



Study on the Motion Characteristics of Cylinders in Sandy Environments

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ABSTRACT

This paper aims to study the motion characteristics of cylinders in sandy environments, particularly focusing on the distance required for a cylinder to roll and eventually stop in the sand. Based on observations from daily life and theoretical assumptions, the experiment investigates the effects of factors such as the length, diameter, material of the cylinder, and the size of sand particles on the rolling distance.

A simple device is designed to simulate the process of a cylinder rolling down a slope and in to the sand, and the rolling distance of the cylinder in the sand is recorded. The experiment employs a controlled variable method to test cylinders of different lengths and diameters under the same conditions (e.g., initial height, sand particle size). The experimental data were recorded in tables and charts for intuitive analysis and comparison.

The rolling distance of a cylinder in the sand is positively correlated with both its length and diameter, as well as its material. Specifically, as the length of the cylinder increases, the rolling distance also increases; similarly, an increase in the diameter of the cylinder results in a corresponding rise in the rolling distance. Additionally, the impact of the cylinder's material on the rolling distance was noted, but a detailed analysis was not conducted due to experimental limitations.

This experiment not only confirms theoretical assumptions but also provides experimental evidence for understanding the motion characteristics of objects in sandy environments. The results have practical implications for designing vehicles for sandy environments and optimising emergency escape lanes. Future research can further explore the effects of cylinder material and sand particle properties on rolling distance, as well as analyse the forces acting on the cylinder during rolling.

Keywords: Sandy Environment, Motion Characteristics of Cylinders, Normal Stress, Tangential Stress

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1. Introduction

On certain steep downhill sections, the car's brake pads become over heated, and the radiator temperature rises, potentially causing brake failure and necessitating emergency braking. Additionally, driving on a thick layer of sand- coveredlanes can lead to increased friction, resulting in a loss of kinetic energy in moving vehicles.

Therefore, I am interested in investigating how long a sandy surface would need to before a passively moving object (such as a ball) to come to a complete stop. What parameters decide this length?



Figure 1. A car driving on a sand covered lane

The ability of an emergency escape lane to bring a vehicle with brake failure to a stop is related to the material used for its surface. The surface materials of escape lanes are typically loose and/or fine gravel. When a heavy vehicle drives on to this surface, the wheels sink in to the sand, and the heavier the vehicle, the deeper it sinks. This increases the resistance acting on the car.

A sand-covered lane leads to kinetic energy loss in moving vehicles. This work examines the parameters that determine how long and deep a sandy surface must be for a passively moving object, such as a ball, to come to a complete stop.

A cylinder rolls down from a specified position on a board into a sandpit, gradually decelerating due to its interaction with the sand until it comes to rest at a particular position.

1. The cylinder sinks to a certain depth in the sand as it rolls.
2. The rolling distance changes regularly when the initial conditions related to the cylinder and the sand are altered.
3. The choice of a cylinder over a ball is due to the ease of deciding the direction of the initial velocity and measuring the straight-line travel distance.

4. In this experiment, the depth to which the cylinder sink sin to the sand pit is relatively shallow. Therefore, in the theoretical analysis, we neglect the pressure and friction forces on the two end faces of the cylinder and mainly focus on the normal and tangential stresses on the cylinder's lateral surface.

2. Experiments

2.1 Design of Experimental Setup

In this experiment, I used a cylinder to simulate a wheel. On the one hand, it can simulate the rolling of a wheel; on the other hand, it has greatest ability during rolling and is less likely to topple over.

Based on the above assumptions, the rolling distance of the cylinder is related to the material, radius, length, initial velocity of the cylinder, and the size of the sand particles. Therefore, my experiment controlled the following variables: the properties of the cylinder (material, length, and diameter) and the diameter of these and particles. According to the experimental design requirements, I purchased sand that met the criteria from the market, selected suitable cylinders from existing materials in the laboratory of the University of Electronic Science and Technology of China, and constructed an experimental apparatus using card board and other materials.

2.2 Selection of experimental equipment

(1)Cylinders:

- Materials: acrylic, wood, iron
- Diameters: 8 cm, 15cm, 16cm, 20cm
- Lengths: 5 cm, 8cm, 10cm,1 2cm

(2) Sand:

- Diameter: 1-2 mm

(3) Other Apparatus:

Below are the schematic diagram and photos of the experimental apparatus I built:



Figure 2. Experimental Apparatus

2.3 Experimental Design

In the experiment, I allowed the cylinder to enter the sand at a specified initial velocity. To control the initial velocity, as shown in the experimental apparatus, I will release the cylinder from rest (initial velocity =0) down a 45° inclined plane with a height of $h=15$ cm. The initial velocity of the cylinder as it enters these and can be determined using the work-energy theorem: $\frac{1}{2}mv^2=mgh$, and the relationship between angular velocity and linear velocity $\omega = v/r$.

The entire experimental process will be recorded using a mobile phone, and the distance the cylinder travels in the sand will be measured with a steel ruler.

3. Experimental Results and Discussion

3.1 Relationship Between Cylinder Length and Rolling Distance

The different lengths of the cylinders will result in varying parameters of contact with the surface, which should affect the rolling distance. To investigate the effect of cylinder length on rolling distance, I selected cylinders with the same material and diameter (8 cm, wood, initial height of 15 cm), varied their lengths, and measured the rolling distance and other relevant parameters.

By maintaining the material and diameter constant, I can isolate the influence of length on the rolling behavior. This approach enables a more accurate analysis of how the cylinder's length affects its rolling distance in the sand.

Cylinder length(cm)	5	8	10	12
Rolling distance (cm)	65.5	67	67	77
Rolling distance (cm)	65	67.5	67	70
Rolling distance (cm)	68	70	68	72
Average value (cm)	66.17	68.17	67.33	73.00

Table1. Relationship Between Cylinder Length and Rolling

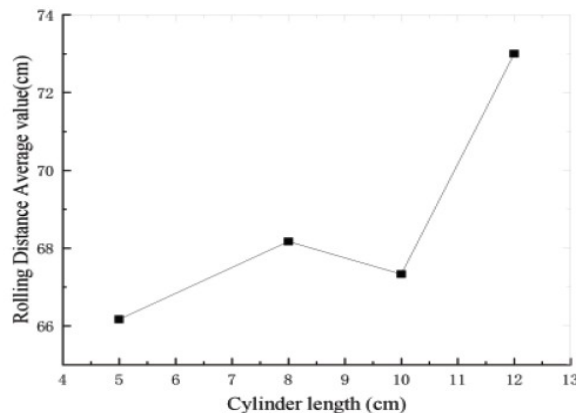


Figure 3. The point-line graph of the relationship between the cylinder length and the rolling distance

In the experiment, the different-coloured curves in the graph represent cylinders of varying lengths: Series1 (5 cm), Series 2 (8cm), Series 3(10cm), and Series 4(12cm). The vertical axis represents distance in centimeters (cm).

To obtain more data, I conducted three tests for each cylinder length and recorded the rolling distances using a steel ruler. The lengths of the cylinders were 5cm, 8cm, 10cm, and 12cm. The average distance was calculated from the recorded data. Using Excel, I plotted the data to create a graph for easier visual analysis.

The rolling distance of the cylinder is proportional to its length.

From the curves, it appears that there is a positive correlation between distance and length; as the length of the cylinder increases, the distance also increases accordingly.

With a limited number of data points, the first data point may have a larger error, while the subsequent three data points exhibit a linear increase.

3.2 Relationship between Cylinder Diameter and Rolling Distance

In this experiment, I chose to vary another parameter to observe its effect. The rolling distance should differ with different cylinder diameters. Therefore, I selected cylinders with the same material (wood) and length (10cm), but with varying diameters, to observe how the rolling distance changes and to investigate whether there is a relationship between the cylinder's diameter and its rolling distance.

I selected three cylinders with diameters of 15cm, 16cm, and 20cm, and used the same setup as in Experiment1. The cylinder rolls down a 45°incline from a height of 15cm, starting from rest (initial velocity = 0). The starting point for distance measurement is the junction of these and the slope. The measurement tool used was a steel ruler.

I conducted three tests for each cylinder diameter, recorded the rolling distances using a steel ruler, and calculated the average distance. The data were then plotted in a line graph using Excel for visual analysis. (Include the tables and graphs here for both experiments.)

By plotting the data in Excel, the relationship between the cylinder diameter And the rolling distance can be analyzed more intuitively.

Diameter (cm)	15	16	20
Rolling distance (cm)	40	50	50
Rolling distance (cm)	45	48	66
Rolling distance (cm)	43.5	49	60
Average value (cm)	42.83	49.00	58.67

Table 2. Relationship Between Cylinder Diameter and Rolling Distance

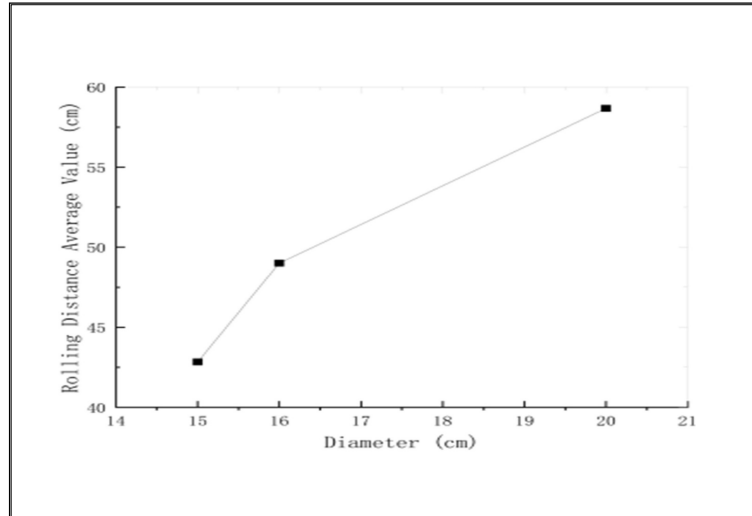


Figure 4. The point-line graph of the relationship between cylinder diameter and rolling distance

The curves of different colors in the figure represent cylinders with various diameters, namely series 1(15cm), series 2(16cm), and series 3(20cm); the vertical axis scale represents distance (cm).

The rolling distance of cylinders is proportional to their diameter.

The charts clearly show a positive correlation between distance and diameter. As the diameter of the cylinder increases, the rolling distance also increases.

3.3 Relationship Between Cylinder Material and Rolling Distance

In the third experiment, I observed the effect of changing the cylinder material while keeping other factors constant and measured the rolling distance and other parameters.

I selected three cylinders made of different materials: metal, wood, and acrylic. The experiment was conducted using the same setup as in Experiment1. The cylinder rolls down a 45° in cline from a height of 15cm, starting from rest (initial velocity=0). The starting point for distance measurement was the junction of the sand and the slope. The measurement tool used was a steel ruler.

I conducted three tests for each cylinder material, recorded the rolling distances using a steel ruler, and calculated the average distance. The data were then plotted in a line graph using Excel for visual analysis.

Since the weights of the wooden and acrylic cylinders are not significantly different, the resulting data show relatively minor differences. However, the substantially heavier metal cylinder exhibited noticeable differences in rolling distance compared to the other two materials.

By plotting the data in Excel, the relationship between the cylinder material and the rolling distance can be analyzed more clearly, providing insights into how different materials influence the behavior of rolling cylinders on sand.

Material	Metal	Wood	Acrylic
Rollingdistance (cm)	9	40	36
Rollingdistance (cm)	7.5	35	36
Rollingdistance (cm)	8	35	36.5
Averagevalue (cm)	8.17	36.67	36.17

Table 3. Relationship Between Cylinder Material and Rolling Distance

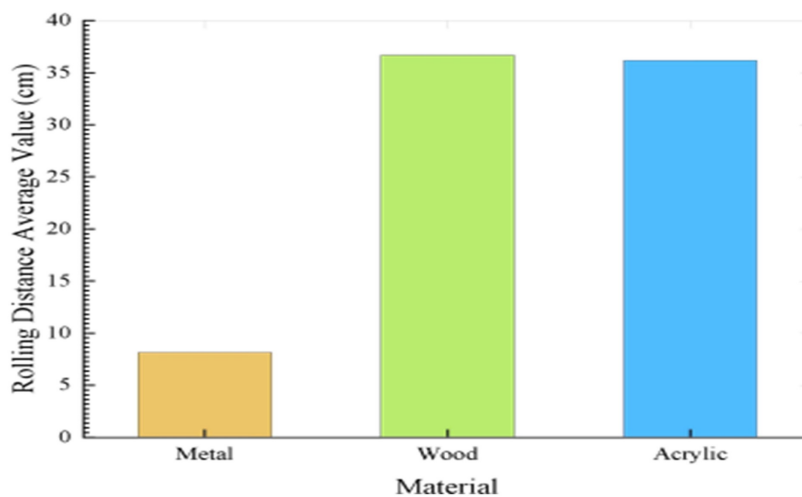


Figure 5. Histogram of cylinder material versus rolling distance

The bar of different colors in the figure represents cylinders with different materials; cylinder parameters: diameter 10cm, length 10cm

The rolling distance of cylinders is inversely proportional to their mass.

The charts clearly show an inverse relationship between the material of the cylinder and the rolling distance. While the differences between wooden and acrylic cylinders are not significant due to their similar weights, it is evident that heavier metal

Cylinders roll shorter distances. This confirms that different materials, which result in various weights, have an inverse relationship with the rolling distance.

3.4 Effect of Different Materials on Experimental Result

In this experiment, another critical factor is the type of sand used. The friction coefficient between the sand and the cylinder plays a crucial role in this process. Therefore, my fourth experiment aims to measure the friction coefficient between the cylinder and the sand.

I use a wooden cylinder and 2mm sand for the measurements, employing a dynamometer as the measuring tool. I increased the weight using weights. Below is the set up for the measurement. On the left side of the image, I placed a wooden board on the sand surface, with weights added to increase its weight, making it easier to rotate the dynamo meter and measure the values.

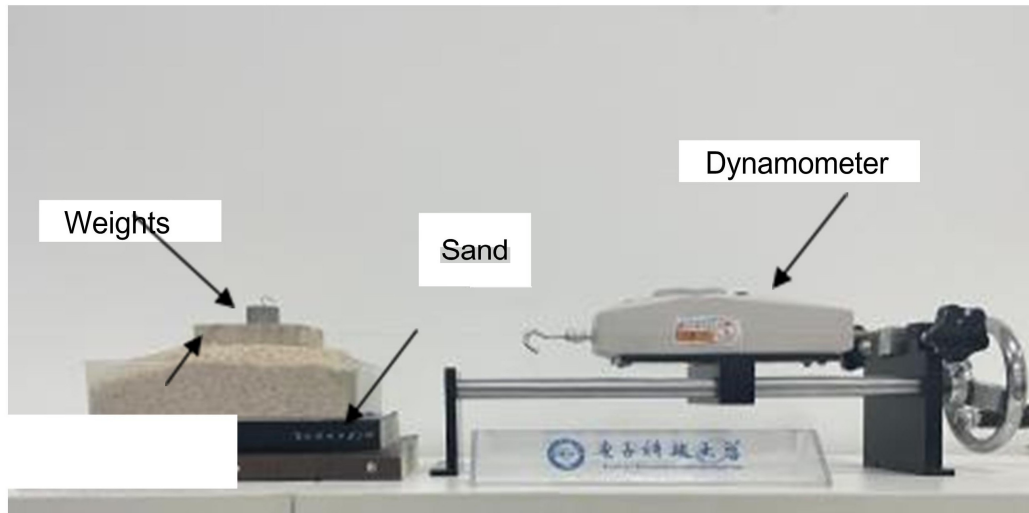


Figure 6. The fourth Experimental apparatus

After adding each weight, I slowly rotated the stepper device, recorded the dynamometer readings and the weights, and calculated the following data:

Friction Coefficients:

- (1) Acrylic board and sand : 0.253
- (2) Steel board and sand : 0.149
- (3) Wooden board and sand : 0.415

The friction between the wooden board and sand is the greatest, followed by the acrylic board, and the least friction is between the steel board and sand.

4. Theoretical Analysis

Due to the shallow sinking of the cylinder in the sand during the experiment, we ignored the pressure and friction on the bottom surface of the cylinder in the theoretical analysis. Instead, we focused on the regular and tangential stresses acting on the side surface of the cylinder.

We can draw a schematic diagram of the side surface of the cylinder as it rolls in to the sand. From the schematic, we can derive the following formulas:

where r is the radius of the cylinder; b is the length of the cylinder; θ_f is the entry angle; θ_r is the exit angle; θ_m is the maximum stress angle.

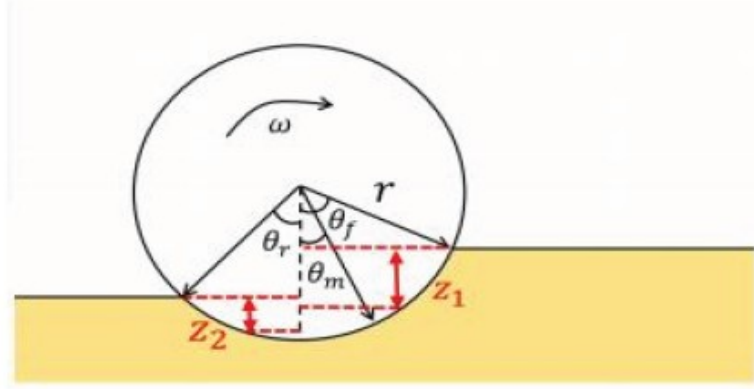


Figure 7. A schematic diagram of the side surface of the cylinder as it rolls into the sands [1]

From the empirical formula [1]

$$p(z) = \left(\frac{k_c}{b} + k_\phi \right) z^n \quad (1)$$

We can get the relationship between several variable sand pressure. K_c the deformation modulus of sand, k_ϕ the modulus of internal friction angle of sand, n soil settlement index. From the above formula, it can be shown that the two normal stresses on the moving cylinder are σ .

$$\sigma = \begin{cases} \left(\frac{k_c}{b} + k_\phi \right) z_1^n & (\theta_m \leq \theta \leq \theta_f) \\ \left(\frac{k_c}{b} + k_\phi \right) z_2^n & (\theta_r \leq \theta \leq \theta_m) \end{cases} \quad (2)$$

From the schematic diagram, we can get the geometric relationship between the front and rear sinking depths and the cylinder radius and angle

$$\begin{aligned} z_1 &= r \cos \theta - r \cos \theta_f \\ z_2 &= r \cos \left[\theta_f - \frac{\theta - \theta_r}{\theta_m - \theta_r} (\theta_f - \theta_m) \right] - r \cos \theta_f \end{aligned} \quad (3)$$

In the geometric relationship θ_m is the maximum stress angle and it depends on θ_f , so we can get the relationship between θ_m and θ_f .

$$\theta_m = (c_1 + c_2 S) \theta_f \quad (4)$$

In the expression, c_1 represents the friction between these and, c_2 represents the friction between the cylinder and the sand, S is the sliprate, where the S is defined as translational sliding rather than rolling. We can get the expression of S .

$$S = 1 - \frac{v}{\omega r} = 1 - \frac{v}{v_t} = \frac{v_t - v}{v_t} = \frac{v_j}{v_t} \quad (5)$$

We can draw a diagram of the effect of normal stress and tangential stress on a cylinder:

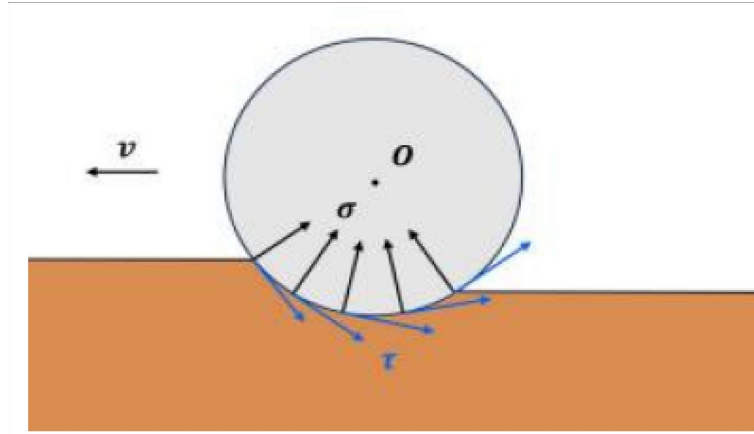


Figure 8. A diagram of the effect of normal stress and tangential stress on a cylinder

We continue to study tangential stress

Since the shear strength of soil τ is within the range where stress changes little, we can express it as a linear function of the normal stress σ on the shear sliding surface,

$$\tau = (c + \sigma \tan \phi)(1 - e^{-k}) \quad (6)$$

(6) The expression of σ is the normal stress in the above formula, the other C is the cohesion of dry sand, ϕ the internal friction angle, K shear modulus of dry sand, J the slip distance during driving.

From the slip rate S the slip distance J can be obtained as:

$$J = \int_{0_0}^t v_j dt = \int_{\theta}^{\theta_f} v_j \frac{d\theta}{\omega} \quad (7)$$

To study the effect of normal stress and tangential stress on the horizontal direction of the cylinder, we decompose the normal stress to obtain their contribution to the horizontal force at the center of mass. The contribution of normal stress to the horizontal force at the center of mass is:

$$R_c = br \int_{\theta_r}^{\theta_f} \sigma \sin \theta d\theta \quad (8)$$

The contribution of tangential stress to the horizontal force at the center of mass is:

$$T_c = br \int_{\theta_r}^{\theta} \tau \cos \theta d\theta \quad (9)$$

We combine the two forces to obtain the resultant force on the center of mass of the cylinder:

$$F_c = T_c - R_c \quad (10)$$

From the geometric relationship, we can get the moment of force on the cylinder:

$$M = br^2 \int_{\theta_r}^{\theta} \tau d\theta \quad (11)$$

We continue to use Newton's second law and we can find the acceleration and angular acceleration

$$F_c = ma \quad (12)$$

$$M = I\beta \quad (13)$$

We put the obtained data in to the expression of the contribution of normal stress and tangential stress to the horizontal direction of the center of mass of the cylinder, and obtain the relationship between the variables in the the variables in the experiment and the normal stress and tangential.

Among which the normal stress contributes to the horizontal force of the center of mass:

$$\begin{aligned}
 R_c &= br \int_{\theta_r}^{\theta_f} \sigma \sin \theta d\theta \\
 &= br \left\{ \int_{\theta_m}^{\theta_f} \left(\frac{k_c}{b} + k_\phi \right) (r \cos \theta - r \cos \theta_f)^n \sin \theta d\theta + \right. \\
 &\quad \left. \int_{\theta_r}^{\theta_m} \left(\frac{k_c}{b} + k_\phi \right) \left[r \cos \left[\theta_f - \frac{\theta - \theta_r}{\theta_m - \theta_r} (\theta_f - \theta_m) \right] - r \cos \theta_f \right]^n \sin \theta d\theta \right\} \\
 T_c &= \left(1 - e^{-\frac{1}{K}} \right) \left\{ \int_{\theta_m}^{\theta_f} \left[c + \left(\frac{k_c}{b} + k_\phi \right) (r \cos \theta - r \cos \theta_f)^n \tan \phi \right] \cos \theta d\theta + \right. \\
 &\quad \left. \int_{\theta_r}^{\theta_m} \left[c + \left(\frac{k_c}{b} + k_\phi \right) \left[r \cos \left[\theta_f - \frac{\theta - \theta_r}{\theta_m - \theta_r} (\theta_f - \theta_m) \right] - r \cos \theta_f \right]^n \tan \phi \right] \cos \theta d\theta \right\}
 \end{aligned} \quad (14)$$

However, since the formula is complicated to solve analytically, we use numerical calculation methods to verify the applicability of the theory to the experiment.

5. Conclusion

From the experiments, the following conclusions can be drawn:

When the length of the cylinder increases, the entry angle, exit angle, and maximum stress angle remain unchanged. However, the mass increases, leading to a larger result ant force on the cylinder's center of mass and a greater moment of force, which in turn increases acceleration. Thus, the rolling distance is greater. According to the experimental data curves, there is a positive correlation between distance and length, where the distance increases with the increase in cylindrical length.

As the diameter of the cylinder increases, the entry angle, exit angle, and maximum stress angle increase. This results in a larger moment of force and increased mass, leading to greater acceleration and, consequently, a longer rolling distance. The experimental data graph indicates a positive correlation between distance and diameter, where the diameter of the cylinder increases, the distance also increases.

When the mass of the cylinder increases, the cylinder sinks deeper in to the sand, increasing the entry angle, exit angle, and maximum stress angle. The diagram clearly shows the inverse relationship between the cylindrical material and the distance travelled, but the weights of the cylinders made of wood and acrylic are not very different, and the difference is not very obvious. However, compared with metal, it can still be judged that the material is different, the weight of the cylinder is different, and the driving distance is inversely proportional to the cylindrical material.

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