Dynamical Modeling and Controlof Quadrotor

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ABSTRACT: *In this paper a dynamical modeling of a quadrotor is discussed with different frame of references.The state equations are derived using gyroscopic and aerodynamics effects. The motor transfer function is derived from theoretical equations of DC motor with an operational delay. Furthermore a control strategy is presented using Proportional Derivative (PD) Controller for the attitude and trajectory control of the Quadrotor (UAV). Finally, simulation results for the PD controller are generated on MATLAB/ Simulink platform by utilizing the dynamical model of Quadrotor (UAV) .Feedback sensor noise is added to the model output to make simulations more realistic. The simulation results confirm that the Quadrotor (UAV) is following the desired trajectory with a maximum error deviation of 0.2 meters.*

Keywords: Quadrotor, Autonomous Control,PID

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1. Introduction

Nowadays modeling and autonomous attitude control of a Quadrotor (UAV) is an area of interest especially for R&D and military organizations. It is a challenging task to work on highly nonlinear systems whereas in some recent research papers modern control techniques for autonomous attitude control has been tested on Quadrotor (UAV) such as Model Predictive Control (MPC), Linear Quadratic Regulator(LQR/LQG), Fuzzy Logic Controllers etc.

Quadrotor (UAV) consists of four rotors that is a type of UAV having capability of Vertical Take-Off & Landing (VTOL). In the last decade, Quadrotor has been used extensively in civilian and military mission such as rescue, monitoring and surveillance. Due to its effectiveness many R&D and military organizations are focusing towards the design and development of Quadrotor. It has tremendous advantages over manned aircraft like hovering, low visibility, easy access to remote areas, take-off and landing in limited/ remote areas. It has some disadvantages which includes higher maneuverability, energy requirements and difficult to control due to the highly nonlinear dynamics.

In this paper, PD controller has been employed in inner and outer loop for nonlinear model of Quadrotor to achieve the desired attitude and trajectory respectively.

2. Dynamical Modeling

Quadrotor structure consists of four rotors assembled in cross configuration in which pair of opposite propellers rotates in opposite direction with respect to other pair as shown in fig 1& 2.

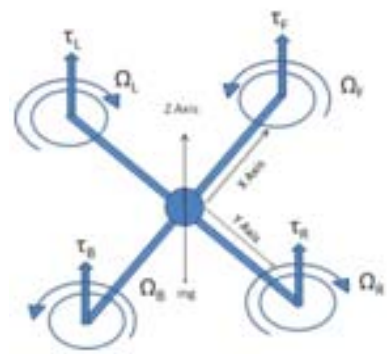

Figure 1. Quadrotor structure

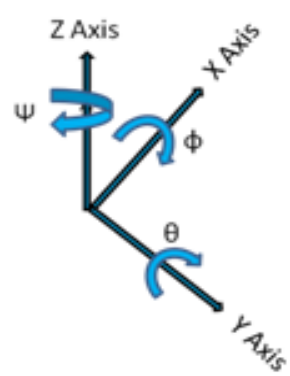

Figure 2. Quadrotor degrees of motion

State variables for Quadrotor's dynamic model are as follows

x = Position along x-axis

y = Position alomg y-axis

z = Position along z-axis

$\dot{x}$ = Velocity in x-axis direction

$\dot{y}$ =Velocity in y-axis direction

$\dot{z}$ =Velocity in z-axis direction

$\ddot{x}$ = Linear Acceleration in x direction

$\ddot{y}$ = Linear Acceleration in y direction

$\ddot{z}$ = Linear Acceleration in z direction

$\emptyset$ = Absolute roll angle

θ = Absolute pitch angle

φ = Absolute yaw angle

$\dot{\emptyset}$ = Roll rate

$\dot{\theta}$ = Pitch ate

$\dot{\varphi}$ = Yaw rate

p = Roll rate in body fixed frame

q = Pitch rate in body fixed frame

r = Yaw rate in body fixed frame

Using the four rotors configuration UAV have 6 Degree of Freedom (DoF) in space according to the inertial frame. Dynamics of UAV are related to the absolute angles ($\emptyset$,θ,φ).

Rotor dynamics are kept simple by introducing a first order transfer function with an operational/ transport delay of 0.1 msec against a Brush-Less DC motor.The rotor dynamics includes a propeller and small gear box.

$$G(s) = \frac{1}{0.1s + 1}$$

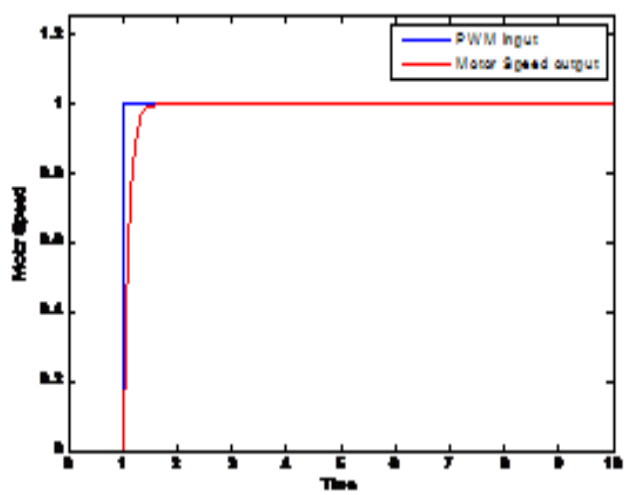

Figure 3. Step response of motor dynamics

1. Altitude

When all rotors are at equal speed they generate equal thrust. consequently the Quadrotor hovers. Increase or decrease of rotors speed cause change in the altitude of the craft depending upon total thrust. Using thrusts of all rotors, Quadrotor motion along z-axis can be controlled as shown in figure below.

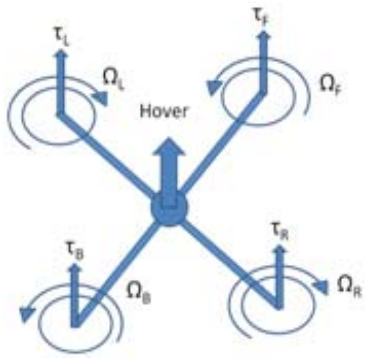

Figure 4. UAV Hovering

2. Roll

When the speed of right or left rotor is increasedordecreased from its nominal value it causes low thrust at slower speed side rotor, as a result roll motion occurs in Quadrotor. Roll motion causes change in y-axis so y-axis motion can be controlled by controlling the roll motion as shown in figure below.

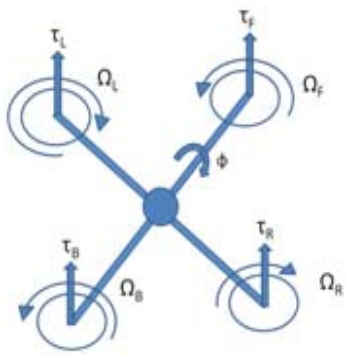

Figure 5. UAV Rolling

3. Pitch

When the speed of the front or back rotor is varied from its nominal value it causes low thrust at slower speed side rotor and as a result pitch motion occurs in Quadrotor. Pitch motion causes change in x-axis therefore x-axis motion can be controlled by controlling the pitch motion as shown in the figure below.

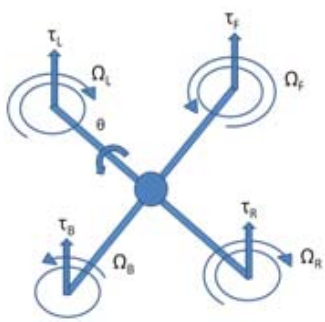

Figure 6. UAV Pitching

4. Yaw

When speed of the front and back or left and right rotors increased/ decreased from its nominal value it causes higher torque at higher speed side rotors and as a result Quadrotor rotates and yaw motion occurs as shown in figure below.

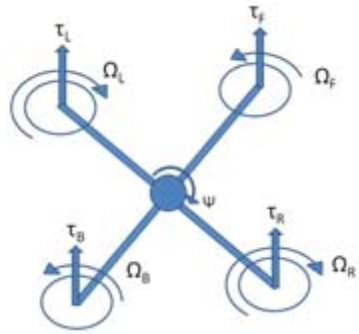

Figure 7. UAV Yawing

Dynamic equations of the Quadrotor are as follows [9].

$$\ddot{x} = -\frac{T}{m}(\cos\emptyset\sin\theta\cos\varphi + \sin\emptyset\sin\varphi) \tag{1}$$

$$\ddot{y} = -\frac{T}{m}(\cos\emptyset\sin\theta\sin\varphi - \sin\emptyset\cos\varphi) \tag{2}$$

$$\ddot{z} = g - \frac{T}{m}(\cos\emptyset\cos\theta) \tag{3}$$

$$\begin{bmatrix}\dot{\emptyset}\\ \dot{\theta}\\ \dot{\varphi}\end{bmatrix} = \begin{bmatrix}1 & \sin\emptyset\tan\theta & \cos\emptyset\tan\theta\\ 0 & \cos\emptyset & -\sin\emptyset\\ 0 & \frac{\sin\emptyset}{\cos\theta} & \frac{\cos\emptyset}{\cos\theta}\end{bmatrix}\begin{bmatrix}p\\ q\\ r\end{bmatrix} \tag{4}$$

$$\begin{bmatrix}\dot{p}\\ \dot{q}\\ \dot{r}\end{bmatrix} = \begin{bmatrix}\left(\frac{J_y - J_z}{J_x}\right)qr\\ \left(\frac{J_z - J_x}{y}\right)pr\\ \left(\frac{J_x - J_y}{J_z}\right)pq\end{bmatrix} + \begin{bmatrix}J_r\theta\cdot\Omega_r\\ -J_r\theta\cdot\Omega_r\\ 0\end{bmatrix} + \begin{bmatrix}\frac{\tau_\varphi}{J_x}\\ \frac{\tau_\theta}{J_y}\\ \frac{\tau_\varphi}{J_z}\end{bmatrix} \tag{5}$$

Where T *and* $\tau_\emptyset, \tau_\theta, \tau_\varphi$ are the required thrust and torques to achieve the desired target. Eq. (1), Eq. (2) & Eq. (3) are simplified form of aerodynamics/ linear acceleration of Quadrotor UAV obtained by employing a translational rotational matrix. Eq. (4) is an inverse of rotational matrix from inertial/ vehicle frame to body fixed frame showing the relationship between absolute angles ($\emptyset, \theta, \varphi$) and angular rates (p, q, r). In Eq. (5) gyro effects are given $(J_y - J_z)qr$ as roll angle, $(J_z - J_x)pr$ pitch angle and $(J_x - J_y)pq$ as yaw angle effect. Where J_x, J_y&J_z represents rotational inertia of Quadrotor along x, y and z axes respectively.

$$\Omega_r = \Omega_R - \Omega_F + \Omega_L - \Omega_B$$

Where Ω representing the rotor speed and Ω_r is the overall residual propeller angular speed.

3. Controller Design

Control design of Quadrotor involves developing a flight control algorithm for under-actuated four rotors responsible for controlling Quadrotor's 6 DoF i.e. its position and orientation. The responsibility for flight control system is to maintain orientation ($\emptyset$, θ,φ) while moving towards the desired location (x_d , y_d, z_d). Control strategy has been developed as shown in figure 3. Inner loop stabilizes the orientation of craft and outer loop drags the craft to the desired location. The System can be rewritten as$\dot{X} = f(U, X)$. The state vector involved in control design is as follows.

State Vector

$$X = [\emptyset\ \dot{\emptyset}\ \theta\ \dot{\theta}\ \varphi\ \dot{\varphi}\ x\ \dot{x}\ y\ \dot{y}\ z\ \dot{z}]$$

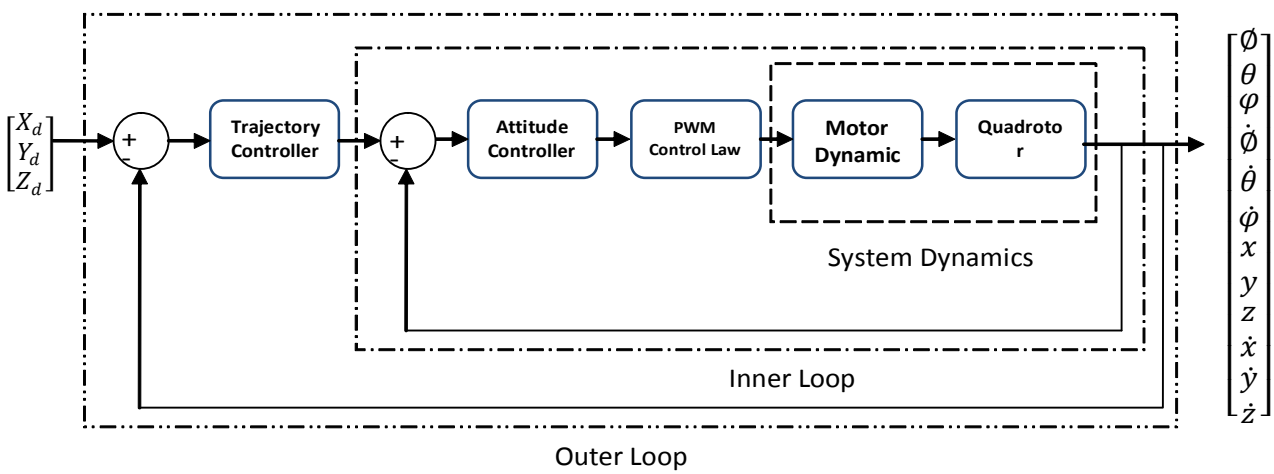

Figure 8. System block diagram

3.1 PD controller

Proportional Derivative control is one of the most implemented control techniques. This control technique requires full state feedback from the sensors. Using state feedback, the controller calculates the difference between desired and current output and adjusts controlled input (U(t)) accordingly. The equation of controller is as follows [3].

$$U(t) = Kp\ e(t) + Kd\ \frac{de(t)}{dt}$$

Where Kp and Kd are proportional and derivative gains respectively. PD control design has been illustrated in figure 8.

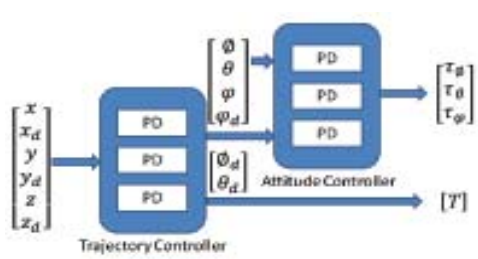

Figure 9. Controller flow chart

3.2 Trajectory Controller in outer loop:

Trajectory controller is a type of PD controller for which mathematical equations are as follows.

$$e_x = x - x_d$$

$$\dot{e}_x = \dot{x} - \dot{x}_d$$

$$U_x = Kp_x\, e_x + Kd_x\, \dot{e}_x \qquad \text{(a)}$$

Similarly for Y-axis and Z-axis motion

$$U_y = Kp_y\, e_y + Kd_y\, \dot{e}_y \qquad \text{(b)}$$

$$U_z = Kp_z\, e_z + Kd_z\, \dot{e}_z \qquad \text{(c)}$$

The flight control algorithm in outer loop proceeded by Equations directly derived from dynamics equations. Simplified forms of Eq. 1 to 3 are as follows [9].

$$\ddot{x} \triangleq -\frac{T}{m}\cos\emptyset\sin\theta \qquad (6)$$

$$\ddot{y} \triangleq \frac{T}{m}\sin\emptyset \cdot \qquad (7)$$

$$\ddot{z} \triangleq g - \frac{T}{m}\cos\emptyset\cos\theta \cdot \qquad (8)$$

$$\begin{bmatrix}\ddot{\emptyset}\\ \ddot{\theta}\\ \ddot{\varphi}\end{bmatrix} = \begin{bmatrix}\frac{\tau_\varphi}{J_x}\\ \frac{\tau_\theta}{J_y}\\ \frac{\tau_\varphi}{J_z}\end{bmatrix} \qquad (9)$$

To obtain

$$\ddot{x} = U_x\,, \ddot{y} = U_y\,, \ddot{z} = U_z$$

Desired roll angle ($\emptyset_d$) and pitch angle (θ_d) can be computed, substituting $\ddot{z} = U_z$ in Eq. 8 for $\frac{T}{m}$.

$$\frac{T}{m} = \frac{g - U_z}{\cos\emptyset\cos\theta} \qquad (10)$$

Solving Eq. 6 & 7 for $\emptyset_d$ and θ_d respectively by substituting Eq. (10).

$$\emptyset_d = \tan^{-1}\left(\frac{U_y\cos\theta_d}{g - U_z}\right) \qquad (11)$$

Where θ_d is as follows

$$\theta_d = \tan^{-1}\left(\frac{U_x}{U_z - g}\right) \qquad (12)$$

3.3 Attitude Controller in inner loop:

Attitude controller is also a PD controller for which mathematical equations are as follows.

$$e_{\emptyset} = \emptyset - \emptyset_d$$

$$\dot{e_{\emptyset}} = \dot{\emptyset} - \dot{\emptyset_d}$$

$$\tau_{\emptyset} = \frac{I_{xx}}{L}(Kp_{\emptyset}\, e_{\emptyset} + Kd_{\emptyset}\, \dot{e}_{\emptyset}) \qquad \text{(d)}$$

Similarly for pitch and yaw angle

$$\tau_{\theta} = \frac{I_{yy}}{L}(Kp_{\theta}\, e_{\theta} + Kd_{\theta}\, \dot{e}_{\theta}) \qquad \text{(e)}$$

$$\tau_{\varphi} = \frac{I_{zz}}{L}(Kp_{\varphi}\, e_{\varphi} + Kd_{\varphi}\, \dot{e}_{\varphi}) \qquad \text{(f)}$$

3.4 PWM Control Law:

Motors of Quadrotor take input as PWM whereas the controller output is in the form of required thrust and torques to stabilize the orientation of the craft and achieve desired/ target position. PWM control law is a translation between thrust and torques to the PWM pulses for rotors. It involves below mathematical equations [4].

$$T_{mot} = K_T\, PWM_{mot}$$

$$\tau_{mot} = K_{\tau}\, PWM_{mot}$$

Where $K_T =$ and K_{τ} are motor specific constants.

$$\begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix} = G \times \begin{bmatrix} T \\ \tau_{\emptyset} \\ \tau_{\theta} \\ \tau_{\varphi} \end{bmatrix}$$

With

$$G = \begin{bmatrix} K_T & K_T & K_T & K_T \\ 0 & -l \times K_T & 0 & l \times K_T \\ l \times K_T & 0 & -l \times K_T & 0 \\ K_{\tau} & K_{\tau} & K_{\tau} & K_{\tau} \end{bmatrix}^{-1}$$

Where l is the length of arm/ wing.

Motors have saturation in terms of speed. It is important to apply saturation a type of nonlinearity to make system close to real.

$$\begin{bmatrix} PWM_F \\ PWM_R \\ PWM_B \\ PWM_L \end{bmatrix} = Sat \begin{bmatrix} U_1 + offset \\ U_2 + offset \\ U_3 + offset \\ U_4 + offset \end{bmatrix}$$

Where *offset* is a prior defined bias to counter balance the weight of Quadrotor and resultant PWM pulses are saturated to the maximum threshold of speed of the motors.

4. Simulation

Simulation of attitude and trajectory control of Quadrotor has been performed in Matlab/ Simulink. Dynamics equation of motion Eq. (1) to Eq. (5) are used to model Quadrotor. The output of system dynamics block is accelerations (linear and angular) and velocity (linear and angular) that are further integrated to obtain velocity (linear and angular) and position (linear and angular). Initial conditions for all state variables are set to zero. Sensor noise is induced in the system to make simulation more realististic.

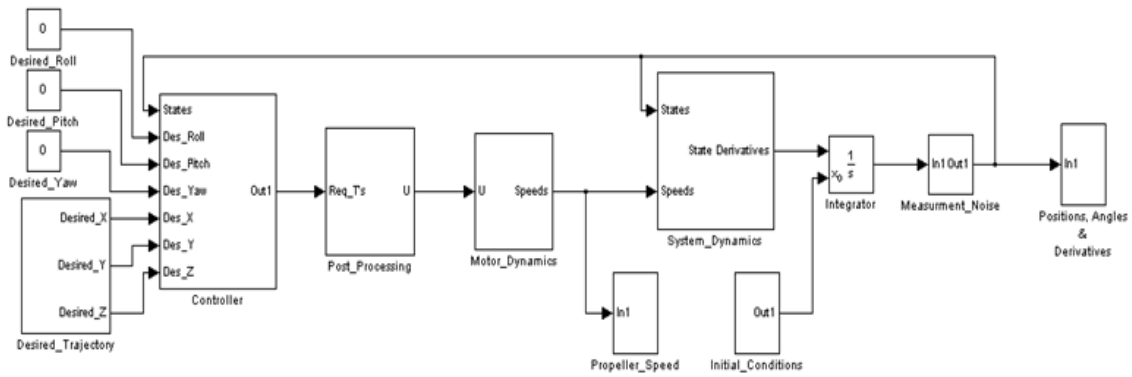

Figure 10. Simulink model

S.no	Parameter	Value	Unit
1	Gravitational acceleration (g)	9.8	m/s^2
2	Mass of Quadrotor (m)	0.4794	Kg
3	Length of wings (l)	0.225	m
4	Rotational Inertia along x-axis (J_x)	0.0086	$Kg.m^2$
5	Rotational Inertia along y-axis (J_y)	0.0086	$Kg.m^2$
6	Rotational Inertia along z-axis (J_z)	0.0172	$Kg.m^2$
7	Residual Rotational Inertia(J_r)	3.7404×10^{-5}	$Kg.m^2$
8	*offset*	0	rad/s
9	Motor constant (K_T)	3.13×10^{-5}	-
10	Motor constant (K_τ)	9×10^{-7}	-

Table 1. Quadrtor's Dynamic Constant Parameter For Simulation

Variable	Parameter Value	
i	$K_{i,p}$	$K_{i,d}$
x	1.18	4
y	1.1	1.5
z	1	1
$\emptyset$	3	1.5
θ	1.5	2
φ	1.2	2

Table 2. PD Controller Parameter Values

The Quadrotor model was tested/ simulated to follow the desired trajectory while the actual response of the model obtained by using above PD controller parameter values is as follows.

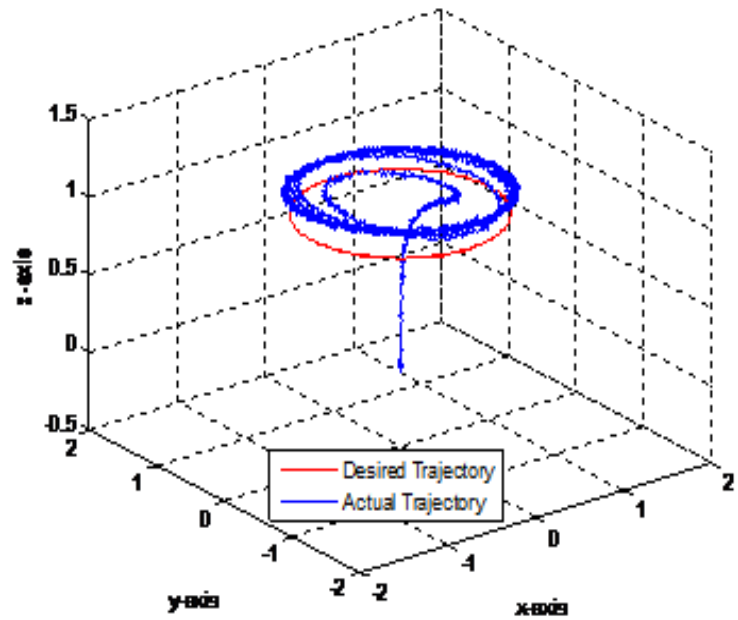

Figure 11. UAV Trajectory results

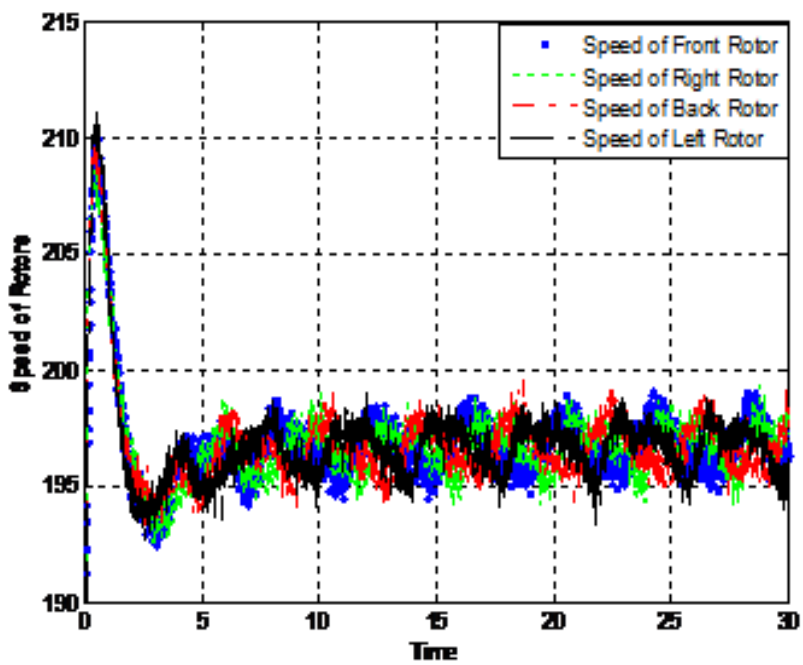

Figure 12. UAV Motors speed

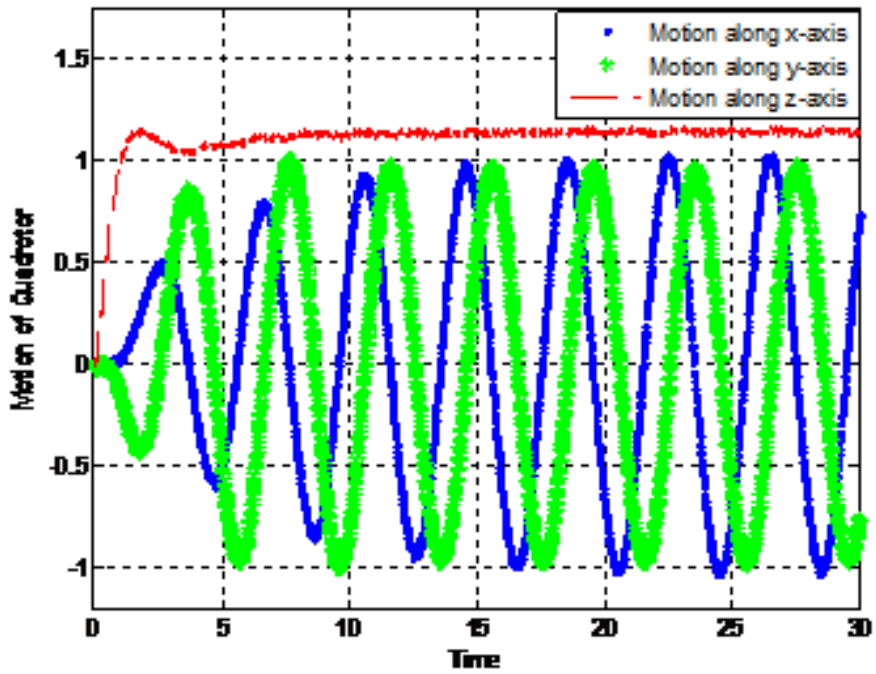

Figure 13. UAV Motion along x, y & z axes

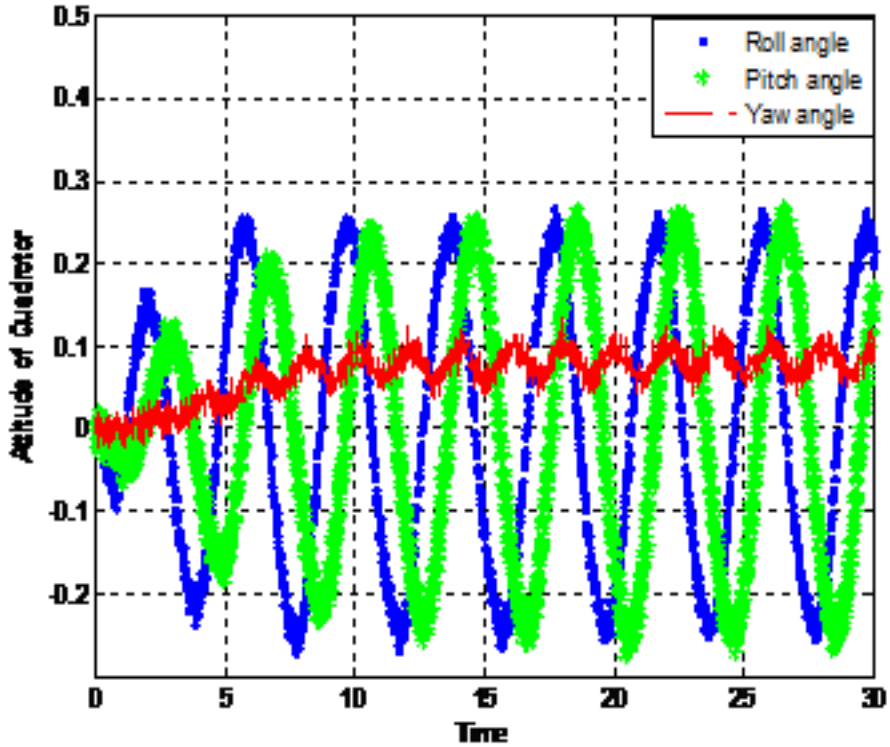

Figure 14. UAV Absolute Attitude angles

The statistical data of above simulation confirm that the trajectory maximum error deviation is 0.2m. In this simulation the performance check of each controller developed for each axis of motion is conducted and results are satisfactory as shown below.

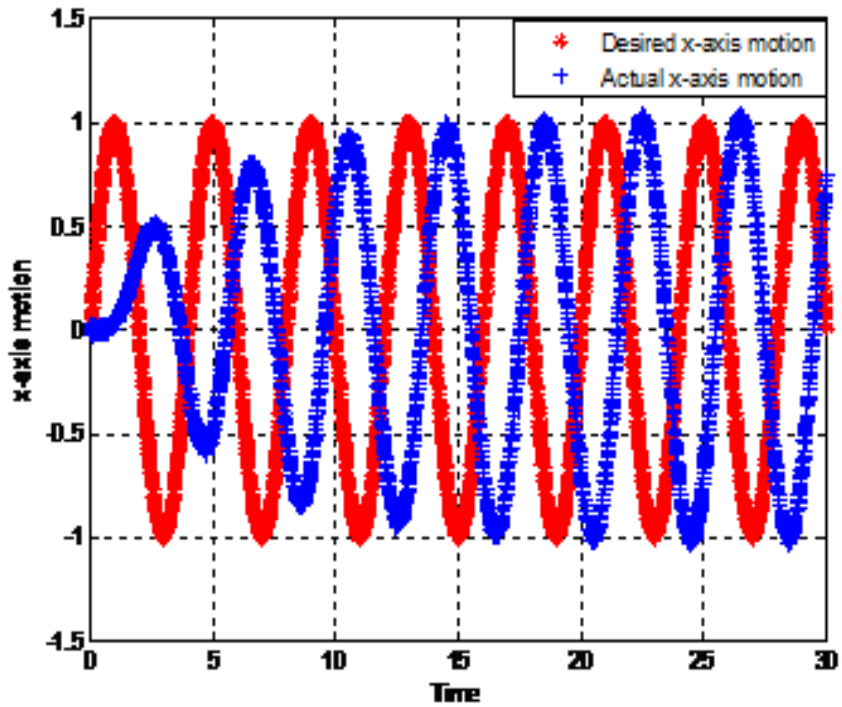

Figure 15. UAV x-axis motion comparison

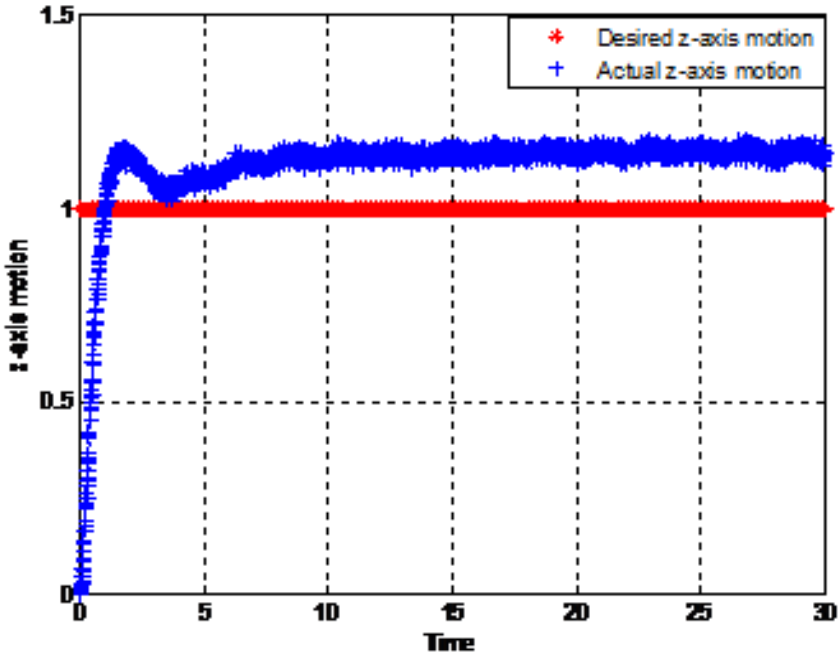

Figure 16. UAV y-axis motion comparison

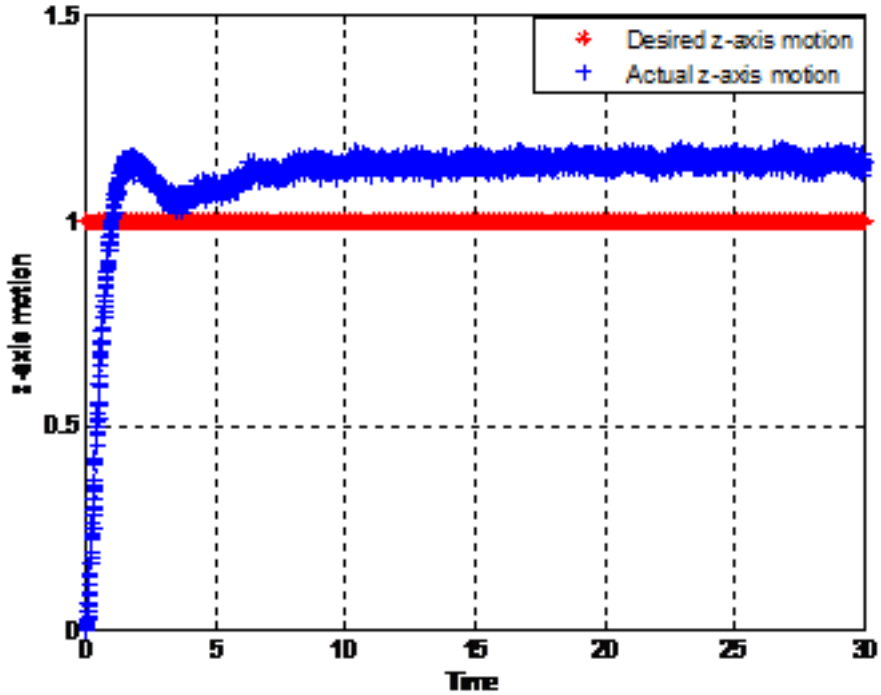

Figure 17. UAV z-axis motion comparison

Controller'sperformance as shown in above figures is acceptable. Whereas the delay in motion of x and y axes is observed which is mainly due to the transport delay induced in motor dynamics.

5. Conclusion

In this paper autonomous attitude and trajectory control problem is being addressed. Quadrotor model used in simulation have 6 DoF with coupled dynamics. The motion of craft primarily the function of attitude angles ($\emptyset$,θ, φ), PD controller is employed to achieve the target position as well as to maintain orientation. According to simulation results, UAV produced good flight trajectories for a beginner's craft design but not up to the mark for professional. The proposed controller has been tested only in simulation environment and the performance of overall system can be optimized using Fuzzy controller instead of PD controller.

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