Array Diagnosis using Compressed Sensing in Near Field

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ABSTRACT: *This paper will present a technique for array diagnosis using a small numberof measured data acquired by a near-field system by making useof the concepts of compressed sensing technique in image processing.Here,the high cost of large array diagnosis in near-field facilitiesis mainly caused by the time required for the data acquisition.So there is a need to decrease the measurement time and at the same time the reconstruction of an array must be satisfactory.The proposed technique uses less number of measurement points compared to other proposed techniques like back-propagation method and standard matrix inversion method.*

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1. Introduction

Near field facilities are nowadays routinely used for array testing. Besides the evaluation of the radiation pattern, another important application is the diagnosis of arrays. The most commonly used method for planar arrays is the back propagation algorithm [1], that is based on the Fourier relationshipbetween the field on the array aperture and on the measurement plane .In order to obtain a sufficiently highresolution, it is necessary to acquire the data on an area in front of the antenna whose dimension allows to collect all the relevant energy radiated by the antenna, otherwise the resolution is seriously degraded.

Using the standard half-wavelength measurement step, the number of measurement points turns out to bevery large. Data acquisition time is consequently the main limitation factor of such a technique. For large arrays the data acquisition could require a significant amount of time, and high costs.

One of the important possible ways to decrease the measurement time is by reducing the number of data.In the last couple of decades an effective theory to reduce the set of data in antenna measurements under a-prior knowledge of the shape of the antenna under test has been developed [4].

In particular, considering the class of antennas contained in a given convex surface, the method allows evaluating the minimum number of information of such a class of radiating sources, assuring a reconstruction of the field from the acquired data within any desired error level. This method has been successfully applied to near-field measurements to reduce the set of data [5]. Using other a-priori information, for example on the spatial correlation of the sources, it is possible to further decrease the number of measurements [6].

With reference to array diagnosis, the goal is to identify the fault elements in the grid of radiating elements. Hopefully, the number of fault elements is small, and this suggests considering as excitation coefficient the difference between the excitation of a reference failure-free array, and the array undertest. Such a "*new* " antenna is a very sparse array, with a small number of radiating elements, allowing a sensible decrease in the amount of information required to identify the (low number of) failures.

The failure-identification algorithm takes advantage of some recent results obtained in the field of compressed sensing, which allows the harmonic estimation of sparse signals using asmall set of data [7], [9]. It is useful to stress that there is a strict connection between Fourier analysis and array synthesis. This connection allowsto use super-resolution algorithms developed for harmonic estimation, like MUSIC and ESPRIT, to perform spatial processing**.**

2. Description Of The Method

Let us consider an Array under Test (AUT) consisting of N radiating elements located in known r_n positions. Let x_n and $f_n(\theta, \Phi)$be the excitation coefficient and the electric-field radiation pattern of the nth radiating element, respectively. A probe having effective height $h(\theta,\Phi)$ is placed in M spatial points. The voltage at the probe output can be expressed by the linear system

$$AX = Y$$

In array diagnosis the goal is to identify the fault elements. The number of the fault elements will be denoted by K. In [2] this goal is reached by inverting the system (1). However, this requires that M ≥ N. In this contribution instead the goal is to identify the failures with M << N, taking advantage that usually K << N.

If we consider the set of the arrays that must be tested, the excitation vectors are highly correlated, since a (hopefully) small number of elements are broken in each array. This suggests to consider the *innovation*vector, obtained subtracting the vector of the excitations of a fault-free array and the vector of the excitation of the AUT. This method parallels a well known method used in video signal processing based on encoding only the innovation in a frame of an image.

If we are only interested in the detection of fault elements, we simply perform the measurements in M points on the measurement plane of the field radiated by the failure-free array, let y^r the vector collecting the measured data, wherein the apex stands for '*reference*'. And x^r be the excitation vector of reference array.

Then the field radiated by the AUT is measured, obtaining the y^d output vector. In the following the vector of the excitations of the AUT will be denoted as x^d .

We consider the new system

$$AX = Y \qquad (1)$$

Where $x = x^r - x^d$
$y = y^r - y^d$

are "*innovation*" vectors. Note that if the number of fault elements K is much smaller than N (as usually happens) we have an equivalent problem involving a highly sparse array. We suppose that M << N. In this case the problem is ill posed, and a regularization procedure is required.

The standard approach for matrix inversion regularization is to introduce *a-priori* information in the inversion. This can be obtained adding a penalty weight related to the norm of the X vector. In particular, a possible choice is the constraint minimization:

$$\min_2 \| X \|_p \text{ Subject to } \| AX - Y \|_2 \ll \varepsilon \quad (2)$$

Where in p stands for the l_p norm, the subscript 2 stands for l_2 norm, and ε is the expected error level due to the noise and the measurement uncertainties.

Usually, the energy of X is used as a-priori information, and the usually adopted norms are l_2. However, in our case a strong *a-priori* information is the sparsification of the array. In absence of noise, the best choice should be the following constraint minimization:

$$\min_2 \| X \|_0 \text{ Subject to } AX = Y \quad (3)$$

Where in is the so called '*zero-norm*', equal to the number of non-zero elements of X.

The zero-norm forces the solution to be sparsified. Norms l_p with have this property $0 \le p \le 1$. However $p < 1$, givesnon convex minimization problems, whose numerical solutionis cumbersome. For this reason l_1, the norm is the most interesting candidate to force a sparsification of the solution [9], [10].

Accordingly, we solve the following constraint minimization:

$$\min_2 \| X \|_1 \text{ Subject to } \| AX - Y \|_2 \le \varepsilon \quad (4)$$

Whose solution has a good stability in presence of noise. Furthermore, from a computational point of view, the above minimization is a convex problem, that has a unique minimum, and for which efficient algorithms are available [13].

The different regularization procedures are available in [13]. Broadly speaking, all the regularization techniques use *a-priori* information on the problem to restrict the search space, increasing the probability of selecting the right solution. The available *a-priori* information generally has different "*degreeof accuracy.*"

The "*best*" regularization technique is the one that gives priority to the most "*accurate*" *a-priori* information. In our specific case, the value of the norm of the solution is not known *a-priori*, so that the best choice is to use *a-prior* information on the noise level.

Summarizing, the strategy proposed in (4) is simple and effective,and requires a minimum amount of *a-priori* information (i.e., the noise level affecting the data).

In array diagnosis, it must be noted that in case of sparse far-fieldmeasurements, A has indeed such abasic structure, and we must expect that $M = O(KlogN)$ measurements are required. In case of near-field measurements the structure of A is more complex, and at the knowledge of the author no exact results are available in the literature.

However, the results obtained from alarge number of numerical simulations, some of them reported in Section III, suggest that the dimensionality of the measurement space increases linearly with the sparsity coefficient K,But only logarithmically with the "*signal domain*" dimensionality N.

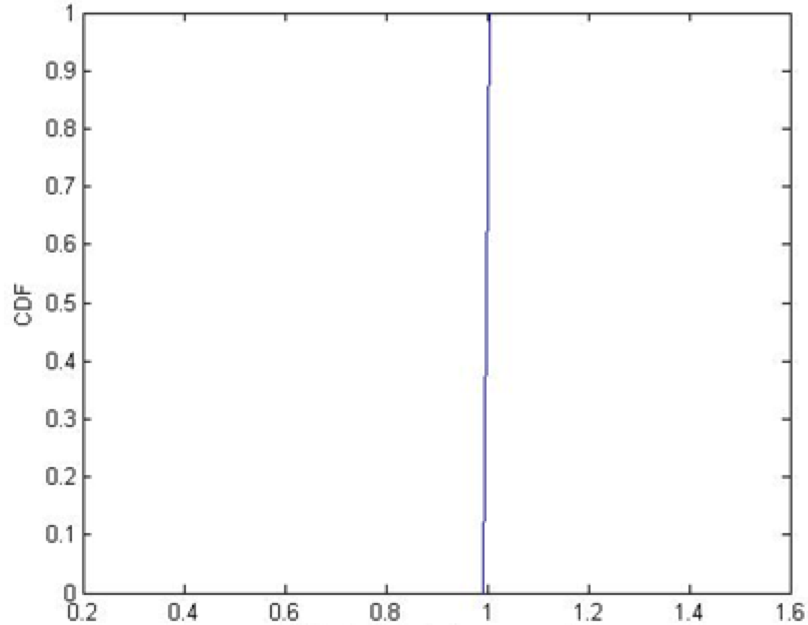
Finally, in real arrays the fault elements are often organized in C clusters. Accordingly, a more appropriate model is the so called (K, C) -*sparse model*, in which K-sparse signals are contained within at most C-clusters. In this case a smaller number of measurements compared to the non-clustered case is generally required [12].

3. Numerical Example

We consider a $13\lambda \; x \; 13\lambda$ uniform planar array of $N = 17 \text{ x} 17 = 289$ radiating elements consisting of short dipoles parallel to the axis placed on a uniform latticewith uniform spacing. All the excitation coefficients of array are equal to one. The electric field is measured using the ideal probe placed on a plane parallel to the aperture of the array. Finally 35dB Gaussian Noise is added to the data in order to simulate Noisy Measurements.

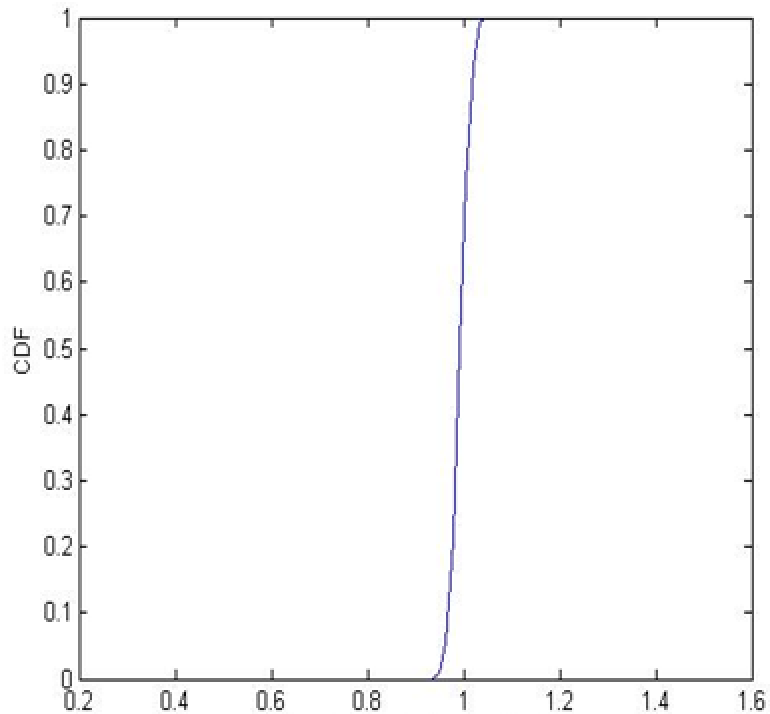
3.1 The CDF of the Excitation Amplitudewithout Noise

Consider all the excitation coefficients of array amplitudes are equal to one. The cumulative distribution function for the estimated excitation amplitudes of array without Noise is as follows



Estimated Excitation Amplitude

Figure 1. CDF of the Estimated Excitation Amplitude



Estimated Excitation Amplitude

Figure 2. The CDF of Noise added Data in Case of No Broken elements

3.2 The CDF of the Estimated Excitation Amplitude with Noise

3.2.1 No Broken Elements in the Array

In order to simulate Noisy measurements, the Gaussian Noise of 35dB is added to the data.Here another consideration is that there is no broken elements in the array i.e. all the excitation coefficients amplitude are equal to one.

The cumulative distribution function was found to the noise added data with all the elements amplitudes are equal to 1.The below plot drawn between estimated Excitation amplitude vs. CDF shows the CDF of Noise added data in case of no broken elements.

3.2.2 Considering Broken Elements of Array

In this case we consider that some of the elements of array are broken i.e. excitation coefficient equal to zero. The measurements are done by taking number of failures randomly from 0 to 3.

The number of measurement points must be in the order of KlogN .Where k =3 number of failures and N = 289 the total number of elements ofarray. The CDF for the Estimated Excitation amplitudes of broken elements are plotted and the graph as follows

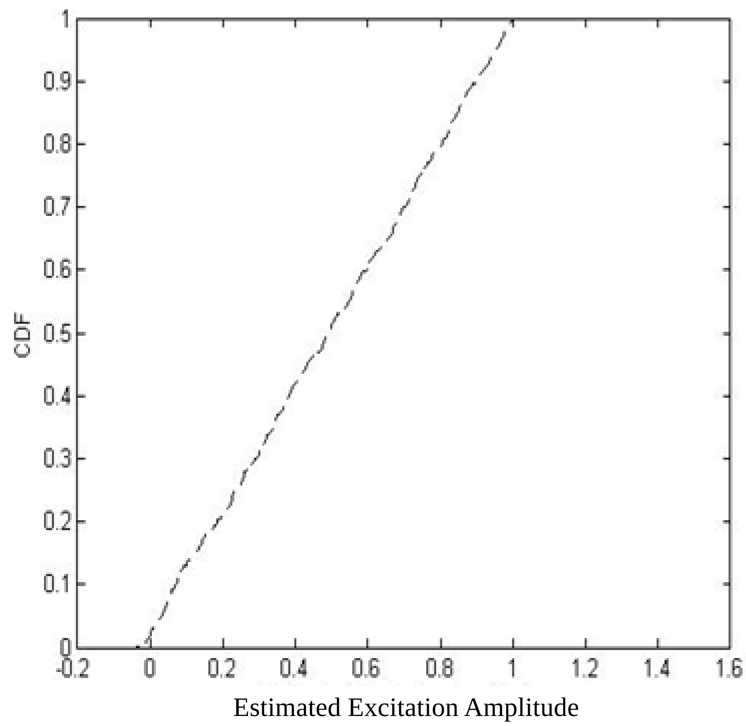


Figure 3. CDF of the Estimated Excitation amplitude having broken elements

3.3 Rice Distribution

For comparison, Let us consider the rician distribution which has a none zero mean. In finding CDF, the rician distribution gives good results.

The Rice Distribution R(v, σ) where $R = \sqrt{X^2 + Y^2}$ and $X = N(\mu_1, \sigma^2)$,$Y = N(\mu_2, \sigma^2)$ are statistically independent Gaussian random variables with common variance σ^2 and mean μ_1 and $\mu_2$with $u_1^2 + u_2^2 = v_2^2$.

3.3.1 CDF of Rice Distribution in Case of No broken Elements

The CDF of the R(0.998,0.034) are plotted showing the good correspondence .The following plot gives the good comparison of CDF of Rice with the Ideal Curve and CDF of Noise added Data.The reconstructed excitation in absence of failure is very close to one in mean (0.998), with a variance 0.011 (≅ -39 dB, i.e., at almost the same level of the noise level -35dB).

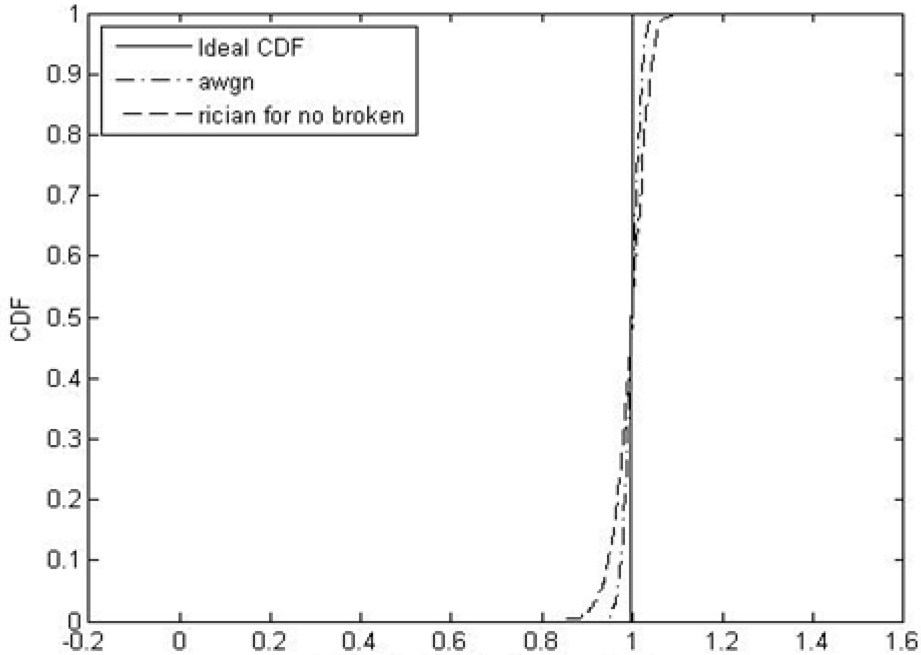


Figure 4. Comparison between CDF of Ideal, Rician,and Noise added data in Case of no broken elements

3.3.2 CDF of Rice Distribution in Case of broken Elements

In the case of broken elements (i.e. excitation amplitude is zero) also, we consider the CDF of the RicianDistribution for Comparison .Here the CDF for the R(0.59,0.20) having the mean 0.62 and 0.036(≅ -29 dB) is plotted. In this case the reconstruction is less satisfactory compared to the no-failure case. This limits the use of the technique for accurate reconstruction of the excitations of fault elements, at least in the case of so few measurement points.

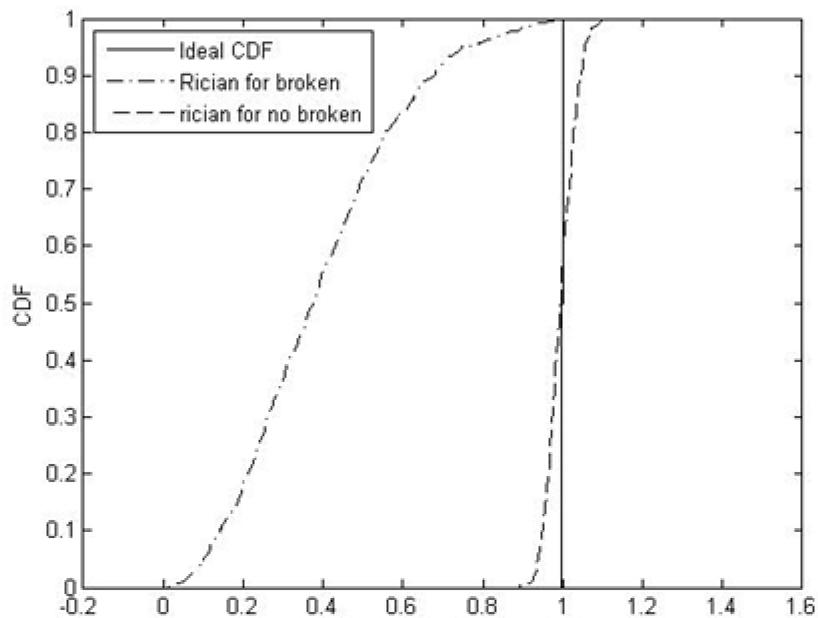


Figure 5. Comparison of CDF of Rician for broken and no broken Cases with the ideal Curve

However, the results suggest that the method is well suited for pass-fail test of the array. For example, using a threshold of 0.85 the probability of false alarm and the probability of a missed failure are both around 5%. A higher threshold allows to further decrease the probability of a missed failure, accepting a higher probability of false alarm. Of course, in case of detection of failures, it is possible to increase the number of measurements, in order to confirm the presence of failures and to obtain a more accurate reconstruction of their excitation coefficients.

4. Conclusion

This paper discusses in detail a simple method for array diagnosis that allows significant decrease in the number of measurements compared to that of elements of array. This method is worthy in the case of large arrays where measurement time is usually very high. The key point of the method is the use of a regularization procedure minimizing the 1-norm of the difference vector between a failure-free excitation vector and the excitation vector of the AUT. This allows to obtain an equivalent sparse array, discarding the pieces of information not of interest for the failure identification problem. The proposed Technique is related to some of the results obtained in '*Compressed Sensing*' usually preferred in data or Image processing.

Further investigations can be done by applying the technique for a maximum number of failures in an array case and then the technique can be evaluated for finding its limitations on accuracy level.

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