Synchronization Control of Delayed Cohen-Grossberg Neural Networks with Diffusion Terms

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ABSTRACT: *Cohen-Grossberg neural networks which include Hopfield neural networks and Cellular neural networks as special cases and have important applications in many fields are researched by many authors. There is few results on synchronization control of delayed Cohen-Grossberg neural networks. In this paper, the synchronization control of delayed Cohen- Grossberg neural networks with diffusion terms is discussed by constructing suitable Lyapunov functional and using inequality techniques. Instead of the linear control term, we consider a class of nonlinear control term. Some sufficient conditions are given to ensure the synchronization control of the drive-response delayed Cohen-Grossberg neural networks with reaction- diffusion terms. A numerical example is presented to verify that the results in the paper are correct.*

Keywords: Synchronization Control, Drive-Response System, Lyapunov Functional, Reaction-Diffusion term

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1. Introduction

Cohen-Grossberg neural networks(CGNNs) were first introduced by Cohen and Grossberg in [7]. The class of networks include the traditional neural networks such as Hopfield neural networks(HNN) and Cellular neural networks(CNN) as special cases and have many good applications in parallel computation, as- sociative memory and optimization problems and so on. Because these applications heavily depend on the dynamical behaviors of the neural networks and the analysis of the dynamical behaviors is the necessary step to design of neural networks, many results on the dynamical behaviors of the networks can be found in [8]-[9].

Since Pecora and Carroll introduced chaos synchronization by proposing the drive-response concept in 1990s[1]-[2], the synchronization of coupled chaos systems and chaos neural networks has been received considerable attention due to its applications in creating secure communication system[3]-[6].

Recently, chaotic behaviors produced by neural networks have also been investigated. Several different approaches including some conventional linear control techniques and advanced nonlinear control schemes to achieve synchronization of neural networks have been proposed[10]-[11]. However, since time delays may led to bifurcation, oscillation, divergence or instability which may be harmful to a system, the study of neural dynamics with consideration of the delayed problem becomes extremely important to manufacture high quality neural networks. Therefore, the models with time-varying delays and continuous distributed delays are more appropriate to the synchronization of neural networks. Synchronization criteria for coupled delayed neural networks with constant time delays and time-varying delays have been proposed in Refs [12]-[13] based on the Lyapunov functional method.

Both in biological and artifcial neural networks, diffusion effect cannot be avoided when electrons are moving in asymmetric electromagnetic field, thus we must consider that the activations vary in space as well as in time. To the best of our knowledge, few authors have considered synchronization control of diffusion CGNN with delays. But it is important in the-ories and applications. In this paper, we will investigate exponential synchronization of a class of reaction-diffusion CGNN with time-varying delays.

This paper is organized as follows. In section 2, model description and main result are given. In section 3, by employing inequality techniques and constructing suitable Lyapunov functional, some sufficient conditions are obtained to ensure the synchronization control of a class of reaction-diffusion CGNN. Section 4 presents a numerical example to verify that the results in this paper are true. Section 5 is the conclusions of the paper.

2. Model description and main result

Consider the following delayed reaction-diffusion Cohen-Grossberg neural networks with Neumann boundary conditions:

$$\begin{aligned}
&\frac{\partial u_i(x,t)}{\partial t}=\sum_{k=1}^{m}\frac{\partial^2 u_i(x,t)}{\partial x_k^2}-a_i(u_i(x,t)).\left[b_i(u_i(x,t))-\sum_{j=1}^{n}c_{ij}g_j(u_j(x,t))-\sum_{j=1}^{n}k_{ij}g_j\left(u_j\left(x,t\text{-}\theta_{ij}(t)\right)\right)-I_i\right],t>0\\
&u_i(x,t)=\varphi_i(x,t),-\tau<t\le 0,x\in\Omega,\frac{\partial u_i(x,t)}{\partial v}=0,\quad \left(x\in\partial\Omega,t>0\right).
\end{aligned}\tag{1}$$

Where $i = 1,..., n$. n is the number of neurons in the networks. Ω is a bounded open domain in R^m with smooth boundary $\partial\Omega$ and $mes\Omega > 0$ denotes the measure of Ω. $\frac{\partial}{\partial v}$ denotes the differentiation in the direction of the outward unit normal to $\partial\Omega$. $u(x,t) = (u_1(x,t),...,u_n(x,t))^T$, $u_i(x,t)$ denotes the state of the ith neural unit at time t and in space $x \in \Omega$. a_i and b_i represent an amplification function and an appropriately behaved function, respectively. c_{ij} and k_{ij} are the coefficients without and with time-varying delays, respectively. $\theta_{ij}(t)$ corresponds to the transmission delays along the axon of the jth neuron form the ith neuron at time t and satisfies

$$0 \le \theta_{ij}(t) \le \tau, \dot{\theta}_{ij}(t) < \rho < 1$$

where $\tau > 0$ and $\rho < 1$ are given constants. $g_i(.)$ is the activation function with $g_i(0) = 0$, $(i = 1,..., n)$. I_i denotes input of the ith neuron. $\varphi(x,t) = (\varphi_1(x,t),...,\varphi_n(x,t))^T$ (where $\varphi_i(x, t)$ is given smooth function defined on Ω x $(-\tau, 0)$ with the following norm

$$\|\varphi\|_2=\sqrt{\sum_{i=1}^{n}\int_\Omega|\varphi_i(x,.)|_\tau^2\,dx}$$

where $|\varphi_i(x,.)|_\tau = \sup_{-\tau<s\le 0}|\varphi_i(x,s)|$.

Throughout the paper, we always assume that system (1) has a smooth solution $u(t, x)$ with the norm

$$\|u(.,t)\|_2=\sqrt{\sum_{i=1}^{n}\int_\Omega|u_i(x,t)|^2\,dx}, \forall t\in(0,+\infty)$$

Let $v = (v_1,..., v_n)^T$ be the control input vector and v_i stands for the external control input that will be appropriately designed for an certain control objective. In this paper, instead of that the control input vector $\tilde{v} = (\tilde{v}_1, ..., \tilde{v}_n)^T$ is assumed to take as the follows.

$$(\tilde{v}_1, ..., \tilde{v}_n)^T = \tilde{M}(u_1(x,t) - \tilde{u}_1(x,t), ..., u_n(x,t) - \tilde{u}_n(x,t))^T$$

where $\tilde{M} = (\tilde{M}_{ij})_{nxn}$ is the controller gain matrix.

We consider the nonlinear control input vector $v = (v_1, ..., v_n)^T$ which satisfies the follows.

$$(v_1, ..., v_n)^T \le M(|u_1(x,t) - \tilde{u}_1(x,t), ..., u_n(x,t) - \tilde{u}_n(x,t)|)^T \tag{2}$$

where $M = (M_{ij})_{nxn}$ is also called the controller gain matrix and will be appropriately chosen for the syn- chronization control in both drive system and response system.

$\tilde{u}_i(x,t)$, $(i = 1, ..., n)$ satisfy the following response neural networks:

$$\begin{aligned} &\frac{\partial \tilde{u}_i(x,t)}{\partial t} = \sum_{k=1}^{m} \frac{\partial^2 \tilde{u}_i(x,t)}{\partial x_k^2} - a_i(\tilde{u}_i(x,t)).[b_i(\tilde{u}_i(x,t)) - \sum_{j=1}^{n} c_{ij} g_j(\tilde{u}_j(x,t)) - \sum_{j=1}^{n} k_{ij} g_j(\tilde{u}_j(x,t-\theta_{ij}(t))) - I_i] + v_i(x,t), t > 0 \\ &\tilde{u}_i(x,t) = \psi_i(x,t), -\tau < t \le 0, x \in \Omega, \frac{\partial \tilde{u}_i(x,t)}{\partial v} = 0, (x \in \partial\Omega, t > 0) \end{aligned} \tag{3}$$

Where $i = 1, ..., n$. $\psi = (\psi_1, ..., \psi_n)^T$, $\psi_i(x, s)$ $(i = 1, ..., n)$ are bounded smooth functions defined on Ω x $(-\tau, 0)$.

In this paper, we always assume that:

($H1$) There exist positive constants m_i and M_i such that $0 < m_i \le a_i(u_i) \le M_i$, $i = 1, ..., n$. Moreover, for any $i = 1, ..., n$, $a_i(.)$ is differentiable and there exists positive constant $\Gamma_i > 0$ such that $\Gamma_i \overset{\Delta}{=} \sup_{x \in R}\{\dot{a}_i(x)\} > 0$.

($H2$) $b_i(.)$ is differentiable and there exists a positive $G_i > 0$ such that $0 \le b_i \le G_i$ for all $i = 1, ..., n$.
Furthermore, $\dot{b}_i(.)$ is the derivative of $b_i(.)$ and $\alpha_i \overset{\Delta}{=} \inf_{x \in R}\{bi(x)\} > 0$.

($H3$) There exist positive constants $\beta_i > 0$ and $\Lambda_i > 0$ for any $i = 1, ..., n$ such that

$$\|g_i\| = \sup|g_i(u)| \le \Lambda_i,\ |g_i(x_i) - g_i(y_i)| \le \beta_i |x_i - y_i|, \forall x_i, y_i \in R.$$

($H4$) There exist constants $\xi_{ij}, \gamma_{ij}, \eta_{ij}$ $(i, j = 1, ..., n)$ such that for all $i = 1, ..., n$, it holds

$$\begin{aligned} &-2\alpha_i m_i + \Gamma_i \left[G_i + |I_i| + \sum_{j=1}^{n} (|k_{ij}| \Lambda_j) \right] + \sum_{j=1}^{n} \left[\left(\beta_j |c_{ij}|\right)^{2\xi_{ij}} + \left(\beta_j |k_{ij}|\right)^{2\gamma_{ij}} \right] \\ &+ \sum_{j=1}^{n} \left[\left(\beta_i |c_{ji}|\right)^{2-2\xi_{ji}} + \frac{1}{1-\rho} \left(\beta_i |k_{ji}|\right)^{2-2\gamma_{ji}} \right] + \sum_{j=1}^{n} \left[|M_{ij}|^{2\eta_{ij}} + |M_{ji}|^{2-2\eta_{ji}} \right] < 0 \end{aligned}$$

Definition 1. *The drive system (1) and the response system (2) are said to be controlled syn chronously if there exist positive constants* $\gamma > 0$ *and* $\varepsilon > 0$ *such that*

$$\|u(., t) - \tilde{u}(., t)\|_2 \le \gamma \|\varphi - \psi\|_2 e^{-\varepsilon t}, \forall t \ge 0,$$

where the constant ε *is said to be the rate of synchronization control.*

The main result is the following theorem:

Theorem 1. *If* $(H1)$ - $(H4)$ *hold, then the drive-response neural networks* (1) *and* (3) *are controlled synchronously.*

Remark 1. *Let* $\xi_{ij} = \gamma_{ij} = \eta_{ij} = \frac{1}{2}$ *, then* $(H4)$ *can be changed into the following* $(H4)_1$*:*

$$-2\alpha_i m_i + \Gamma_i\left[G_i + |I_i| + \sum_{j=1}^{n}\left(|k_{ij}|\Lambda_j\right)\right] + \sum_{j=1}^{n}\left[\left(\beta_j |c_{ij}|\right) + \left(\beta_j |k_{ij}|\right)\right]$$
$$+\sum_{j=1}^{n}\left[\left(\beta_j |c_{ji}|\right) + \frac{1}{1-\rho}\left(\beta_i |k_{ji}|\right)\right] + \sum_{j=1}^{n}\left[|M_{ij}| + |M_{ji}|\right] < 0$$

Then we have the following corollary:

Corollary 1. *If* $(H1)$-$(H3)$ *hold, furthermore,* $(H4)_1$ *holds, then the drive-response neural networks* (1) *and* (3) *are controlled synchronously.*

3. The synchronization of the drive and response neural networks

Let us define the synchronization error signal $\varepsilon_i(x, t) \overset{\Delta}{=} u_i(x, t) - \tilde{u}_i(x, t)$, where $u_i(x, t)$ and $\tilde{u}_i(x, t)$ are the ith state variable of the drive and response neural networks, respectively. Therefore, the dynamics error between (1) and (3) can be expressed as the follows.

$$\frac{\partial \varepsilon_i(x,t)}{\partial t} = \sum_{k=1}^{m}\frac{\partial^2 \varepsilon_i(x,t)}{\partial x_k^2} - a_i\left(u_i(x,t)\right).[b_i(u_i(x,t)) - \sum_{j=1}^{n} c_{ij} g_j(u_j(x,t)) - \sum_{j=1}^{n} k_{ij} g_j(u_j(x,t-\theta_{ij}(t)) - I_i)]$$
$$+a_i(\tilde{u}_i(x,t)).[b_i(\tilde{u}_i(x,t)) - \sum_{j=1}^{n} c_{ij} g_j(\tilde{u}_j(x,t)) - \sum_{j=1}^{n} k_{ij} g_j(\tilde{u}_j(x,t-\theta_{ij}(t)) - I_i)]$$
$$-v_i(x,t), \varepsilon_i(x,t) = \varphi_i(x,t) - \psi_i(x,t), \quad -\tau < t \le 0, \frac{\partial \varepsilon_i(x,t)}{\partial v} = 0, (x \in \partial\Omega)$$

Where $i = 1,..., n$ and the control input vector $v(x, t) = (v_1(x, t), ..., v_n(x, t))^T$ which satisfies the condition (2) has been mentioned above.

Then we have

$$\frac{\partial \varepsilon_i(x,t)}{\partial t} = \sum_{k=1}^{m}\frac{\partial^2 \varepsilon_i(x,t)}{\partial x_k^2} - \left(a_i\left(u_i(x,t)\right) - a_i(\tilde{u}_i(x,t))\right).\left[b_i(u_i(x,t)) - \sum_{j=1}^{n} c_{ij} g_j(u_j(x,t)) - \sum_{j=1}^{n} k_{ij} g_j(u_j(x,t-\theta_{ij}(t)) - I_i)\right]$$
$$-a_i(\tilde{u}_i(x,t)).\left[b_i(u_i(x,t)) - b_i(\tilde{u}_i(x,t)) - \sum_{j=1}^{n} c_{ij}\left(g_j(u_j(x,t)) - g_j(\tilde{u}_j(x,t))\right) - \sum_{j=1}^{n} k_{ij}\left(g_j(u_j(x,t-\theta_{ij}(t))) - g_j(\tilde{u}_j(x,t-\theta_{ij}(t)))\right)\right]$$
$$-v_i(x,t),\ \varepsilon_i(x,t) = \varphi_i(x,t) - \psi_i(x,t), \frac{\partial \varepsilon_i(x,t)}{\partial v} = 0, (x \in \partial\Omega) \tag{4}$$

By $(H4)$, there exists a sufficiently small positive constant $\lambda < \min_i\{a_i m_i\}$ such that

$$W_i = \lambda - \alpha_i m_i + \frac{1}{2}\left\{\Gamma_i\left[G_i + |I_i| + \sum_{j=1}^{n}\left(|k_{ij}|\Lambda_j\right)\right] + \sum_{j=1}^{n}\left[\left(\beta_j |c_{ij}|\right)^{2\xi_{ij}} + \left(\beta_j |k_{ij}|\right)^{2\gamma_{ij}}\right]\right.$$
$$\left. + \sum_{j=1}^{n}\left[\left(\beta_j |c_{ji}|\right)^{2-2\xi_{ji}} + \frac{e^{2\lambda\tau}}{1-\rho}\left(\beta_j |k_{ji}|\right)^{2-2\gamma_{ji}}\right] + \sum_{j=1}^{n}\left[|M_{ij}|^{2\eta_{ij}} + |M_{ji}|^{2-2\eta_{ji}}\right]\right\} \le 0 \tag{5}$$

Taking Liapunov funcional as follows:

$$V(t)=\sum_{i=1}^{n}\int_{\Omega}\left[\varepsilon_i^2(x,t)e^{2\lambda\tau}+\sum_{j=1}^{n}\frac{\left(\beta_j\left|k_{ji}\right|\right)^{2-2\gamma_{ij}}}{1-\rho}\cdot\int_{t-\theta_{ij}(t)}^{t}\varepsilon_j^2(x,s)e^{2\lambda(s+\tau)}ds\right]dx$$

Calculating $D+V(t)$ along system (4), we have

$$D+V(t)\le 2e^{2\lambda t}\sum_{i=1}^{n}W_i\int_{\Omega}\varepsilon_i^2(x,t)\,dx\le 0,$$

where W_i is defined in (5). It means that

$$V(t)\le V(0)=\sum_{i=1}^{n}\int_{\Omega}\left[\varepsilon_i^2(x,0)+\sum_{j=1}^{n}\frac{\left(\beta_j\left|k_{ji}\right|\right)^{2-2\gamma_{ij}}}{1-\rho}\int_{-\theta_{ij}(0)}^{0}\varepsilon_j^2(x,s)e^{2\lambda(s+\tau)}ds\right]dx$$

Let

$$\Upsilon=1+\frac{e^{2\lambda\tau}-1}{2\lambda(1-\rho)}\sum_{i,j=1}^{n}\left(\beta_j\left|k_{ji}\right|\right)^{2-2\gamma_{ij}}>1$$

For $V(t)\ge\left\|\varepsilon(.,t)\right\|_2^2 e^{2\lambda t}$, it holds:

$$\left\|\varepsilon(.,t)\right\|_2^2\le\Upsilon\left\|\varphi(.,t)-\psi\right\|_2^2 e^{-2\lambda t},$$

that is

$$\left\|u(.,t)-\tilde{u}(.,t)\right\|_2^2\le\Upsilon\left\|\varphi-\psi\right\|_2^2 e^{-2\lambda t}$$

by Definition 1, we know that the drive-response neural networks can be controlled synchronously. So we finish the proof of Theorem 1.

4. Example

Consider delayed Cohen-Grossberg neural networks with reaction-diffusion terms

$$\begin{aligned}
\frac{\partial u_1(x,t)}{\partial t}&=\frac{\partial^2 u_1(x,t)}{\partial x^2}-a_1(u_1(x,t))[b_1(u_1(x,t))-\sum_{j=1}^{2}c_{1j}g_j(u_j(x,t))-\sum_{j=1}^{2}k_{1j}g_j(u_j(x,t-t_j(t)))+I_1(t)],\\
\frac{\partial u_2(x,t)}{\partial t}&=\frac{\partial^2 u_2(x,t)}{\partial x^2}-a_2(u_2(x,t))[b_2(u_2(x,t))-\sum_{j=1}^{2}c_{2j}g_j(u_j(x,t))-\sum_{j=1}^{2}k_{2j}g_j(u_j(x,t-t_j(t)))+I_2(t)],\\
u_i(x,t)&=\varphi_i(x,t),-\tau\le t\le 0,\ x\in\Omega,\ l=1,2\ \ \frac{\partial u_i(x,t)}{\partial\nu}=0,\qquad x\in\partial\Omega,\ i=1,2.
\end{aligned}\tag{6}$$

The corresponding response neural networks can be expressed as follows.

$$\begin{aligned}
\frac{\partial \tilde{u}_1(x,t)}{\partial t}&=\frac{\partial^2 \tilde{u}_1(x,t)}{\partial x^2}-a_1(\tilde{u}_1(x,t))[b_1(\tilde{u}_1(x,t))-\sum_{j=1}^{2}c_{1j}g_j(\tilde{u}_j(x,t))-\sum_{j=1}^{2}k_{1j}g_j(\tilde{u}_j(x,t-t_j(t)))+I_1(t)+\nu_1(x,t)],\\
\frac{\partial \tilde{u}_2(x,t)}{\partial t}&=\frac{\partial^2 \tilde{u}_2(x,t)}{\partial x^2}-a_2(\tilde{u}_2(x,t))[b_2(\tilde{u}_2(x,t))-\sum_{j=1}^{2}c_{2j}g_j(\tilde{u}_j(x,t))-\sum_{j=1}^{2}k_{2j}g_j(\tilde{u}_j(x,t-t_j(t)))+I_2(t)+\nu_2(x,t)],\\
\tilde{u}_i(x,t)&=\tilde{\varphi}_i(x,t),-\tau\le t\le 0,\ x\in\Omega,\ l=1,2\ \ \frac{\partial \tilde{u}_i(x,t)}{\partial\nu}=0,\qquad x\in\partial\Omega,\ i=1,2.
\end{aligned}\tag{7}$$

Let Ω be a bounded open domain with $|\Omega|\le 1$.

Let $a_1=6+\cos u_1(x,t)$ and $a_2=3-\cos u_2(x,t)$ satisfy (H1) with $m_1=5$, $M_1=7$, $m_2=2$, $M_2=4$, $\Gamma_1=1$, $\Gamma_2=1$.

Let $b_1=5u_1(x,t)+\frac{1}{2}\tanh u_1(x,t)$ and $b_2=6u_2(x,t)+\frac{1}{2}\sin u_2(x,t)$ satisfy (H2) with $\alpha_1=\frac{9}{2}$ $\alpha_2=\frac{11}{2}$.

Let $g_1 = \sin u_1(x,t)$ and $g_2 = \sin u_2(x,t)$ satisfy (H3) with $\beta_1 = 1$, $\beta_2 = 1$, $\gamma_{ij} = 1$, $i, j = 1, 2$ and $\Lambda_1 = 1$, $\Lambda_2 = 1$.

Let $c_{ij} = 0$, $i, j = 1,2$ and $k_{11} = \frac{1}{20}$, $k_{12} = \frac{1}{60}$, $k_{21} = \frac{1}{50}$, $k_{22} = \frac{1}{10}$.

Let $\rho = 1/2$, $I_1 = 2 + \frac{t}{1+t}$ and $I_2 = 1 - \frac{3t}{1+t}$ be bounded functions. And the control input vector $(v_1(x,t), (v_2(x,t))^T$ satisfy the following condition

$$(v_1, v_2)^T \leq M\left(\left|u_1(x,t) - \tilde{u}_1(x,t)\right|, \left|u_2(x,t) - \tilde{u}_2(x,t)\right|\right)^T ,$$

where $M_{11} = 1/3$, $M_{12} = 1/6$, $M_{21} = 1/6$, $M_{22} = 1/3$.

By simple calculation, we can choose

$$\xi_{ij} = \gamma_{ij} = \eta_{ij} = \frac{1}{2}, (i, j = 1,2)$$

Such that

$$-2\alpha_1 m_1 + \sum_{j=1}^{2} \Gamma_1\left(\left(\left|c_{1j}\right| + \left|k_{1j}\right|\right)\Lambda_j + \left|I_1\right|\right) + \sum_{j=1}^{2}\left(M_1\left|k_{1j}\right|\beta_j + M_j\left|k_{j1}\right|\beta_1\right) < 0.$$

Similarly, we have

$$-2\alpha_2 m_2 + \sum_{j=1}^{2} \Gamma_1\left(\left(\left|c_{2j}\right| + \left|k_{2j}\right|\right)\Lambda_j + \left|I_2\right|\right) + \sum_{j=1}^{2}\left(M_2\left|k_{2j}\right|\beta_j + M_j\left|k_{j2}\right|\beta_2\right) < 0.$$

5. Conclusion

In this paper, the synchronization control of delayed Cohen-Grossberg neural networks with diffusion terms is studied. Some sufficient conditions expressed by algebra inequalities have been given to ensure the synchronization control of Cohen-Grossberg neural networks with diffusion terms. The methods used here can be used to deal with the exponential synchronization of general problems. The result obtained in this paper is still true for other delayed reaction-diffusion neural networks.

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