A New Searching Algorithm Using Quantum Systems

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ABSTRACT: *To search for an element in an unsorted database, it requires O(N) operations using classical computers, where it takes only O ($\sqrt{N}$) operations in quantum systems. In this paper, we provide an in-depth look at the existing quantum searching algorithms and emphasize the quantum entanglement feature to propose a new technique for sorting N elements and thus improve the sorting process. This paper presents two sort algorithms over quantum computing system.*

Keywords: Quantum Searching Algorithms, Quantum Entanglement, Element Probability

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1. Introduction

Quantum computation has gained a lot of attention especially for the need of searching unsorted database in a faster way rather than using the classical search mechanisms. Many search problems have been investigated in the computer science field, which can be solved by speeding up the searching algorithms [1]. Grover introduced one of the most important quantum searching algorithms, which is faster than the well-known classical searching algorithms [2].

In classical computers, each bit can be represented by either zero or one, whereas quantum systems employ a Qubit that contains both zero and one simultaneously with different probabilities. This phenomenon is called superposition, which helps to perform several tasks at the same time and much faster than traditional computing [3], [4]. Grover's algorithm can be very beneficial for data searching from unsorted dataset in a diverse nature. For searching in N elements, Grover's algorithm takes only $O(\sqrt{n})$ steps compared to $O(N)$ steps in classical computers [5, 6].

Quantum entanglement technique is considered being one of the most important properties of a quantum computer, which enables Qubits to simultaneously handle two states. An entangled state can only be accomplished, when two qubits or more are able to communicate with each other. In other words, we can specify bit (s) depending on the measured value of the other bit (s). It is obvious that in case of two qubits, have four combination values 00, 01,10,11 are conveyed simultaneously (superposition). Moreover, quantum systems should be able to allocate any combination patterns for these possible values. For example, when combining the values of 00 and 11, if one state of any single qubit is defined, the second state of the related qubit automatically will be known which is called entangled states. Entangled states indicated that these states could not be separated into different states. Thus, any entangled state always has to be a superposition state; however, a superposition state is not always an entangled state [7].

Since, quantum computing systems are still in their infancy phases of growth, the experimental trials of searching algorithms need quantum hardware to perform tasks simultaneously. The proposed algorithm in this paper combines some of the quantum features with searching techniques in order to speed up the searching process in conventional environments. This paper provides a new technique that employs quantum entanglement property to sort N elements.

This paper is organized as follows. Section II, presents the related work of existing quantum searching algorithms. In section II, quantum-searching applications are discussed. Section III, provides the proposed algorithms to sort static data set by using entanglement bits. Also, it offers analysis of the proposed algorithms. Finally, section IV, conclusion of our algorithms.

2. Related work

Authors in [8] demonstrated the hardware implementation of the main quantum operators that are used in quantum algorithm gates stimulation on classical computers. The authors used efficient software for the purpose of simulation. The paper presents the hardware used to perform all the functions needed in Grover's quantum search algorithm. In addition, [9] provides a numerical simulation of Grover's quantum search algorithm using microwave devices and frequencies as an implementation solution due to the technical problems and the high cost of quantum components. The paper concluded that, at a microwave resonator, an electromagnetic field can be 0 and 1 depending on the selection of the predetermined field level.

Poto *et. al.* [10] introduced a new algorithm called quantum existence testing algorithm, which is an exceptional case of quantum counting. This algorithm is useful for searching structured and unstructured databases because it uses classical logarithmic techniques. The authors in [11] discussed new approaches that solve the problem of finding the minimum and the maximum values among a set of integers with time complexity of $O(1)$. This approach is successful and effective in the case of using low size quantum registers.

Furthermore, quantum computers are more powerful than classical computers due to the entanglement principle. One of the most interesting quantum computers is duality quantum computer. In this kind of computers, the gates are not unitary which give more flexibility especially for unsorted database search [12]. Another quantum search algorithm called Quantum walk based search algorithm is described in [13]. This algorithm can solve many problems such as element distinctness, restricted range associativity, matrix product verification and group commutatively [13].

Earlier designs of parallel algorithms were constructed in [14]. It is shown that if two oracles have been performed simultaneously on a group of entangled qubits, it would speed up the computation by $\sqrt{2}$ times faster than the sequential quantum search.

Moreover, for any fixed $\sqrt{N}$, the probability of parallel search is a 22 thousand times faster compared with sequential search. A new parallel search algorithm was proposed in [15] where the authors introduced a new technique to improve the performance of the existing quantum search algorithms in case of non-uniform data. The study concluded that the parallel technique demonstrated its ability to handle almost all cases whether the data is uniform or not. Moreover, the paper discussed some hard algorithmic problems such as order finding, factoring and many others that require excessive resources for solving them.

3. Proposed Algorithm

In our proposed algorithm we sort an array of N elements using quantum properties. Entanglement of bits is one of the quantum powerful characteristic where we can determine one bit if we know the other one. For example in {00, 11}, if the first bit is zero, then the second bit will be zero. By using these properties, we can sort our data in ascending or descending order. Our methodology depends on 3 bits and adds an extra bit to the most significant bit and then we separate them into 2 groups by using the divide and conquer algorithm to solve the problem. For example in quicksort and merge sort algorithms, both of them divide the problem into smaller pieces until the smallest unit is reached which enables us to sort the data.

Our sorting algorithm consists of the following steps as shown in Figure 1:

1. Receive N numbers.

2. Convert all Numbers into binary representation.

3. Insert 1 to the most significant bit for each number

4. Create two sets {11, 10} where we know the first bit if we know the second bit. In our state, if the value is 0, then first bit will

be 1. If the second bit is 1, then first will be 1.

5. Now we divide our number set into 2 groups.

6. Remove the most 2 significant bits.

7. Repeat step number 3 until we have a single bit then we apply quantum single bit comparator gate.

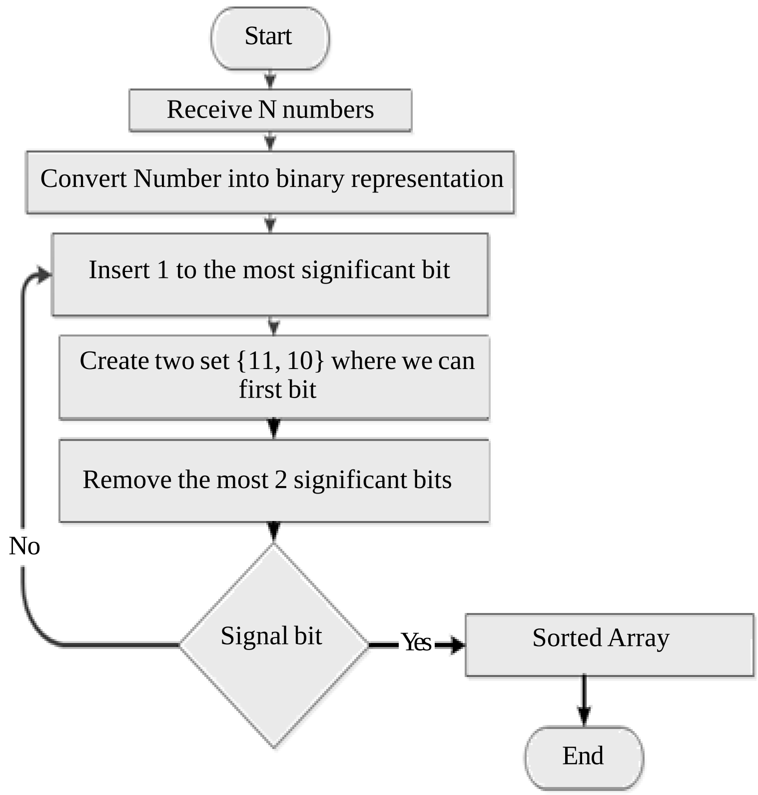

Figure 1. Sorted Algorithm {11, 10} Flow chart

Example:-

Step 1:- {0, 7, 2, 3, 1, 4, 5, 6}

Step 2: {000, 111, 010, 011, 001, 100, 101, 110}

Step 3: {1000, 1111, 1010, 1011, 1001, 1100, 1101, 1110}

Step 4: Classify our value into 2 sets {10, 11} where 10 < 11

Step 5: Set 1 : = { 1000, 1010, 1011, 1001} = {0, 2, 3, 1}.
Set 2 : = {1111,1100,1101,1110} = {7, 4, 5, 6}.

Step 6: We remove the most 2 significant bits then
Set 1 := { 00, 10, 11, 01}
Set 2 := { 11, 00, 01, 10}

Step 7: Repeat step 3 until we have single bits.

To improve our algorithm, we will use Toffoli Quantum gate where the first input is 1 and the second input will be {0, 1} as shown in Figure 2. Details of the example are shown in Figure 3.

By using special design for Toffoli gate, we can check the second bit in a very fast strategy where we have two Outputs and the result will be in which half it belongs. Table 1 Shows the output results and input value for each step.

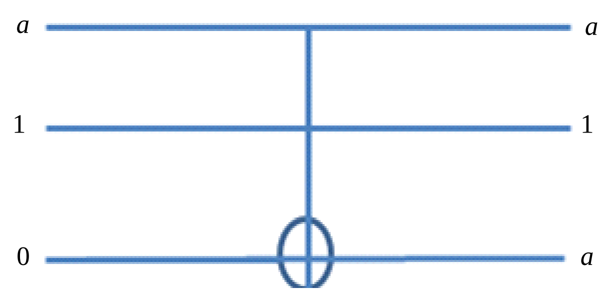

Figure 2.Toffoli Gate

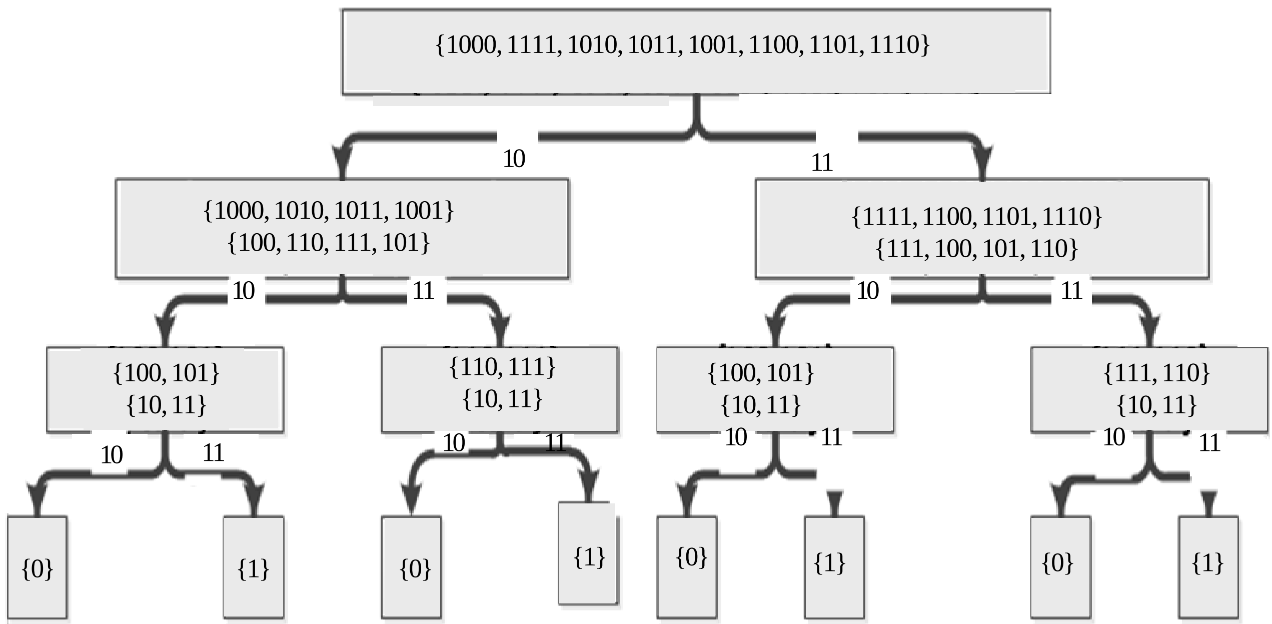

Figure 3. Tree representation {10, 11}

The number of compressions in our algorithm for N elements will be $N \log N$. In worst case analysis, it will be N^2 and in the best case will be 1. But if we improve it by using QuPhaRegisters for each branch of our decision tree, it will produce $(N-2) \log (N-2)$. In this case, we will deal with some parallelism mechanism. In our algorithm each level of our decision tree will add 1 to the most significant bit.

The second algorithm depends on the other entanglement set {00, 01} by inserting 0 in the most significant bit in the number after its binary presentation. The Algorithm works as follows:

1. Receive N numbers.
2. Convert all Numbers into binary representation.
3. Insert 0 to the most significant bit for each number
4. Create two sets {00, 01} where if we know the second bit, we finds the first bit. In our state, if the second bit is Zero then first bit is Zero. If the second bit is 1, then first bit is zero.
5. Divide the number set into two groups.
6. Remove the most two significant bits.
7. Repeat step number 3 until we have a single bit then we apply the quantum single bit comparator gate.

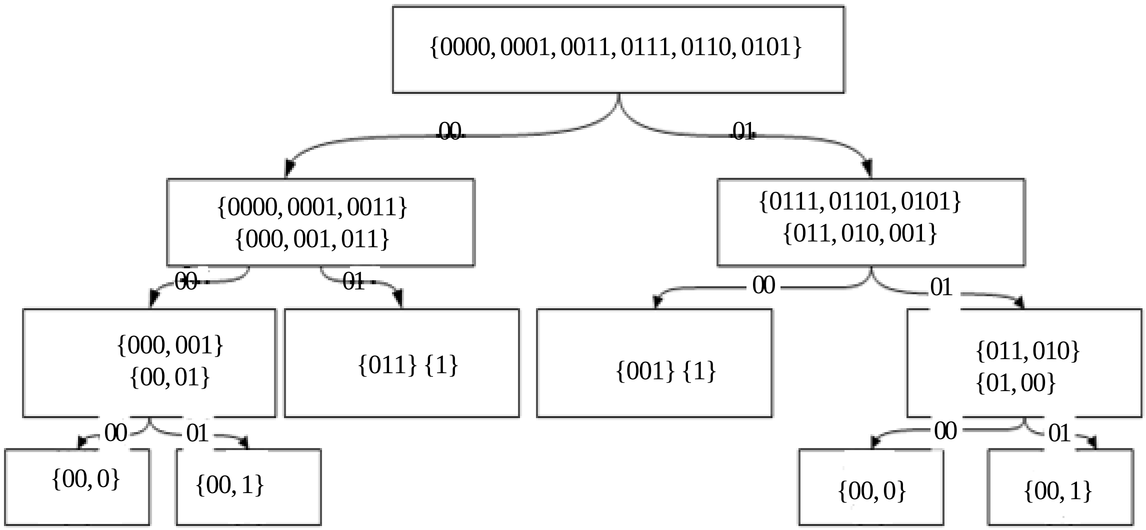

Figure 4.Tree representation {00, 01}

The main advantage of those algorithms is their ability to apply over simple computer gates. The first algorithm adds {1} to the most significant bit, then we can create AND logic gate between the most significant bits. Depending on the AND result, we can divide the set into two groups. The second algorithm adds 0 then checks the two most significant bits to recognize which group they belong. By employing the OR logic gate, we can identify the number to which group it belongs.

Take for example {0, 5, 4, 7}, which is represented in 4 bits binary as follows:

{000, 101, 100, 111}➔{0000, 0101, 0100, 0111}

The algorithm proceeds as follows after applying OR operation:-

{0000}➔0 then remove the first two most significant bits ➔{00}. Insert 0 to the most significant bit then OR gate again ➔{000} ➔again left side. Remove the first two most significant bits ➔0 which is the smallest number where all OR gate results is 0. This number can classify in level zero (smallest value).

{0101}➔1 then remove the first two most significant bits{01}. Insert 0 to the most significant bit then OR gate again➔{001}➔ again left side. Remove the first two most significant bits ➔1 then by OR with zero the number takes place to its correct position.

In Table 2, we present some of the classical sorting algorithms and compare with our algorithm. Quantum Tree algorithm can reduce time complexity like quick sort. Moreover applying parallelism on the next input for tree level, improves time complexity.

Input *a*	0	1
Value of *b*	1	1
Output	0	1
Conclusion	10	11

Table 1. Output results and input value

4. Conclusion

In this paper, we presented some quantum searching algorithms and discussed some quantum concepts. Many applications

Sort Algorithms	Time Complexity	Operation
Bubble Sort	$O\ (N^2)$	Exchanging
Insertion Sort	$O\ (N^2)$	Insertion
Merge Sort	$O\ (Nlog\ n)$	Merging
Heap sort	$O\ (Nlog\ n)$	Selection
Quick sort	$O\ (Nlog\ n)$	Partition
Proposed algorithms	$O\ (N-2)\log\ (n-2))$	Partition

Table 2. Big O for sorting algorithms

apply sorting data algorithms. The main idea of our paper represents Quantum efficiency for sorting data. By applying divide and conquer concept, we employ an interesting Quantum concept which is entanglement of bits. Then we partition our data into two groups for the next level, by repeating the division process we reach a sorted data array. The presented algorithm although has $O\ (n\ logn)$ time complexity and performs very well on smaller data sets.

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