

A Novel Proxy Blind Signature Scheme for Mobile Devices

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ABSTRACT: A proxy blind signature scheme is a special form of blind signature which allows a designated person called proxy signer to sign on behalf of two or more original signers without knowing the content of the message or document. It combines the advantages of proxy signature and blind signature scheme and satisfies the security properties of both proxy and blind signature scheme. This paper describes an efficient simple proxy blind signature scheme. The security of the scheme is based on Elliptic Curve Discrete Logarithm Problem (ECDLP). ECC offers security equivalent to RSA and DSA while using far smaller key sizes. The benefits of this higher-strength per-bit include higher speeds, lower power consumption, bandwidth savings, storage efficiencies, and smaller certificates. This can be implemented in low power and small processor mobile devices such as smart card, PDA etc.

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1. Introduction

Blind signature scheme was first introduced by Chaum [3]. It is a protocol for obtaining a signature from a signer on any message, without revealing any information about the message or its signature. In 1996, Mamo et al proposed the concept of proxy signature [1]. In proxy signature scheme, the original signer delegates his signing capacity to a proxy signer who can sign a message submitted on behalf of the original signer. A verifier can validate its correctness and can distinguish between a normal signature and a proxy signature. In multi-proxy signature scheme, an original signer is allowed to authorize a group of proxy members to generate the multi signature on behalf of the original signer. In 2000, Hwang et al. proposed the first multi-proxy signature scheme [4]. A proxy blind signature scheme is a digital signature scheme that ensures the properties of proxy signature and blind signature. In a proxy blind signature, an original signer delegates his signing capacity to proxy signer.

2. Background

In this section we brief overview of prime field, Elliptic Curve over that field and Elliptic Curve Discrete Logarithm Problem.

2.1 The finite field F_p

Let p be a prime number. The finite field F_p is comprised of the set of integers $0, 1, 2, \dots, p-1$ with the following arithmetic operations [5] [6] [7]:

- Addition: If $a, b \in F_p$, then $a + b = r$, where r is the remainder when $a + b$ is divided by p and $0 \leq r \leq p-1$. This is known as addition modulo p .

- Multiplication: If $a, b \in F_p$, then $a \cdot b = s$, where s is the remainder when $a \cdot b$ is divided by p and $0 \leq s \leq p-1$. This is known as multiplication modulo p .

Inversion: If a is a non-zero element in Fp , the inverse of a modulo p , denoted a^{-1} , is the unique integer $c \in Fp$ for which $a.c = 1$.

2.2 Elliptic Curve over Fp

Let $p \geq 3$ be a prime number. Let $a, b \in Fp$ be such that $4a^3 + 27b^2 \neq 0$ in Fp . An elliptic curve E over Fp defined by the parameters a and b is the set of all solutions $(x, y), x, y \in Fp$, to the equation $y^2 = x^3 + ax + b$, together with an extra point O , the point at infinity. The set of points $E(Fp)$ forms an Abelian group with the following addition rules [9]:

1. Identity : $P + O = O + P = P$, for all $P \in E(Fp)$

2. Negative : if $P(x, y) \in E(Fp)$ then $(x, y) + (x, -y) = O$, The point $(x, -y)$ is denoted as $-P$ called negative of P .

3. Point addition: Let $P(x_1, y_1), Q(x_2, y_2) \in E(Fp)$, then $P + Q = R \in E(Fp)$ and coordinate (x_3, y_3) of R is given by $x_3 = \lambda^2 - x_1 - x_2$ and $y_3 = \lambda(x_1 - x_3) - y_1$ where $\lambda = \frac{y_2 - y_1}{x_2 - x_1}$

4. Point doubling : Let $P(x_1, y_1) \in E(K)$ where $P = -P$ then $2P = (x_3, y_3)$ where $x_3 = \left(\frac{3x_1^2 + a}{2y_1}\right) - 2x_1$ and $y_3 = \left(\frac{3x_1^2 + a}{2y_1}\right)(x_3 - x_1) - y_1$

2.3 Elliptic Curve Discrete Logarithm Problem (ECDLP)

Given an elliptic curve E defined over a finite field F_p , a point $P \in E(F_p)$ of order n , and a point $Q \in \langle P \rangle$, find the integer $l \in [0, n - 1]$ such that $Q = lP$. The integer l is called discrete logarithm of Q to base P , denoted $l = \log_P Q$ [9].

3. Preliminaries

3.1 Notations

Common notations used in this paper as follows.

- p : the order of underlying finite field.
- F_p : the underlying finite field of order p
- E : elliptic curve defined on finite field F_p with large order.
- G : the group of elliptic curve points on E .
- P : a point in $E(F_p)$ with order n , where n is a large prime number.
- $H(\cdot)$: a secure one-way hash function.
- d : the secret key of the original signer S to be chosen randomly from $[1, n - 1]$.
- Q is the public key of the original signer S , where $Q = d \cdot P$.
- \parallel : Concatenation operation between two bit strings.

4. Proxy Signature and Proxy Blind Signature

A proxy blind signature is a digital signature scheme that ensures the properties of proxy signature and blind signature schemes. Proxy blind signature scheme is an extension of proxy signature, which allows a single designated proxy signer to generate a blind signature on behalf of group of original signers. A proxy blind signature scheme consists of the following three phases:

- Proxy key generation
- Proxy blind signature scheme
- Signature verification

5. Security properties

The security properties described in [2] for a secure blind signature scheme are as follows:

- **Distinguishability** : The proxy blind signature must be distinguishable from the ordinary signature.
- **Strong unforgeability**: Only the designated proxy signer can create the proxy blind signature for the original signer.
- **Non-repudiation**: The proxy signer can not claim that the proxy signature is disputed or illegally signed by the original signer.

-**Verifiability:** The proxy blind signature can be verified by everyone. After verification, the verifier can be convinced of the original signer's agreement on the signed message.

-**Stong undeniably:** Due to fact that the delegation information is signed by the original signer and the proxy signature are generated by the proxy signer's secret key. Both the signer can not deny their behavior.

-**Unlinkability:** When the signer is revealed, the proxy signer can not identify the association between the message and the blind signature he generated.

-**Secret key dependencies:** Proxy key or delegation pair can be computed only by the original signer's secret key.

-**Prevention of misuse :** The proxy signer cannot use the proxy secret key for purposes other than generating valid proxy signatures. In case of misuse, the responsibility of the proxy signer should be determined explicitly.

6. Proposed Protocol

The protocol involves three entities : Original signer S , Proxy signer P_s and verifier V . It is described as follows.

6.1 Proxy Phase

-Proxy generation : The original signer S selects random integer k in the interval $[1, n - 1]$. Computes $R = k \cdot P = (x_1, y_1)$ and $r = x_1 \bmod n$. Where x_1 is regarded as an integer between 0 and $q - 1$. Then computes $s = (d + k \cdot r) \bmod n$ and computes $Q_p = s \cdot P$.

-Proxy delivery : The original signer S sends (s, r) to the proxy signer P_s and make Q_p public.

- Proxy Verification: After receiving the secret key pairs (s, r) , the proxy signer P_s checks the validity of the secret key pairs (s, r) with the following equation.

$$Q_p = s \cdot P + r \cdot R \quad (1)$$

6.2 Signing Phase

-The Proxy signer S_p chooses random integer $t \in [1, n - 1]$ and computes $U = t \cdot P$ and sends it to the verifier V .

- After receiving the verifier chooses randomly $\alpha, \beta \in [1, n - 1]$ and computes the following

$$R = U + \alpha \cdot P - \beta \cdot Q_p \quad (2)$$

$$\tilde{e} = H(\tilde{R} \| M) \quad (3)$$

$$e = (\tilde{e} + \beta) \bmod n \quad (4)$$

and verifier V sends e to the proxy signer S_p .

- After receiving e , S_p computes the following

$$\tilde{s} = (t - s \cdot e) \bmod n \quad (5)$$

and sends it to V .

- Now V computes

$$s_p = (\tilde{s} + \alpha) \bmod n \quad (6)$$

The tuples (M, s_p, \tilde{e}) is the proxy blind signature.

6.3 Verification Phase

The verifier V computes the following equation.

$$\gamma = H((s_p \cdot P + \tilde{e} \cdot Q_p) \| M) \quad (7)$$

and verifies the validity of proxy blind signature (M, s_p, \tilde{e}) with the equality $\gamma = \tilde{e}$.

6. Security Analysis

Theorem 1 : *It is infeasible for adversary A to derive signer's private key from all available public information.*

Proof : Assume that the adversary A wants to derive signer's private key d from his public key Q , he has to solve ECDLP problem which is computationally infeasible. Similarly, the adversary will encounter the same difficulty as she/he tries to obtain proxy signer's private key.

Theorem 2 : *Proxy signature is distinguishable from original signer's normal signature.*

Proof: Since proxy key is different from original signer's private key and proxy keys created by different proxy signers are different from each other, any proxy signature is distinguishable from original signer's normal signature and different proxy signer's signature are distinguishable.

Theorem 3: *The scheme satisfies Unlinkability security requirement*

Proof: In verification stage, the signer checks only whether $\gamma = H((s_p \cdot P + \tilde{e} \cdot Q_p) // M)$ holds. He does not know the original signer's private key and proxy signer's private key. Thus the signer knows neither the message nor the signature associated with the signature scheme.

7. Correctness

Theorem 4 : *The proxy blind signature (M, s_p, \tilde{e}) is universally verifiable by using the system public parameters.*

Proof: The of correctness of the signature is verified as follows We have to prove that

$$\begin{aligned}
 H((s_p \cdot P + \tilde{e} \cdot Q_p) // M) &= H(\tilde{R} // M) \\
 i:e \text{ to show } s_p \cdot P + \tilde{e} \cdot Q_p &= \tilde{R} \\
 &= (\tilde{s} + \alpha) \cdot P + \tilde{e} \cdot Q_p \\
 &= s \cdot P + \alpha \cdot P + \tilde{e} \cdot Q_p \\
 &= (t - s \cdot e) \cdot P + \alpha P + \tilde{e} \cdot Q_p \\
 &= t \cdot P - e \cdot Q_p + \alpha \cdot P + \tilde{e} \cdot Q_p \\
 &= t \cdot P - (\tilde{e} + \beta) \cdot Q_p + \alpha P + \tilde{e} \cdot Q_p \\
 &= t \cdot P - \tilde{e} \cdot Q_p - \beta \cdot Q_p + \alpha \cdot P + \tilde{e} \cdot Q_p \\
 &= t \cdot P - \beta \cdot Q_p + \alpha \cdot P \\
 &= U - \beta \cdot Q_p + \alpha \cdot P \\
 &= \tilde{R}
 \end{aligned}$$

8. Conclusion

The security of the scheme is hardness of solving ECDLP. The primary reason for the attractiveness of ECC over systems such as RSA and DSA is that the best algorithm known for solving the underlying mathematical problem namely, the ECDLP takes fully exponential time. In contrast, sub-exponential time algorithms are known for underlying mathematical problems on which RSA and DSA are based, namely the integer factorization (IFP) and the discrete logarithm (DLP) problems. This means that the algorithms for solving the ECDLP become infeasible much more rapidly as the problem size increases more than those algorithms for the IFP and DLP. In this proposed scheme it is infeasible or adversary to derive signer's private key from all available public information. This protocol also achieves the security like requirements distinguishability, strong unforgeability, non-repudiation, strong undeniability and unlinkability.

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