Security Analysis of High Dimensional Chaotic-Based Cryptosystem via its Key Sensitivity Study

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ABSTRACT: *Stabilization conditions are proposed in this paper leading to master and slave hyperchaotic systems synchronization. They are, essentially, based on the use of a new state feedback controller and of the aggregation techniques for stability study associated to the arrow form matrix for system description. Indeed, the synchronization property is considered as the main way guaranteeing the design of one high dimensional cryptosystem. The security of this proposed communication scheme is investigated through its key sensitivity study. Numerical simulation results point up the efficiency of these contributions as well as the success of image signal transmission for the considered cryptosystem, by means of two identical 4-D chaotic Lorenz Stenflo maps as transmitter and receiver keys.*

Keywords: Cryptosystem, High Dimensional Chaotic Systems, Synchronization, Aggregation Techniques, Arrow form Matrix, State Feedback, Security Analysis, Key Sensitivity

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1. Introduction

The synchronization phenomenon is an interesting and well-known property of chaotic systems. Since its introduction by Pecora and Carrol in 1990 [1], chaos synchronization has attracted increasing interest in both theory and applications [2-4], as far as several fields are concerned. As a matter of fact, the synchronization of chaotic systems has been successfully applied in secure communication and image encryption, information processing, life science [5-12], and so on. Recently, chaos synchronization has been studied from various angles and a variety of different synchronization phenomena have been discovered, such as generalized synchronization [13-14], phase synchronization [15], lag synchronization [16], anti-synchronization [17], hybrid synchronization [18-20], observer-based synchronization [21-23], etc.

The chaos-based encryption has suggested, from several applications, new and efficient way to deal with the intractable problem of fast and highly secure image encoding [23-26].

Without a doubt, chaotic systems have many important properties, such as the sensitive dependence on initial conditions and system parameters, pseudorandom property, no periodicity and topological transitivity, etc. In such a way, most properties meet some requirements, such as diffusion and mixing, in the sense of cryptography.

As a result, chaotic cryptosystems have more useful and practical applications [23-26].

The most important contribution, in this paper, consists on the original approach leading to synchronize hyperchaotic systems and its application in the field of secure image transmission. The proposed algorithm is based on pixel scrambling where the randomness of hyperchaos is used to mix up the position of the data. Indeed, the position of the data is knotted in the order of randomness of the elements obtained from the hyperchaotic system and again rearranged back to their original position in decryption process. The same algorithm is tested with the 4-D Lorenz Stenflo chaotic systems and performance analysis is done to put in prominent position the efficiency of the chosen map as a cryptosystem.

The outline of this paper is as follows: synchronization behaviour of two identical 4-D Lorenz Stenflo systems is, firstly, studied. Then, the proposed approach dealing with the proposed nonlinear state feedback design viewpoints, relatively to the coupled master-slave Lorenz Stenflo hyperchaotic system, is developed. Afterwards, the problem of synchronization between two hyperchaotic systems is investigated through the concept of secure image transmission. Finally, the high security of the proposed cryptosystem is performed.

2. Problem Statement

Throughout the present paper, the use of the aggregation techniques [28-29] associated to the arrow form matrix [3, 20-21, 30-32], is, firstly, applied to synchronize two identical Lorenz Stenflo systems. Then, a simple secure image communication scheme based on the hyperchaotic Lorenz Stenflo system and chaos synchronization is proposed, showing that the developed approach guarantees high security and can be easily implemented.

3. New State Feedback Control Law Synchronizing Two Coupled Lorenz Stenflo Hyperchaotic Systems [26]

The stability study of the dynamical error system is considered, in this part, in order to synchronize two identical 4-D chaotic Lorenz Stenflo systems.

3.1 Error System Description

The studied hyperchaotic system is formulated by Stenflo from a low frequency short wavelength gravity wave equation. It is described by the following nonlinear differential equations [4]:

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \\ \dot{x}_4(t) \end{bmatrix} = \begin{bmatrix} -\alpha & \alpha & 0 & \gamma \\ r - x_3(t) & -1 & 0 & 0 \\ x_2(t) & 0 & -\beta & 0 \\ -1 & 0 & 0 & -\alpha \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \end{bmatrix} \tag{1}$$

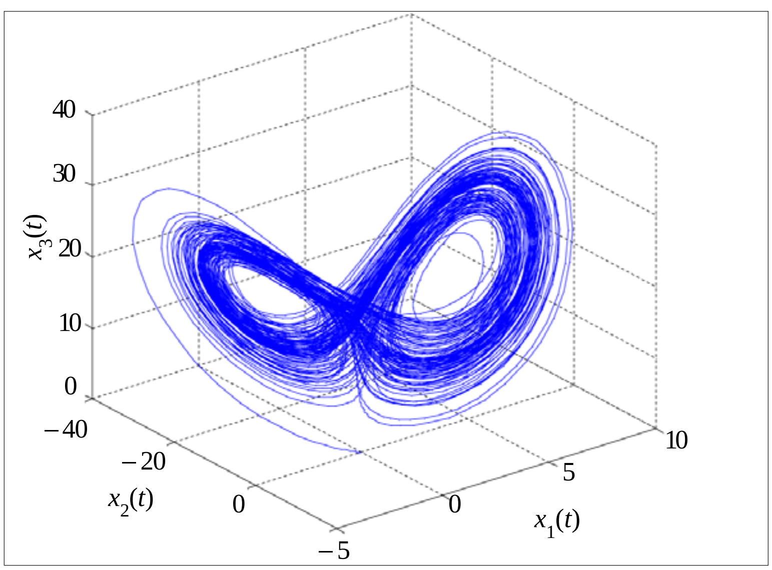

Figure 1. 3-Dimensional view of the Lorenz Stenflo attractors

where x_1, x_2, x_3 and x_4 are state variables, $x(t) = [x_1(t)\ x_2(t)\ x_3(t)\ x_4(t)]^T$, and r, α, γ and β are, respectively, the Rayleigh number, Prandtl number, rotation number and geometric parameter.

It is crucial to denote that the considered Lorenz Stenflo system exhibits a chaotic attractor, shown in Figure 1., for the following parameter values: $\alpha = 1$, $\beta = 0.7$, $\gamma = 1.5$ and $r = 26$.

Consider a master Lorenz Stenflo system given by:

$$\begin{cases} \dot{x}_{m1}(t) = \alpha(x_{m2}(t) - x_{m1}(t) + \gamma x_{m4}(t)) \\ \dot{x}_{m2}(t) = x_{m1}(t)\,(r - x_{m3}(t)) - x_{m2}(t) \\ \dot{x}_{m3}(t) = x_{m1}(t)\,x_{m2}(t) - \beta x_{m3}(t) \\ \dot{x}_{m4}(t) = -x_{m1}(t) - \alpha x_{m4}(t) \end{cases} \tag{2}$$

which drives a slave Lorenz Stenflo system described by:

$$\begin{cases} \dot{x}_{s1}(t) = \alpha(x_{s2}(t) - x_{s1}(t) + \gamma x_{s4}(t)) \\ \dot{x}_{s2}(t) = x_{s1}(t)\,(r - x_{s3}(t)) - x_{s2}(t) + u_1(t) \\ \dot{x}_{s3}(t) = x_{s1}(t)\,x_{s2}(t) - \beta x_{s3}(t) + u_2(t) \\ \dot{x}_{s4}(t) = -x_{s1}(t) - \alpha x_{s4}(t) \end{cases} \tag{3}$$

$u_i(t)$, $\forall i = 1, 2$, are the appropriate control functions to be determined.

At this stage, let the dynamical state error vector $e_S(t)$, $e_S(t) = [e_{S1}(t)\ \ e_{S2}(t)\ \ e_{S3}(t)\ \ e_{S4}(t)]^T$, be such that:

$$e_{Si}(t) = x_{si}(t) - x_{mi}(t) \quad \forall i = 1, \ldots, 4 \tag{4}$$

and leading to the error dynamics equations below:

$$\begin{cases} \dot{e}_{S1}(t) = \alpha(e_{S2}(t) - e_{S1}(t)) + \gamma e_{S4}(t) \\ \dot{e}_{S2}(t) = (r - x_{s3}(t))\,e_{S1}(t) - e_{S2}(t) - x_{m1}(t)\,e_{S3}(t) + u_1(t) \\ \dot{e}_{S3}(t) = -x_{m2}(t)\,e_{S1}(t) + x_{s1}(t)\,e_{S2}(t) - \beta e_{S3}(t) + u_2(t) \\ \dot{e}_{S4}(t) = -e_{S1}(t) - \alpha e_{S4}(t) \end{cases} \tag{5}$$

which can be rewritten in the following form:

$$\begin{bmatrix} \dot{e}_{S1}(t) \\ \dot{e}_{S2}(t) \\ \dot{e}_{S3}(t) \\ \dot{e}_{S4}(t) \end{bmatrix} = \begin{bmatrix} \alpha(e_{S2}(t) - e_{S1}(t) + \gamma e_{S4}(t)) \\ (r - x_{s3}(t))\,e_{S1}(t) - e_{S2}(t) - x_{m1}(t)\,e_{S3}(t) \\ -x_{m2}(t)\,e_{S1}(t) + x_{s1}(t)\,e_{S2}(t) - \beta e_{S3}(t) \\ -e_{S1}(t) - \alpha e_{S4}(t) \end{bmatrix} + Bu(t) \tag{6}$$

with:

$$B = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \tag{7}$$

The synchronization of the coupled master-slave Lorenz Stenflo hyperchaotic dynamical system needs the stabilization of the resulting error system (5), which can be achieved when the proposed following nonlinear active feedback control laws, in the form:

$$u_i(t) = -\sum_{j=1}^{4} k_{ij}(.)\, e_{Sj}(t)\ \forall i = 1, 2 \tag{8}$$

are applied; $K(.) = \{K_{ij}(.)\}$, $i = 1, 2$ and $j = 1, ..., 4$ is the instantaneous control gain matrix.

It comes:

$$\begin{cases} \dot{e}_{S1}(t) = \alpha(e_{S2}(t) - e_{S1}(t) + \gamma e_{S4}(t)) \\ \dot{e}_{S2}(t) = (r - x_{s3}(t) - k_{11}(.))\, e_{S1}(t) - (1 + k_{12}(.))\, e_{S2}(t) - (x_{m1}(t) + k_{12}(.))\, e_{S3}(t) - k_{14}(.)\, e_{S4}(t) \\ \dot{e}_{S3}(t) = -(x_{m2}(t) + k_{21}(.))\, e_{S1}(t) + (x_{s1}(t) - k_{22}(.))\, e_{S2}(t) - (\beta + k_{23}(.))\, e_{S3}(t) - k_{24}(.)\, e_{S4}(t) \\ \dot{e}_{S4}(t) = -e_{S1}(t) - \alpha\, e_{S4}(t) \end{cases} \tag{9}$$

Therefore, the controlled dynamical 4-D error system can be described by:

$$\dot{e}_S(t) = A_c(.)\, e_S(t) \tag{10}$$

with:

$$A_c(.) = \begin{bmatrix} -\alpha & \alpha & 0 & \gamma \\ \begin{pmatrix} r - x_3(t) \\ -k_{11}(.) \end{pmatrix} & -\begin{pmatrix} 1 \\ +k_{12}(.) \end{pmatrix} & -\begin{pmatrix} x_{m3}(t) \\ +k_{13}(.) \end{pmatrix} & -k_{14}(.) \\ -\begin{pmatrix} x_{m2}(t) \\ -k_{12}(.) \end{pmatrix} & \begin{pmatrix} x_{s1}(t) \\ -k_{22}(.) \end{pmatrix} & -\begin{pmatrix} \beta \\ +k_{23}(.) \end{pmatrix} & -k_{24}(.) \\ -1 & 0 & 0 & -\alpha \end{bmatrix} \tag{11}$$

So, the nonlinear matrix elements $a_{c_{ij}}(.)$, $A_c(.) = \{a_{c_{ij}}(.)\}$, $\forall i, j = 1, ..., 4$, depend on system's parameters, control gains, slave and master state variables.

Now, in order to study the stability of the closed-loop error system (9), our task is restricted to choose the control gains in order to simplify the complexity of the dynamical error system, in one hand, and to make efficient the following proposed stability method, in the other hand.

3.2 Proposed Sufficient Stability Conditions

When the obtained error system (5) is stabilized by the state feedback control law , u defined by (8), the error will converge to zero as ; $t \to +\infty$; then, the master and slave hyperchaotic Lorenz Stenflo systems (2) and (3) will be globally synchronized.

Taking into account the importance of arrow form choice for instantaneous characteristic matrices, to obtain useful sufficient stability conditions for nonlinear systems, as shown in [3-4, 17-18, 21-23, 26-30], let design a suitable state feedback controller, so that the closed-loop error system (10) being described by the following nonlinear differential equations form:

$$\begin{cases} \dot{e}_{S1}(t) = a_{c_{11}}(.)\, e_{S1}(t) + a_{c_{12}}(.)\, e_{S2}(t) + a_{c_{13}}(.)\, e_{S3}(t) + a_{c_{14}}(.)\, e_{S4}(t) \\ \dot{e}_{S2}(t) = a_{c_{21}}(.)\, e_{S1}(t) + a_{c_{22}}(.)\, e_{S2}(t) \\ \dot{e}_{S3}(t) = a_{c_{31}}(.)\, e_{S1}(t) + a_{c_{33}}(.)\, e_{S3}(t) \\ \dot{e}_{S4}(t) = a_{c_{41}}(.)\, e_{S1}(t) + a_{c_{44}}(.)\, e_{S4}(t) \end{cases} \tag{12}$$

which leads to an instantaneous characteristic matrix under the arrow form, such that non zero elements are located in its main diagonal, its first row and its first column.

The application of the aggregation techniques [26-27], for the stability study, associated to the arrow form matrix [3-4,17-18,21-

23, 28-30], for the system description, leads to the following theorem.

Theorem. *The error system* (5) *is stabilized by the proposed nonlinear state feedback control law* (8), *if the matrix* $A_c(.)$, *defined by* (11), *is under the arrow form and such that*:

i. the nonlinear elements are isolated in either one row or one column of the matrix $A_c(.)$,

ii. the diagonal elements, $a_{c_{ii}}(.)$, *of the matrix* $A_c(.)$ *are such that*:

$$a_{c_{ii}}(.) < 0 \ \forall i = 2, 3, 4 \tag{13}$$

iii. there exist $\varepsilon > 0$ *for which*:

$$a_{c_{11}}(.) - \sum_{j=1}^{4} (|a_{c_{1i}}(.)\, a_{c_{i1}}(.)|)\, a_{c_{ii}}^{-1}(.) \leq -\varepsilon \tag{14}$$

Proof. The overvaluing system $M(A_c(.))$ associated to the vectorial norm $p(z) = [|z_1| \ |z_2| \ |z_3| \ |z_4|]^T$ for which $[z_1 \ z_2 \ z_3 \ z_4]^T$, is defined, in this case, by the following system of differential equations:

$$z(t) = M(A_c(.))\, z(t) \tag{15}$$

such that the elements $m_{ij}(.)$ of $M(A_c(.))$ are deduced from the ones of the matrix $A_c(.)$ by substituting the off-diagonal elements by their absolute values, such that:

$$\begin{cases} m_{ii}(.) = a_{c_{ii}}(.) \ \ \forall i = 1, ..., 4 \\ m_{ij}(.) = |\, a_{c_{ij}}(.)\, | \ \ \forall i, j = 1, ..., 4, \ i \neq j \end{cases} \tag{16}$$

The error system (5) is then stabilized by the proposed control law (8), if the matrix $M(A_c(.))$ is the opposite of an $M-$ matrix [27], or if, by application of the aggregation techniques [26], the sufficient stability conditions, for $\varepsilon > 0$, are formulated in the subsequent manner:

$$\begin{cases} a_{c_{ii}}(.) \leq -\varepsilon \ \ \forall i = 2, 3, 4 \\ (-1)^4 = \det(M(A_c(.))) \geq \varepsilon \end{cases} \tag{17}$$

The development of the first member of the last inequality (17):

$$(-1)^4 = \det(M(A_c(.))) = (-1)\,(a_{c_{11}}(.) - \sum_{j=1}^{4} (|a_{c_{1i}}(.)\, a_{c_{i1}}(.)|)\, a_{c_{ii}}^{-1}(.))\,(-1)^3 \prod_{j=2}^{4} a_{c_{jj}}(.) \tag{18}$$

achieves easily the proof of the above-mentioned theorem.

3.3 Application of the Proposed Stability Conditions to Synchronize the Coupled Master-Slave 4-D Lorenz Stenflo Chaotic System

The concept of chaos synchronization emerged much later, not until the gradual realization of the usefulness of chaos by scientists and engineers. Synchrony is the simplest effect of coupled identical systems: two identical systems display the same dynamical pattern in their common phase space [4]. For that reason, the developed state feedback control technique is applied, in this subsection, to achieve chaos synchronization of two identical 4-D Lorenz Stenflo systems.

In fact, the characterization of the closed-loop error system (10) by an arrow form matrix is easily checked by choosing the correction parameters, in the instantaneous characteristic matrix (11), $k_{13}(.)$, $k_{14}(.)$, $k_{22}(.)$ and $k_{24}(.)$ such that:

$$\begin{cases} k_{13}(.) = -x_{m1}(t) \\ k_{14} = k_{24} = 0 \\ k_{22}(.) = x_{s1}(t) \end{cases} \tag{19}$$

To satisfy the assumption (*i*) as well as the constraints (13) of the above-mentioned theorem, the two gain parameters k_{12} and k_{23}

are chosen linear, such that:

$$\begin{cases} k_{12} > -1 \\ k_{23} > -\beta \end{cases} \tag{20}$$

Then, it remains only to fulfil the condition (14), expressed in this case as follows:

$$\left(-\alpha - \left(\begin{array}{c} |\,\alpha\,(r - x_{s3}(t) - k_{11}(.))\,|\,(-1 - k_{12})^{-1} \\ +\,|\,\gamma\,|\,(-\alpha)^{-1} \end{array}\right)\right) < 0 \tag{21}$$

to guarantee the asymptotic stability of the dynamical error system (9).

Hence, $\forall\, k_{21}(.)$ and for $\alpha > 0$, one possible choice of the other gain parameters is given through the instantaneous control gain matrix $K(.)$ as:

$$K(.) = \begin{bmatrix} -x_{s3}(t) & 1 & -x_{m1}(t) & 0 \\ 0 & x_{s1}(t) & \beta & 0 \end{bmatrix} \tag{22}$$

This choice ensures that the synchronization between master and slave systems is achieved. This is also confirmed by the exponential convergence of the synchronization quality defined by the error propagation on the error states:

$$\|e_S(t)\| = \sqrt{e_{S1}^2(t) + e_{S2}^2(t) + e_{S3}^2(t) + e_{S4}^2(t)} \tag{23}$$

Figure 2 and Figure 3 show the error dynamics in the uncontrolled state, while Figure 4. illustrates the error dynamics when controller is switched on.

Obviously, the two hyperchaotic Lorenz Stenflo systems evolve in the same direction as well as the same amplitude; that's to say, they are globally asymptotically synchronized by means of the proposed nonlinear state feedback controller.

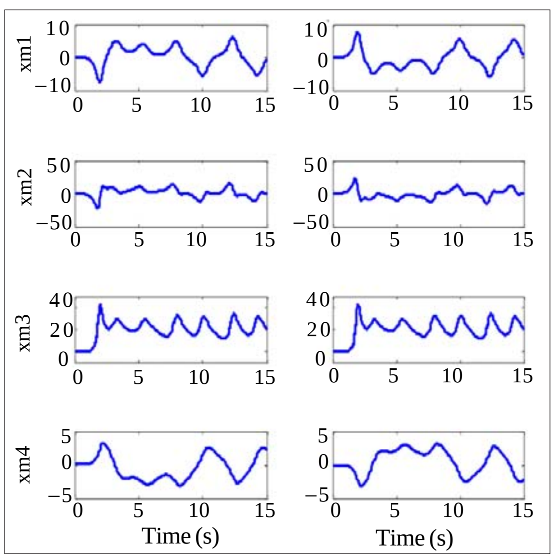

Figure 2. Evolutions of master and slave states of the Lorenz Stenflo system when controller is switched off

4. Feedback-Based Synchronization of 4-D Chaotic Systems for Secure Image Transmission – Basic Idea

In this section, the problem of feedback-based synchronization between two identical hyperchaotic systems is applied to a new chaos-based image cryptosystem, to illustrate the feasibility of the theoretical proposed approach. The input of the considered

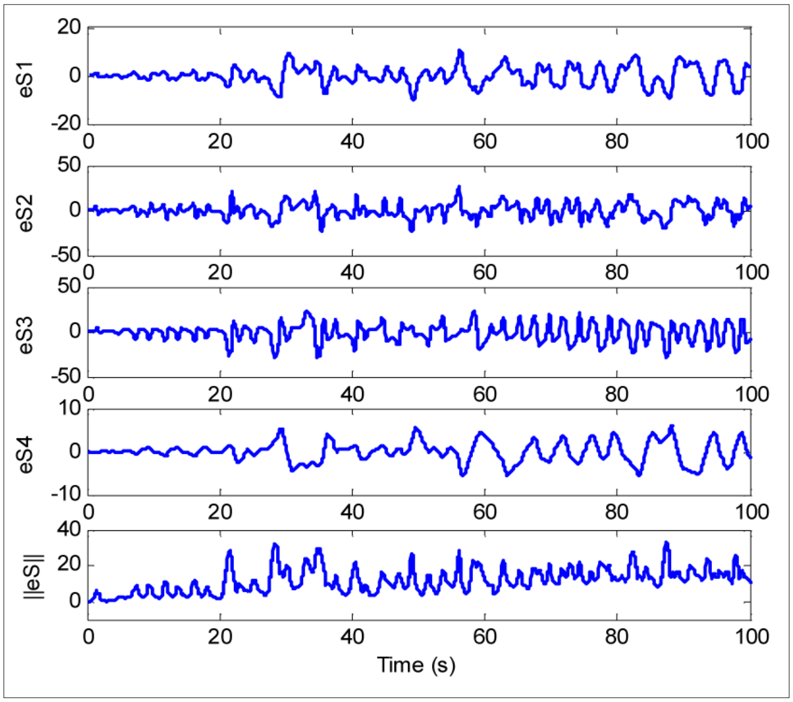

Figure 3. Error dynamics between two coupled Lorenz Stenflo systems when controller is deactivated

cryptosystem is the plain image which is to be encrypted. The cryptosystem consists of two stages. The first step is the confusion stage and the second one is the diffusion stage, Figure 5 (a) and Figure 5 (b). Among several hyperchaotic dynamic systems, the Lorenz Stenflo one is selected and it is applied to the digital color image encoding. The second step of the masking process is to encrypt the shuffled image by changing its pixel values based on the 4-D Lorenz Stenflo chaotic system. This is referred to as the diffusion stage. The resulting image is the cipher image.

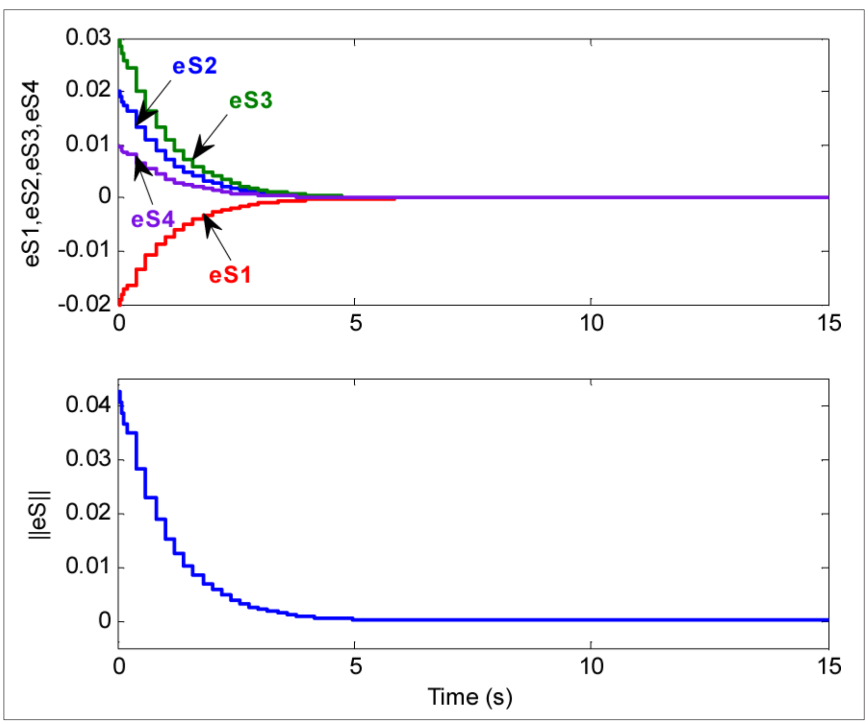

Figure 4. Error dynamics of the coupled master-slave Lorenz Stenflo system when controller is switched on

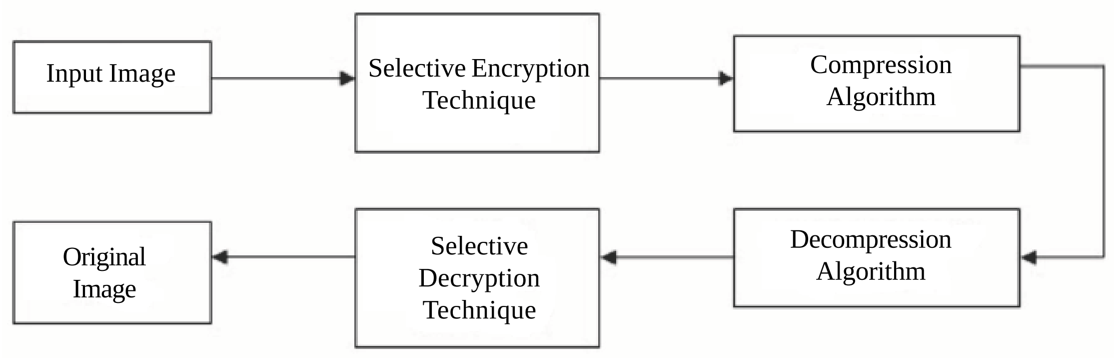

Figure 5 (a). Selective encryption procedure

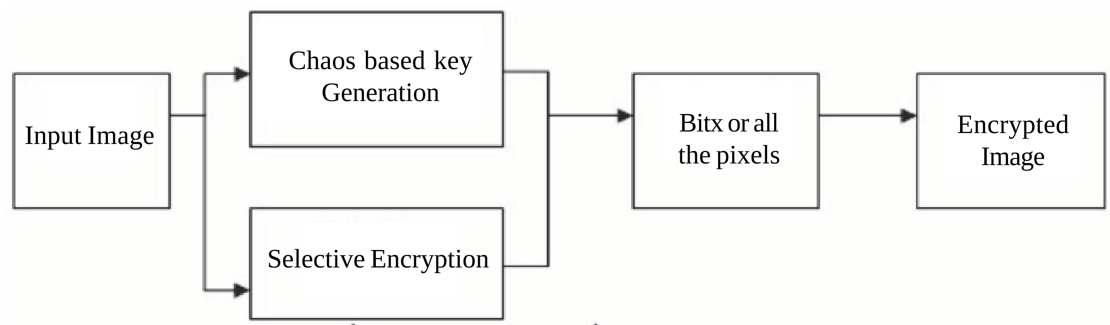

Figure 5 (b). Encryption proposed technique

Systems parameters of the master Lorenz Stenflo hyperchaotic system, painstaking as transmitter system, are chosen to further enhance the complexity of the considered cryptosystem and thereby improving the security of the image diffusion process.

Firstly, we form a vector with three layers in the RGB format containing the image colors. After that, the chaotic signal of the

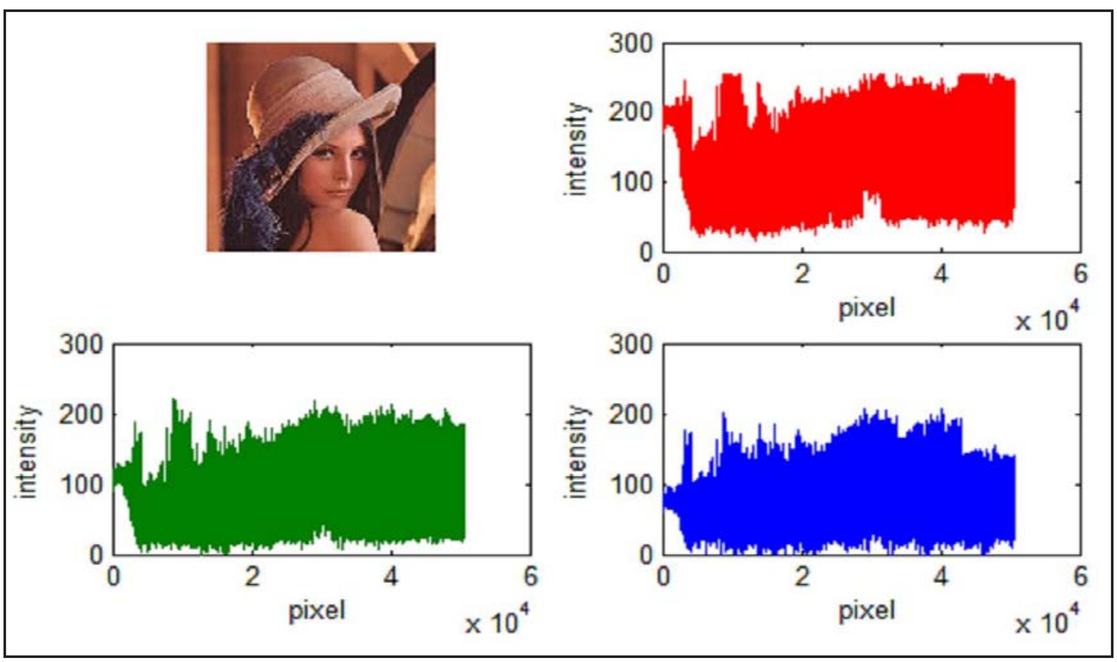

Figure 6 (a). Original Lena image and its actual RGB intensity plots

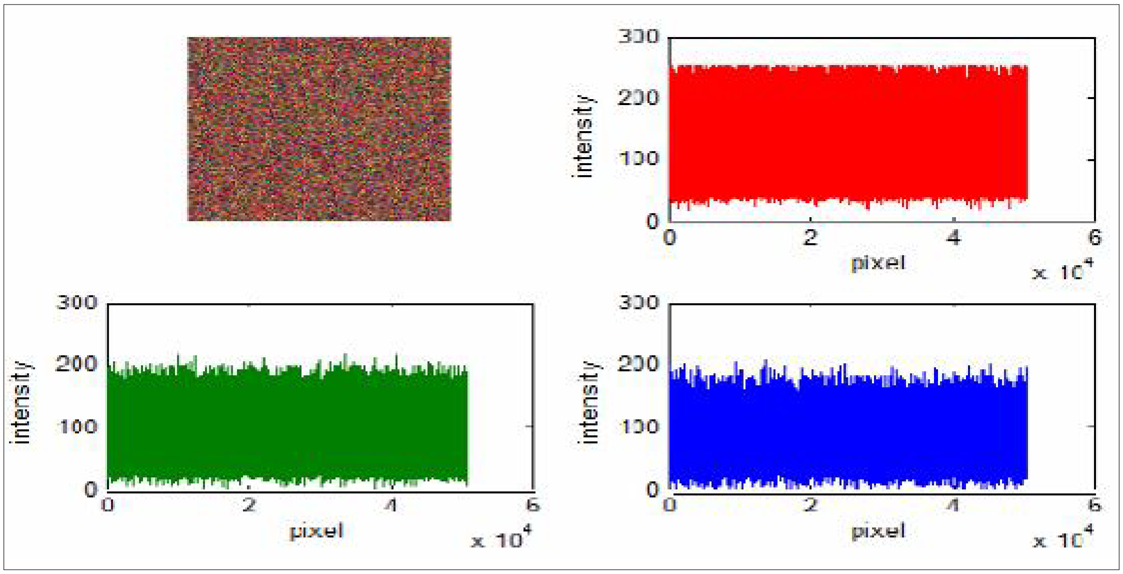

Figure 6 (b). Encoded Lena image and its RGB intensity plots after encryption

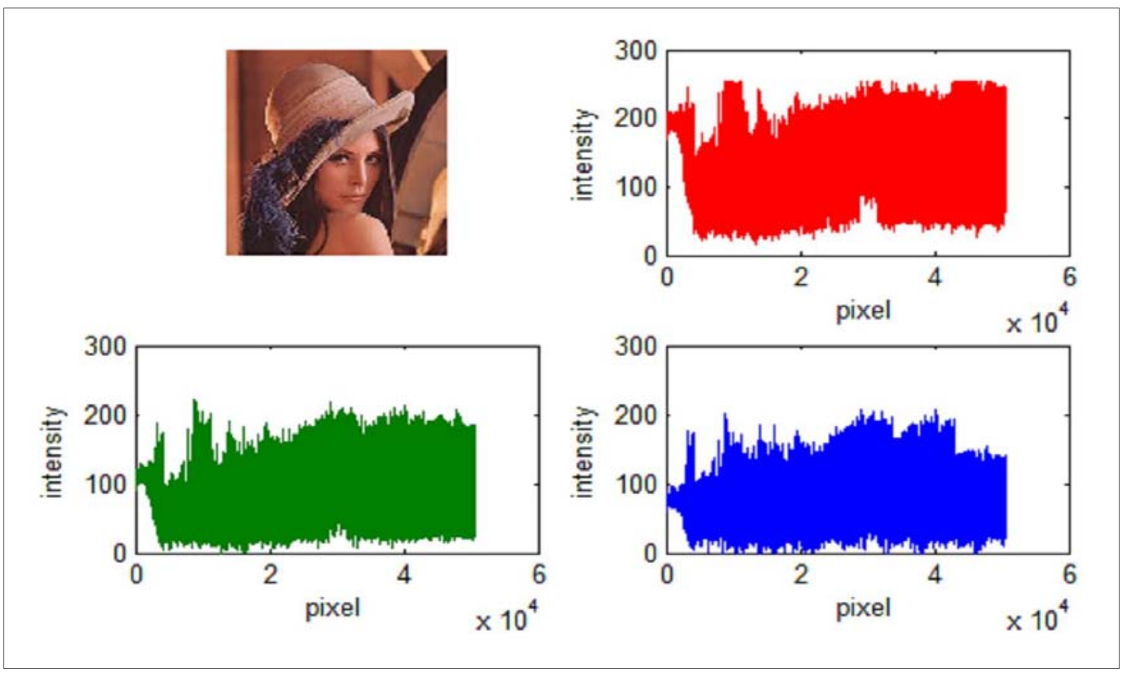

Figure 6 (c). Decoded Lena image and its RGB intensity plots after decryption

master transmitter system is added to the image, to further enhance the complexity of the considered cryptosystem and thereby improving the security of the image transmission process. Subsequently and thanks to the synchronization property, the image is successfully recovered through the subtraction between the encrypted image and the slave chaotic signal. At last, the three layers are joined in order to form the color image.

Experimental results of the proposed image encryption scheme are shown in this section using Lena image, Figure 6 (a). The image encryption is based on the Lorenz Stenflo hyperchaotic system and generated elements have been stored within the 4-D chaotic matrix.

The pixel position permuted of Lena image, after applying Lorenz Stenflo hyperchaotic system, is obtained and shown, as the diffused image, in Figure 6 (b).

In the stage of decryption, the parameters of the slave hyperchaotic system, assumed to be as the receiver system, are set to their real values, thanks to the successful hyperchaotic synchronization property obtained between master and slave systems, which assures the perfect depiction of the original transmitted Lena image, Figure 6 (c). Figure 6 (a), 6 (b). and 6 (c). show the original image, pixel scrambled encrypted image and decrypted image with their RGB levels.

5. Performance Analysis

To measure the performance of the proposed encryption algorithm, we have calculated the correlation coefficient, in the first part of this Section. Moreover, to prove that the decryption process is possible only with one specific key, simulations are done with slightly varied keys, in the second part of it.

5.1. Correlation Coefficient

The correlation coefficient *r* is the measure of extent and direction of linear combination of two random variables. If two variables are closely related, the correlation coefficient is close to the value 1. But, if the coefficient is close to 0, the two variables are not related.

The correlation coefficient *r* can be calculated by the following formula:

$$r = \frac{\sum_i (X_i - X_m)(Y_i - Y_m)}{\sqrt{\sum_i (X_i - X_m)^2}\sqrt{\sum_i (Y_i - Y_m)^2}} \tag{24}$$

where X_i, X_m, Y_i and Y_m represent, respectively, the pixel intensity of the original image, the mean value of the original image intensity, the pixel intensity of the encrypted image and the mean value of the encrypted image intensity.

The correlation values are calculated for both original and encrypted images with Lorenz Stenflo map and shown in the Table below, from which, it is clearly illustrated that the obtained correlation coefficient values are very near to zero with the considered map.

With this we can conclude that for this encryption algorithm, based on the Lorenz Stenflo map, the correlation coefficient values are very low and near to zero. This proves that the algorithm leads to a satisfactory secured encryption process.

Figure 7 (a). Original Lena image

Figure 7 (b). Decrypted Lena image using original key $\beta = 0.7000$

Figure 7 (c). Decrypted Lena image with slightly different key $\beta = 0.7001$

Image	Correlation of R	Correlation of G	Correlation of B
Lena.jpg	$1.327e^{-005}$	$8.7905e^{-006}$	$7.7947e^{-006}$

Table. Correlation coefficient values with Lorenz Stenflo map

5.2. Key Sensitivity

A good Encryption algorithm should be very much sensitive to the key. A slight variation in the key should result in totally different image in the rebuilding process at the destination end.

The experiment is carried using the Lorenz Stenflo map with its actual initial condition β = 0.7000. Figure 7 (a). and Figure 7 (b), considered as original key, and slightly different value β = 0. 7001 . From the obtained results, it is clear that a slight variation, say 0.0001, results in totally different decrypted image as shown in Figure 7 (c).

It is worth noting that in our algorithm we have used one single Lorenz Stenflo map, thus key space is less in comparison to works where three chaos maps are used in order to achieve encryption, bringing into play a key space three times more that the necessary key space of our proposed algorithm. Consequently, memory requirement is fewer in this proposed cryptosystem and better for the applications like wireless communications. It is also noticeable that, in this algorithm, the initial conditions assumed to generate the chaotic map acts as the key. Furthermore, the security is compromised even without precise knowledge of the hyperchaotic systems parameters used.

6. Conclusion

In this paper, the synchronization is achieved for two identical coupled 4-D chaotic systems. Under some structural assumptions of the master system and based on aggregation techniques associated to the arrow form matrix, a state feedback-based slave system is designed to assure that the property of synchronization is fruitfully reached.

Additionally, the Lorenz Stenflo hyperchaotic system is taken as an example to demonstrate the effectiveness of this proposed synchronization scheme and the simulation results show that image encoding and decoding are so good and the considered cryptosystem has high-quality of security. The obtained values clearly signify the importance of this algorithm in the application of image encryption, especially for wireless communications.

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