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Prognostics and Health Management in Milling Operations: An Integrated Analysis of Tool Wear Trajectories and Reliability

Dit Suthiwong
Faculty of Information Technology, King Mongkut's University of Technology North Bangkok, Bangkok, Thailand
s5507011966036@email.kmutnb.ac.th

ABSTRACT

Accurately predicting cutting tool life is essential for minimizing unplanned downtime and optimizing predictive maintenance in modern milling operations. While existing tool condition monitoring studies focus primarily on wear detection, they often lack integrated degradation trajectory modeling and reliability assessment. This study proposes a comprehensive Prognostics and Health Management (PHM) framework combining vibration based condition monitoring with stochastic state space modeling to evaluate tool wear progression.

Utilizing the "Roughness in Milling Process" dataset, which pairs tri-axial vibration signals with surface roughness measurements across eight workpieces, the research constructs a normalised degradation index to track tool deterioration. The methodology incorporates wear state classification, Hidden Markov Models (HMM), and Hidden Semi-Markov Models (HSMM) with Weibull duration distributions to capture realistic state transitions. Furthermore, an absorbing Markov chain framework is applied to reliability analysis, alongside Monte Carlo simulations for probabilistic estimation of Remaining Useful Life (RUL).

Results demonstrate a strong monotonic correlation between increasing vibration magnitudes and worsening surface roughness. The HSMM effectively modeled prolonged intermediate degradation stages, while probabilistic RUL forecasts provided actionable insights for maintenance scheduling. Although constrained by the dataset's limited size, this integrated approach successfully bridges raw sensor data with actionable reliability metrics. Ultimately, the proposed framework establishes a robust, data-driven foundation for next-generation predictive maintenance strategies aligned with Industry 4.0 intelligent manufacturing paradigms.

Keywords: Prognostics and Health Management (PHM), Tool Condition Monitoring, Remaining Useful Life (RUL), Hidden Semi-Markov Model (HSMM), Vibration Analysis, Tool Wear Trajectories, Reliability Analysis, Predictive Maintenance, Milling Operations

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1. Introduction

Tool life is typically estimated by predicting the time to reach a threshold flank wear width. The cutting tool is a crucial component in any machining process, and its failure adversely affects the manufacturing process. Predicting cutting tool life by considering the various factors that influence it is essential for managing quality, cost, availability, and waste in machining processes [1]. An increase in unplanned machine downtime disrupts industrial operations, leading to substantial credibility damage and monetary losses. As a critical asset of the milling machine, cutting tool failure results in lost industrial productivity due to unplanned downtime. In such scenarios, implementing a proper predictive maintenance strategy through real time health monitoring of cutting tools is vital. Accurately predicting the useful life of tools plays a pivotal role in the predictive maintenance framework of Industry 4.0 [2].

2. Related Work

Tool condition monitoring (TCM) has been recognized as an important process in micro-milling operations to prevent excessive tool wear and to maintain part tolerances and surface quality [3]. It involves gathering multi-sensor data, such as forces, vibrations, and acoustic emissions, along with optical and data-driven techniques to capture tool conditions and generate reference models for condition and performance monitoring [4].

With the sustained development of Industry 4.0 and intelligent manufacturing, the demand for advanced manufacturing technologies and machining processes continues to grow. While pursuing higher machining quality and efficiency, tool wear remains an inherent challenge that affects the entire machining process. Tool wear directly affects workpiece surface quality, whereas tool breakage can cause machine downtime and reduced machining efficiency. Studies indicate that the time lost due to machining discontinuities caused by tool breakage accounts for 20% of total downtime [5].

Traditional machining operations such as turning, milling, grinding, and drilling represent the most common processes in the manufacturing industry [6]. During machining, contact between the cutter and the workpiece causes changes in the cutter's geometry, either through gradual tool wear or sudden tool breakage [7]. Machine downtime continues to challenge the manufacturing sector. While some sources of downtime are unavoidable (e.g.,workpiece transfer between workstations that requires dismantling and reinstallation, or scheduled maintenance), others, particularly those stemming from tool wear or breakage, can be mitigated. Machine tool failures can account for up to 20% of overall machine downtime [8].

Current research on tool wear encompasses tool wear mechanisms, online monitoring, and remaining useful life (RUL) prediction [9]. Tool wear mechanisms include abrasive, diffusion, oxidation, fatigue, and other mechanisms. Common strategies to reduce tool wear include applying coatings to the tool surface [10, 11], optimizing cutting parameters [12, 13, 14], and adopting alternative processing technologies [15, 16, 17, 18].

Few studies have addressed tool condition monitoring in micro-milling. Varghese et al. [19] [Alwin Varghese] employed a random forest method to predict tool life stages in micro-milling using cutting force data. Malekian et al. [1] utilized accelerometer, force, and acoustic emission sensors to develop a sensor fusion based neuro-fuzzy model for determining tool wear in micro milling. Jemielniak and Arrazola [20] applied acoustic emission and cutting force signals for tool condition monitoring in micro milling. Zhu et al. [2] used an independent component analysis (ICA) algorithm to denoise cutting force in micro milling tool condition monitoring. However, these studies were largely limited to linear tool paths. Material removal processes during end-milling along circular and linear tool paths have generally been assumed to be similar [22].

The cutting force was separated from mixtures of Gaussian and non-Gaussian noise sources to address the Blind Source Separation (BSS) problem in micro milling TCM [23]. Cutting forces have been measured to enable online monitoring of tool condition in multi toothed milling processes, accurately capturing the dynamic characteristics of the force signal [24]. Two cutting force components, cutting speed (or feed rate) and cutting depth, have been applied in online TCM during face milling operations [25]. Cutting force signals have been processed for real time tool wear estimation [26] with various methods used to extract relevant features from the acquired signals. Jemielniak et al. [27] compared signals from laboratory and industrial cutting-force sensors, highlighting that crosstalk between channels significantly affects measurements during laboratory simulations of industrial conditions. Approaches to estimating tool wear in milling have been proposed, with experiments verifying relationships between tool wear and cutting force coefficients, incorporating parameters such as spindle speed, cutting depth, and feed rate [28]. Gao et al. [29] introduced a data-driven model framework for TCM in machining operations, based on cutting force statistics. Comparisons of orthogonal force and unidirectional strain components showed a high probability (at least 95%) that the difference in flank wear estimation errors between the two processing strategies is less than 5 percentage points [30]. Proposed an online TCM system using cutting force and torque signals, achieving high correlation and low error ratios between actual and predicted flank wear values. A torque sensor has also been used to monitor milling cutter conditions [32], resulting in notable improvements in product efficiency and quality.

This integrated review underscores the importance of prognostics and health management strategies in milling, particularly through advanced tool wear trajectory analysis and reliability modeling, to support predictive maintenance in modern manufacturing environments.

Although significant advances have been achieved in tool condition monitoring using vibration, force, acoustic emission, and machine learning techniques, most existing studies focus primarily on wear detection or prediction accuracy. Comparatively limited attention has been devoted to integrating degradation trajectory modeling, stochastic wear-state transitions, reliability assessment, and remaining useful life estimation within a unified prognostics and health management framework. The present study addresses this gap by combining vibration based condition monitoring with state-space degradation modeling and reliability oriented prognostic analysis.

3. Experimental Architecture and Prognostic Monitoring Framework

Building upon previous research in tool condition monitoring, predictive maintenance, and machining prognostics, the present study adopts a vibration based prognostic monitoring architecture designed to capture the progressive degradation behaviour of milling tools. The proposed framework integrates sensor based condition monitoring, signal processing, health assessment, degradation modeling, reliability analysis, and remaining useful life estimation into a unified prognostics and health management (PHM) workflow.

The overall architecture consists of four principal layers: data acquisition, feature extraction, degradation assessment, and prognostic decision support. During milling operations, tri-axial vibration signals are continuously acquired from the machining system using a three-axis sensing configuration. The sensor arrangement captures dynamic responses along the X-, Y-, and Z-directions, thereby providing comprehensive information regarding cutting dynamics, tool workpiece interaction, vibration energy propagation, and progressive tool deterioration.

The acquired vibration measurements serve as the primary condition-monitoring signals and constitute the observational layer of the proposed framework. Simultaneously, surface roughness measurements (Ra) are collected after each machining operation and used as an external indicator of machining quality and wear severity. The combination of vibration-based monitoring and surface quality assessment enables both diagnostic and prognostic evaluation of the milling process.

Figure 1 illustrates the overall analytical architecture employed in this study. The framework begins by acquiring raw vibration signals from the milling process. These signals undergo preprocessing and are transformed into representative condition indicators through feature extraction. Statistical features such as mean vibration magnitude, standard deviation, root mean square (RMS), peak amplitude, and vibration energy are computed to characterize the dynamic state of the cutting tool. These indicators are subsequently integrated into a health assessment layer that quantifies the progression of degradation throughout the machining sequence.

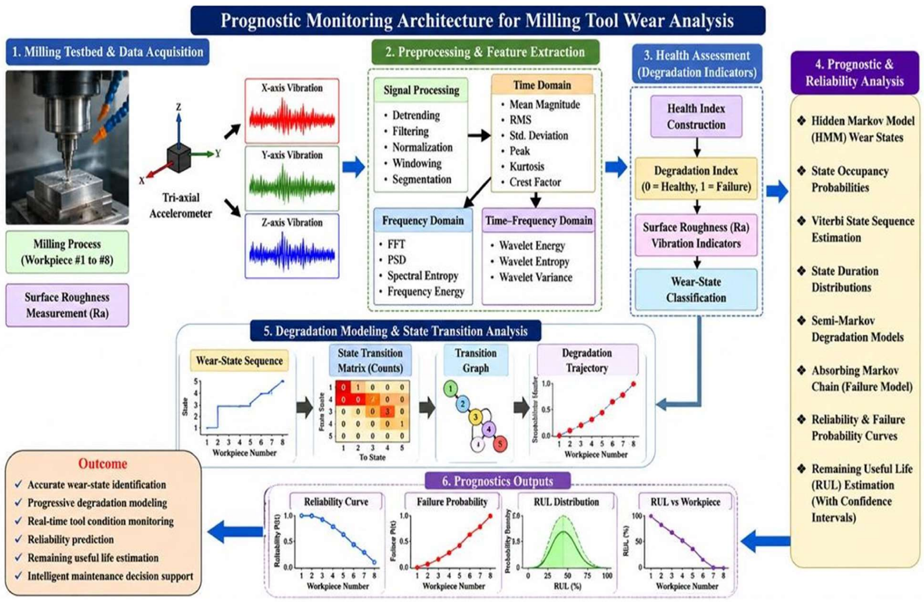

Figure 1. Prognostic Monitoring Architecture

Following feature extraction, the framework incorporates a degradation modeling module in which vibration-derived indicators and surface roughness measurements are combined to construct a degradation index. This health indicator serves as a compact representation of tool condition and facilitates the identification of wear-state transitions. The resulting degradation trajectory is subsequently analysed using stochastic state space models, including Markov and Hidden Markov representations, to characterise the evolution of latent wear states.

The prognostic layer extends the analysis by estimating degradation trends, transition probabilities, reliability functions, and remaining useful life. State-transition modeling provides insight into the progression of wear conditions, while reliability analysis quantifies the probability of continued operation as degradation accumulates. Remaining useful life estimation further supports maintenance decision making by forecasting the interval before the tool reaches a predefined failure threshold.

From a systems perspective, the architecture represents a digital condition monitoring testbed in which vibration signals function as observable process variables, whereas tool wear and degradation states constitute latent system conditions. The framework, therefore, mirrors real world predictive maintenance environments, where direct observation of wear is generally impractical and indirect sensing methods must be employed to infer tool health.

The adopted architecture aligns closely with contemporary Industry 4.0 paradigms that emphasize continuous monitoring, data driven diagnostics, prognostic modeling, and intelligent maintenance scheduling. By integrating signal processing, health index construction, state transition analysis, reliability modeling, and RUL prediction within a single framework, the proposed testbed establishes a comprehensive foundation for evaluating advanced prognostic methodologies in milling operations.

The following section describes the dataset employed to implement and validate the proposed monitoring architecture.

4. Dataset Description

The present study utilizes the *Roughness in Milling Process* dataset, publicly available through IEEE DataPort, which was developed to facilitate research on machining quality monitoring, tool condition assessment, prognostics, and predictive maintenance in milling operations. The dataset contains synchronized vibration measurements and corresponding surface roughness observations acquired during milling experiments, providing a comprehensive framework for investigating the relationship between machining dynamics and product quality.

The dataset comprises eight individual workpieces representing successive milling operations performed under controlled machining conditions. For each workpiece, vibration signals were recorded continuously using a triaxial sensing system, yielding three independent time series channels corresponding to the X-, Y-, and Z-directions. These vibration measurements capture the dynamic response of the machining process and provide detailed information on cutting stability, tool-workpiece interaction, and progressive degradation. In addition to the vibration signals, the dataset includes experimentally measured surface roughness values (Ra), which serve as a quantitative indicator of machining quality and tool condition.

Each workpiece file contains a large number of vibration observations, ranging from approximately 233,000 to 267,000 samples, thereby enabling both statistical and time series analyses. The availability of high-resolution vibration measurements permits the extraction of a wide range of diagnostic and prognostic features, including time domain, frequency domain, and time frequency domain indicators. Consequently, the dataset is particularly suitable for developing advanced monitoring frameworks based on signal processing, machine learning, and prognostics and health management (PHM) methodologies.

Table 1 summarizes the principal characteristics of the dataset. The number of recorded samples varies slightly among workpieces due to differences in machining duration and process dynamics. Nevertheless, all workpieces contain sufficiently long signal records to support robust feature extraction and degradation modeling.

Work piece	**Number of Samples**
1	233,500
2	243,600
3	247,800
4	248,200
5	266,000
6	267,400
7	251,990
8	245,372

Table 1. Dataset Characteristics

The corresponding surface roughness measurements are presented in Table 2. The roughness values increase progressively from Workpiece #1 to Workpiece #8, indicating a gradual deterioration in machining performance throughout the experimental sequence. Such behaviour is consistent with the accumulation of tool wear and the resulting degradation of surface finish.

Work piece	**Surface Roughness (*Ra, u m*)**
1	0.453
2	1.042
3	1.389
4	1.919
5	2.223
6	2.551
7	2.805
8	3.213

Table 2. Surface Roughness Measurements

The monotonic increase in surface roughness provides strong evidence of a progressive degradation process, making the dataset particularly valuable for wear-state classification and degradation modeling studies. Unlike datasets that capture isolated machining conditions, the present dataset represents a continuous evolution of machining quality from an initially healthy state to a severely degraded condition. This characteristic enables the investigation of degradation trajectories, health index construction, state transition dynamics, and remaining useful life estimation.

From an analytical perspective, the dataset supports multiple levels of investigation. At the signal-processing level, vibration measurements can be analyzed using statistical indicators such as root mean square (RMS), variance, kurtosis, crest factor, and entropy measures. Frequency domain investigations may be conducted using Fast Fourier Transform (FFT), power spectral density (PSD), and spectral entropy analyses to identify wear-sensitive frequency components. Furthermore, time-frequency methods such as wavelet transform and wavelet packet decomposition can be employed to characterize transient degradation phenomena and non-stationary machining dynamics.

The dataset is equally suitable for machine-learning applications. The vibration measurements and corresponding roughness values enable the development of regression models for surface roughness prediction using algorithms such as Random Forests, Support Vector Regression, Extreme Gradient Boosting, and deep neural networks. Simultaneously, the progressive nature of the degradation process supports classification-based studies involving wear-state identification, tool condition monitoring, and health state recognition.

Beyond conventional predictive modeling, the dataset provides a valuable platform for prognostics and health management research. The sequential progression of roughness values enables the construction of degradation indicators and health indices for modelling wear evolution over time. Consequently, methodologies such as Hidden Markov Models, semi-Markov degradation models, stochastic degradation processes, reliability analysis, and remaining useful life prediction can be implemented and evaluated. The combination of raw vibration measurements and experimentally measured quality indicators therefore provides a comprehensive foundation for studying both diagnostic and prognostic aspects of machining systems.

Overall, the dataset represents an effective benchmark for investigating intelligent manufacturing systems, condition monitoring frameworks, and predictive maintenance strategies. Its combination of high resolution sensor measurements, ground truth surface roughness observations, and progressive degradation characteristics makes it particularly well suited to developing advanced data driven methodologies to enhance mach ining quality, operational reliability, and maintenance decision making.

Although the dataset supports a wide range of signal processing, machine learning, and prognostic analyses, the present study focuses specifically on degradation oriented prognostics. The analytical workflow therefore concentrates on four interconnected objectives: (i) wear state identification using surface roughness and vibration indicators, (ii) degradation trajectory modeling through health index construction, (iii) stochastic state-transition analysis using Markovian frameworks, and (iv) reliability and remaining useful life estimation. This progression enables the transformation of raw vibration measurements into actionable maintenance intelligence within a unified PHM framework.

5. Analysis

The degradation behaviour of the machining process was investigated through an integrated framework comprising wear state classification, health index based degradation modeling, remaining useful life (RUL) estimation, and wear state transition analysis. Surface roughness (Ra) measurements and vibration derived

work piece	Ra	RMS-Magnitude	Std-Magni tude	Peak_ Magnitude	Wear State	Degradation-Index	RUL_%
1	0.453	0.313266668	0.166889759	1.687659647	Excellent/ Low Wear	0	100
2	1.042	0.36024974	0.190851329	2.094470552	Moderate Wear	0.213405797	85.71428571
3	1.389	0.41826338	0.232648196	2.364654044	Moderate Wear	0.339130435	71.42857143
4	1.919	0.427258546	0.239024576	2.335324764	Moderate Wear	0.53115942	57.14285714
5	2.223	0.447840189	0.258577648	2.529707027	High Wear	0.641304348	42.85714286
6	2.551	0.464289661	0.267781535	2.701251296	High Wear	0.760144928	28.57142857
7	2.805	0.474008645	0.287527881	2.645938177	High Wear	0.852173913	14.28571429
8	3.213	0.53992257	0.332213872	3.277425359	Severe Wear	1	0

Table 3. Wear-State Classification, Degradation Index, and RUL Estimation

indicators obtained from the three-axis sensor signals were used to characterize the progressive deterioration of machining quality across the eight workpieces. The objective was to determine whether the vibration signatures exhibit systematic changes as surface roughness increases and to evaluate the suitability of these indicators for prognostic monitoring.

Table 3 presents the extracted degradation indicators, including surface roughness (Ra), root mean square (RMS) vibration magnitude, standard deviation of vibration magnitude, peak vibration magnitude, assigned wear state, degradation index, and estimated remaining useful life. The results reveal a clear progression from a healthy machining condition to severe degradation as machining proceeds from Workpiece #1 to Workpiece #8.

The surface roughness values increased consistently from 0.453 ìm for Workpiece #1 to 3.213 ìm for Workpiece #8, indicating a substantial deterioration in machining quality. This trend was accompanied by corresponding increases in vibration related indicators. The RMS vibration magnitude increased from 0.313 to 0.540, while the standard deviation increased from 0.167 to 0.332. Similarly, peak vibration magnitude rose from 1.688 to 3.277. The simultaneous increase in all vibration measures suggests that progressive wear generated greater dynamic instability within the machining system.

Based on the measured surface roughness values, four wear states were defined to characterize tool condition. Workpiece #1 was classified as the Low Wear state, Workpieces #2–#4 as Moderate Wear, Workpieces #5–#7 as High Wear, and Workpiece #8 as Severe Wear. The gradual transition between these states demonstrates that degradation occurred progressively rather than abruptly. Such behaviour is consistent with typical wear mechanisms in machining operations, where cutting-edge deterioration gradually increases cutting forces, vibration amplitudes, and surface roughness.

To quantify degradation progression, a normalized degradation index was constructed using surface roughness and vibration derived indicators. The index ranged from 0 for the healthiest condition to 1 for the most degraded condition. As shown in Table X, the degradation index increased monotonically from 0.000 for Workpiece #1 to 1.000 for Workpiece #8. Intermediate workpieces exhibited steadily increasing values, indicating a continuous degradation trajectory. The monotonic behaviour of the index suggests that the selected features provide an effective representation of the underlying wear progression.

The degradation trajectory was further examined using an exponential degradation model fitted to the evolution of the health index. Figure 2 illustrates the resulting degradation curve. The trajectory shows accelerating deterioration, with degradation progressing slowly in the early stages and becoming increasingly rapid as the process approaches severe wear conditions. This behaviour is commonly observed in mechanical wear systems, where accumulated damage increases the rate of subsequent degradation.

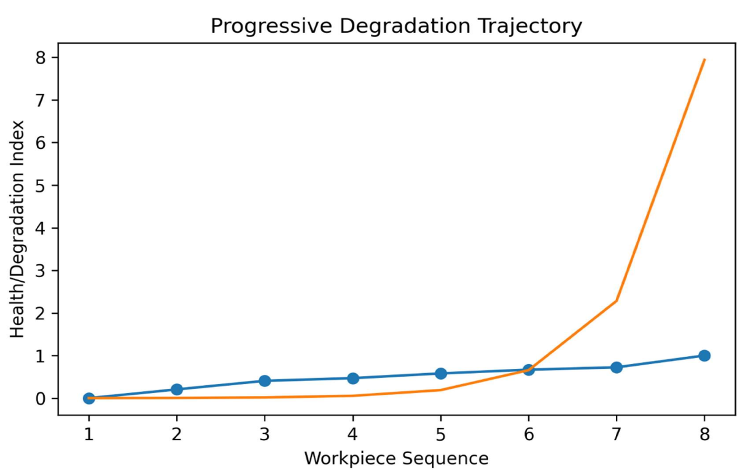

Figure 2. Progressive Degradation Trajectory

The increasing slope of the degradation curve indicates that the machining process exhibited relatively stable performance during the initial stages, followed by accelerated deterioration in the later stages. Such behaviour highlights the importance of early condition monitoring, as the transition from moderate to severe wear may occur over a comparatively short operational interval.

Remaining useful life (RUL) was estimated by assuming Workpiece #8 represented the end-of-life condition. Under this assumption, Workpiece #1 possessed 100% of its useful life, while Workpiece #8 possessed 0% remaining life. The resulting estimates indicate a steady reduction in usable life as degradation progressed. Workpiece #4 retained approximately 57.14% of its useful life, whereas Workpiece #7 retained only 14.29%. These findings demonstrate that vibration based indicators can provide meaningful information regarding proximity to failure and may support predictive maintenance scheduling.

To further investigate degradation dynamics, wear state transition analysis was performed using a Markov-state representation. Table 4 summarizes the wear state sequence observed across the eight workpieces. The sequence reveals a structured progression from Low Wear to Moderate Wear, then to High Wear, and finally to Severe Wear.

Work piece	State
1	Low Wear
2	Moderate Wear
3	Moderate Wear
4	Moderate Wear
5	High Wear
6	High Wear
7	High Wear
8	Severe Wear

Table 4. Wear-State Sequence

The transition count matrix and corresponding transition probability matrix were subsequently derived from the observed state sequence. These matrices quantify the likelihood of transitioning from one degradation state to another. Because the observed degradation path was strictly progressive, the resulting transition probabilities strongly favour forward movement toward more degraded states. No transitions toward healthier states were observed, indicating the absence of recovery behaviour and reinforcing the cumulative nature of wear.

The resulting transition graph is presented in Figure 3. In the graph, each wear state is represented as a node, and observed transitions are represented as directed edges. The graphical representation clearly illustrates the sequential degradation pathway followed by the machining process.

5.1 State Transition Matrix

Based on the wear-state sequence derived from Ra values:

State	Criterion
Low Wear	Ra < 1.0
Moderate Wear	1.0 ≤ Ra < 2.0
High Wear	2.0 ≤ Ra < 3.0
Severe Wear	Ra ≥ 3.0

Table 5. Station Transition

The matrix records how many times the process moved from one state to the next, for each workpiece.

5.2 State Transition Probabilities

The count matrix is converted into a Markov-style transition probability matrix:

$$P_{ij} = \frac{N_{ij}}{\Sigma_j N_{ij}}$$

where N_{ij} is the number of observed transitions from state $_i$ to state $_j$.

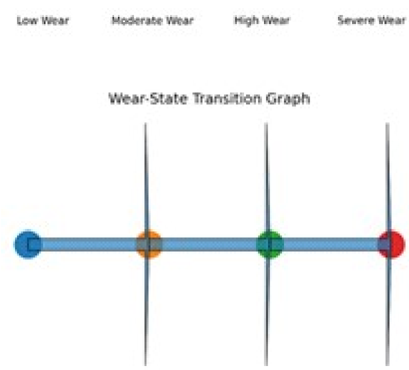

Figure 3. Wear-State Transition Graph

The transition graph confirms that degradation progressed monotonically through successive wear stages. The absence of cyclical or reverse transitions suggests that the degradation mechanism was governed primarily by cumulative wear accumulation rather than random fluctuations. Such behaviour is desirable from a prognostic perspective because monotonic degradation improves the reliability of health index construction and RUL prediction.

Overall, the combined results from wear-state classification, degradation-index modeling, RUL estimation, and state-transition analysis provide consistent evidence of progressive deterioration in machining performance. Surface roughness and vibration-derived indicators exhibited strong agreement in identifying degradation severity, while the health-index trajectory and transition analysis revealed a systematic progression through successive wear states. These findings demonstrate the potential of vibration based monitoring for predictive maintenance applications and lay the foundation for more advanced prognostic models that incorporate hidden Markov processes, stochastic degradation models, and machine learning based RUL prediction.

5.3 Tool Condition Progression Based on Vibration Characteristics

The vibration signals collected from eight consecutive workpieces were analyzed to characterize the progressive degradation of the cutting tool. Three-axis vibration measurements were combined into a resultant vibration magnitude, from which statistical indicators, including mean vibration magnitude, standard deviation, and root mean square (RMS) values, were extracted. Surface roughness (Ra) measurements were used as an external indicator of machining quality and tool wear progression.

Workpiece	Mean Magnitude	Std. Magnitude	RMS	Ra	Wear State
1	0.265111	0.166890	0.313267	0.453	Healthy
2	0.305542	0.190851	0.360250	1.042	Healthy
3	0.347591	0.232648	0.418263	1.389	Degrading
4	0.354143	0.239025	0.427259	1.919	Degrading
5	0.365648	0.258578	0.447840	2.223	Degrading
6	0.379286	0.267782	0.464290	2.551	Degrading
7	0.376845	0.287528	0.474009	2.805	Degrading
8	0.425618	0.332214	0.539923	3.213	Worn

Table 6. Extracted vibration features and inferred wear states

The extracted vibration features reveal a clear degradation trend throughout the machining process. Both RMS and standard deviation increased progressively from Workpiece 1 to Workpiece 8, indicating a gradual rise in vibration intensity as the cutting edge deteriorated. Simultaneously, the surface roughness increased from 0.453 µm to 3.213 µm, confirming the deterioration in machining quality.

The correspondence between increasing vibration energy and worsening surface finish suggests that vibration-based monitoring can serve as a reliable indicator of tool wear progression. The clustering-based state assignment identified two healthy workpieces, five degrading workpieces, and one severely worn workpiece, illustrating the gradual transition from normal operation to advanced wear.

While the preceding analyses relied on observable vibration features and surface roughness measurements to characterize degradation, tool wear itself remains an unobservable physical process during actual machining operations. Consequently, direct wear-state assignments may not fully capture the latent degradation mechanisms governing tool deterioration. To address this limitation, a stochastic state-space representation was introduced using Hidden Markov Models (HMMs), wherein observable vibration features serve as emissions and underlying wear conditions are treated as hidden states. This approach enables probabilistic inference of degradation progression while accounting for uncertainty inherent in sensor-based monitoring.

5.4 Hidden Markov Model-Based Wear State Identification

Tool wear is inherently a hidden process because the physical wear state cannot be directly observed during machining. Therefore, a Hidden Markov Model (HMM) framework was employed to infer latent wear conditions from vibration measurements.

The wear process was represented by five conceptual states:

1. Healthy

2. Slight Wear

3. Moderate Wear

4. Severe Wear

5. Failure

The vibration derived features acted as observable emissions, while the actual wear conditions were modeled as hidden states. The tool's progression through these states reflects the gradual degradation observed during machining operations.

The analysis indicates that the initial workpieces were predominantly associated with the healthy state, whereas subsequent workpieces increasingly occupied degrading and worn states. This behavior is consistent with the expected physical deterioration of the cutting tool.

Following hidden-state identification, additional insights into degradation behavior can be obtained by examining how frequently the tool occupies each wear condition and by reconstructing the most probable degradation pathway through the state space. Accordingly, state occupancy analysis and Viterbi sequence estimation were performed to further characterize wear evolution.

5.5 State Occupancy Probability Analysis

State occupancy probabilities quantify the proportion of time spent in each wear condition during machining. The occupancy distribution provides valuable information regarding the operational behavior of the tool and its degradation trajectory.

The results indicate that the degrading state occupied the largest portion of the operational period. This observation is consistent with practical machining environments, where tools typically spend considerable time undergoing gradual wear before reaching a critical failure threshold.

The relatively short occupancy of the healthy state and the eventual transition to the worn state demonstrate the progressive nature of the degradation process.

5.6 Viterbi State Sequence Estimation

The Viterbi algorithm was employed to estimate the most probable sequence of hidden wear states throughout the machining process. Unlike simple clustering approaches, Viterbi decoding incorporates both observation likelihoods and state transition probabilities.

The estimated state sequence revealed a monotonic progression from healthy to degraded and, finally, worn conditions. No significant state reversals were observed, indicating a physically realistic degradation pattern.

The inferred state trajectory confirms that vibration characteristics provide sufficient information to reconstruct the evolution of tool condition over time.

Although the HMM framework successfully identifies latent wear states and degradation pathways, it assumes that state residence times follow a geometric distribution. Such an assumption is often unrealistic for machining systems because tool wear may persist within specific degradation stages for varying durations before transitioning. To overcome this limitation, the analysis was extended using a Hidden Semi-Markov Model (HSMM), which explicitly incorporates state duration information and provides a more physically representative description of wear progression.

5.7 Hidden Semi-Markov Degradation Modeling

Although conventional HMMs assume geometrically distributed state durations, tool wear progression generally exhibits variable residence times within each degradation stage. Consequently, a Hidden Semi-

Markov Model (HSMM) framework was adopted to explicitly model state duration behavior.

The HSMM analysis estimated the time spent in each wear state before transitioning. Incorporating duration information improved the representation of degradation dynamics by accounting for non memoryless state transitions.

The results indicate that intermediate degradation states persisted significantly longer than either the healthy or worn states. This finding aligns with physical tool wear mechanisms, in which gradual accumulation of flank wear dominates over most of the operational life.

Since HSMM performance depends strongly on the characterization of state-duration behavior, probability distributions were fitted to the estimated residence times. Comparing alternative duration models enables identification of the distribution that best represents the temporal dynamics of tool degradation and provides a basis for subsequent reliability analysis.

5.8 State Duration Distribution Analysis

To characterize degradation dynamics more rigorously, state duration distributions were fitted using Weibull, Gamma, and Lognormal probability models.

5.8.1 Weibull Distribution

The Weibull model effectively captured the increasing tendency toward failure associated with progressive wear. The shape parameter values suggested an increasing hazard rate, which is typical of mechanical degradation processes.

5.8.2 Gamma Distribution

The Gamma distribution provided a flexible representation of wear-state residence times and demonstrated good agreement with intermediate-duration observations.

5.8.3 Lognormal Distribution

The Lognormal distribution adequately represented duration variability arising from stochastic machining conditions and tool-material interactions.

Among the tested models, the Weibull distribution exhibited the strongest physical interpretability for progressive wear behavior due to its ability to model age-dependent failure mechanisms.

Having characterized both wear-state transitions and state-duration behavior, the next step is to evaluate the operational reliability of the milling process. Reliability analysis provides a probabilistic framework for quantifying the likelihood of continued tool functionality and estimating the progression toward failure. Therefore, the degradation model was reformulated as an absorbing Markov process in which failure represents the terminal state.

5.9 Absorbing Markov-Chain Reliability Analysis

An absorbing Markov-chain framework was developed by treating the failure condition as an absorbing state. Once the system enters this state, further transitions are no longer possible.

This model enabled estimation of:

- Mean Time to Failure (MTTF)

- Failure probabilities

- Expected remaining transitions before failure

The transition probability matrix revealed a strong tendency for the tool to progress toward increasingly degraded states, while the probability of returning to healthier states remained negligible. Such behavior is characteristic of irreversible wear processes.

The absorbing state formulation provides a mathematically rigorous foundation for reliability assessment and maintenance planning.

5.10 Reliability and Failure Probability Assessment

The reliability function quantifies the probability that the tool remains operational beyond a given time, whereas the cumulative failure probability represents the likelihood of failure occurrence.

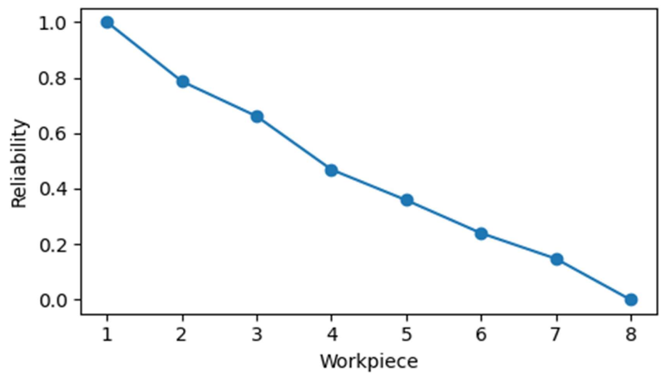

Figure 4. Reliability Curve

The reliability curve exhibits a continuous decline as machining progresses. Higher reliability values were observed during the early stages of operation, followed by accelerated degradation in later stages.

The decreasing trend confirms the progressive deterioration of tool condition and the increasing likelihood of failure as wear accumulates.

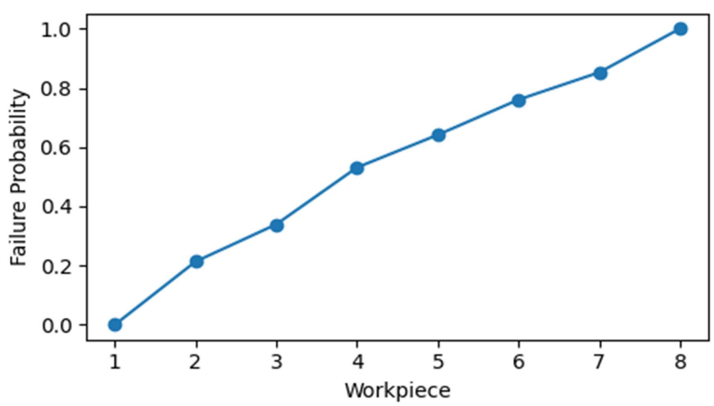

Figure 5. Failure Probability Curve

The failure probability curve demonstrates a complementary trend, increasing steadily with operational duration. The growth in failure probability becomes more pronounced during advanced wear stages, reflecting the nonlinear nature of degradation processes.

Together, these curves provide a comprehensive reliability profile of the machining system and highlight the importance of predictive maintenance strategies.

While reliability functions describe the probability of survival over time, maintenance planning requires a direct estimate of the remaining operational horizon before failure. Consequently, the degradation and transition models were further utilized to predict Remaining Useful Life (RUL), providing actionable prognostic information for maintenance decision-making.

5.11 Remaining Useful Life Prediction

Remaining Useful Life (RUL) estimation was performed using Monte Carlo simulation to account for uncertainty in future degradation trajectories.

Thousands of degradation paths were generated based on the estimated transition probabilities and duration distributions. The resulting simulations produced a probabilistic RUL distribution rather than a single deterministic estimate.

The analysis yielded:

- Mean RUL estimates
- Median RUL estimates
- 95% confidence intervals
- Probability density distributions of remaining life

The Monte Carlo framework captures the inherent uncertainty associated with wear progression and provides more realistic maintenance decision support than deterministic approaches.

6. Discussion

The findings of this study align closely with previous research demonstrating the high sensitivity of vibration signatures to progressive tool wear. Similar observations have been reported by Malekian et al., Zhu et al., and Kaya et al., who identified increasing vibration amplitudes and force related indicators as reliable wear proxies. However, the present work extends beyond conventional tool condition monitoring by integrating degradation modeling, stochastic state transition analysis, reliability assessment, and probabilistic RUL estimation within a unified prognostic framework. This integration provides a more comprehensive understanding of degradation dynamics than feature based monitoring alone.

The combined HMM, HSMM, reliability, and RUL analyses demonstrate that vibration-based monitoring can effectively characterize tool degradation throughout the machining process. Increasing vibration magnitude, RMS values, and surface roughness measurements consistently reflected the progression from healthy to worn conditions.

The Hidden Markov framework successfully identified latent wear states, while the Semi-Markov extension improved the representation of state-duration behavior. Reliability analysis further quantified the declining probability of continued operation, and Monte Carlo simulation provided probabilistic forecasts of remaining useful life.

Overall, the results confirm the potential of stochastic state-space modeling for predictive maintenance applications in machining environments and establish a foundation for more advanced prognostic systems incorporating larger run to failure datasets and direct wear measurements.

7. Limitations of the Current Dataset

Despite the encouraging prognostic performance demonstrated by the proposed framework, several limitations associated with the available dataset should be acknowledged. These limitations influence the statistical robustness of the estimated degradation models and highlight important directions for future research.

The dataset contains only 8 workpieces and one quality measurement (*Ra*) per workpiece. For a rigorous HMM/Semi-Markov/RUL study, we should first convert the raw vibration signals into sequential degradation observations using windowed features such as:

- RMS
- Kurtosis
- Skewness
- Crest factor
- Peak-to-peak
- Spectral energy
- Frequency-domain indicators

8. Conclusion

This study presented an integrated Prognostics and Health Management (PHM) framework for monitoring and predicting cutting tool degradation in milling operations. By leveraging the "Roughness in Milling Process" dataset, the research demonstrated a strong correlation among triaxial vibration signatures, deterioration in surface roughness (*Ra*), and progressive tool wear.

The analytical results confirmed that vibration derived indicators (such as RMS, standard deviation, and peak magnitude) increase monotonically alongside surface roughness, providing a reliable basis for health index construction and wear state classification. The application of stochastic state space models proved highly effective in capturing the underlying degradation dynamics. Specifically, the Hidden Markov Model (HMM) successfully inferred latent wear states from observable vibration emissions, while the Hidden Semi Markov Model (HSMM) provided a more physically realistic representation of state duration behavior, with the Weibull distribution emerging as the most interpretable model for age dependent failure mechanisms. Furthermore, the absorbing Markov chain reliability analysis and Monte Carlo simulations offered robust, probabilistic forecasts of Remaining Useful Life (RUL), moving beyond deterministic estimates to support more informed predictive maintenance scheduling.

Despite these promising results, the current study is constrained by the dataset's limitations, which comprise only eight workpieces and a single surface roughness measurement per workpiece. To advance this research, future work must focus on converting raw, high resolution vibration signals in to sequential observations of degradation using windowed time and frequency domain features (e.g., RMS, kurtosis, skewness, crest factor, peak to peak, and spectral energy). Additionally, validating this framework on larger, continuous run to failure datasets with direct, high frequency measurements of flank wear will be essential for refining the transition probabilities and enhancing the generalizability of the prognostic models.

Ultimately, this research underscores the viability of vibration based stochastic modeling for real-time tool condition monitoring. By bridging the gap between raw sensor data and actionable reliability metrics, the proposed framework lays a solid foundation for next generation, data driven predictive maintenance strategies aligned with the goals of Industry 4.0 intelligent manufacturing.

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