $\mathbf{C} = (0.9, 0.7, 0.5, 0.3, 0.1)$, so we can obtain the comprehensive evaluation value W according to the Equation 17.

$$W = \mathbf{BC}^T = 0.7019$$

At this point, $W = 0.1, 0.3, 0.5, 0.9$ respectively note the effectiveness results of information based missile defense mission are far from ideal, not very ideal, ideal, comparatively ideal, and very ideal.

By the above computational process, we can get that the comprehensive evaluation value W is 0.7019. It means the effectiveness of information system in this missile defense mission is comparatively favorable.

6. Conclusions

From the view of integrity, accuracy and timeliness to evaluate information acquisition, processing and delivery, this paper establishes the effective evaluation index system to assess the information system in missile defense. By means of multi-level gray evaluation approach, we comprehend the three stages of missile defense and get the value of evaluation, which validates the feasibility and rationality in some degree. Further, we are in the process of perfection the index calculation methods.

References

[1] Ang, G., Zengfu, W., Huibo, Z., Feng, Y., Quan, P., Weiguo, L. (2011). Review of the research on dodaf architecture, *Computers & Operations Research*, 6 (5) 461–465.

[2] Perry, W., Signori, D., Boon, J. (2004). Exploring information superiority: a methodology for measuring the quality of information and its impact on shared awareness. Rand Corp, 1467.

[3] Force, A. (2002). Future warfighting concept, Canberra. prepared by Policy Guidance and Analysis Division.

[4] Huibo, Z. (2011). Research on Several Key Technologies of Anti-ballistic Missile Warning System Structure, Ph.D. dissertation, Northwestern Ploytechnical University.

[5] Crovella, M., Bestavros, A. (1997). Self-similarity in world wide web traffic: evidence and possible causes, *IEEE/ACM Transactions on Networking*, 5 (6) 835–846.

[6] Bouaynaya, N., Schonfeld, D. (2008). Theoretical foundations of spatiallyvariant mathematical morphology part ii: Gray-level images, IEEE Transactions on Pattern Analysis and Machine Intelligence, 30 (5) 837–850.

[7] Shujing, S., Tianbao, G., Chao, W. (2009). Comprehensive evaluation model of real estate investment based on gray system theory, *In*: Information Technology and Applications. IFITA'09. International Forum on, 1, 304–308.

[8] Deng, H., Yeh, C., Willis, R. (2000). Inter-company comparison using modified topsis with objective weights, *Computers & Operations Research*, 27 (10) 963–973.

[9] Jun, W. (2011). Research on Several Key Technologies of ISR and Design and Implementation of Simulation Platform, Ph.D. dissertation, Northwestern Ploytechnical University.